

# Mathematical modeling of an axial compressor in a gas turbine cycle

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## Abstract

Contemporary thermal diagnostic systems of power units require advanced computational tools, including mathematical models. These models should have a simple structure and short computing time. These conditions are satisfied by models that include mass and energy balances as well as additional empirical functions whose coefficients are estimated by using the measurement results. This paper presents a simulation model of an axial compressor which forms part of a gas turbine unit with a rated output of 125.4 MW. The model was developed with the use of a generalized compressor map, describing the relationship between corrected air mass flow rate, pressure ratio, isentropic efficiency and corrected rotational speed. The model encompasses the empirical formula for compressor internal efficiency, which additionally considers the effect of variation of the angle of inlet guide vanes. The unknown values of empirical coefficients which appear in this equation were estimated on the basis of operating data. The calculation results obtained were compared with the measurement results. The quality of the model prediction was evaluated and conclusions were drawn.

**Keywords:** mathematical modelling, gas turbine, axial compressor, empirical functions, inlet guide vanes of compressor

## 1. Introduction

Current thermal diagnostics systems of power plants require mathematical models of thermal processes. The mathematical model can be developed on the basis of physical laws (analytical model) and as an approximation of measured data (empirical model). The advantage of using analytical models is the ability to accurately understand the process mechanism. These processes are often complex, which makes it impossible to develop a model using only the process laws of physics. In such cases, empirical models are frequently used, which are easier to develop than analytical models [1]. However, their scope of applicability is limited to the operating parameters for which the model was calibrated. Good results are obtained by combining analytical and empirical models.

This article presents a simulation model of an axial compressor which forms part of a gas turbine unit with a rated output of 125.4 MW.

The model encompasses a generalized compressor map and empirical equations describing isentropic efficiency of the compressor and pressure drop in an air filter. The unknown values of empirical coefficients were estimated on the basis of measurement results. As a result, the current technical condition of the modeled machine is taken into account.

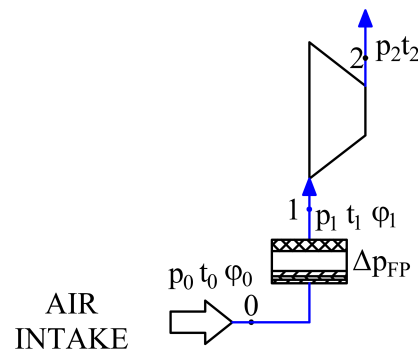


Figure 1: Schematic diagram of the axial compressor

The model can be used to calculate mass flow rate, thermal parameters of compressed air and energy assessment indexes, e.g., compressor efficiency.

## 2. Mathematical model of an axial compressor

Fig. 1 presents a schematic diagram of the modelled axial compressor.

The mathematical model of the axial compressor encompasses:

- generalized compressor map,

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- empirical equation describing isentropic efficiency,
- empirical equation describing pressure drop in an air filter.

The generalized compressor map is used to calculate air mass flow rate and air pressure downstream of the compressor. The temperature of the compressed air is computed from the empirical equation describing isentropic efficiency.

### 2.1. Compressor map

In order to calculate the mass flow rate and thermal parameters of compressed air, access to a real compressor map is required. However, this information is often inaccessible [2].

The problem of the lack of precise maps of the compressor may be solved by adopting generalised maps, which are presented in a dimensionless system and with the assumption that the geometry of the blade system is fixed [2, 3]. Moreover, the analyzed compressor is additionally equipped with an inlet guide vanes (IGV) system, which changes the geometry of the blade system during operation.

The generalized map was used to develop the mathematical model of the compressor. The parameters present in the compressor map were defined as follows [2–6]:

- corrected air mass flow rate:

$$\dot{m}_{corr} = \frac{\dot{m}_1 \cdot \sqrt{T_1}}{p_1} \left( \frac{\dot{m}_1 \cdot \sqrt{T_1}}{p_1} \right)_{nom} \quad (1)$$

where:  $\dot{m}_1$ – air mass flow rate, kg/s,  $T_1$ – compressor air inlet temperature K,  $p_1$ – compressor air inlet pressure kPa, the index “nom” refers to nominal parameters.

- pressure ratio:

$$PR = \frac{\pi_2}{\pi_{2nom}} = \frac{\left( \frac{p_2}{p_1} \right)}{\left( \frac{p_2}{p_1} \right)_{nom}} \quad (2)$$

where:  $p_2$ – compressor air inlet pressure, kPa.

- corrected rotational speed:

$$N_{corr} = \frac{\left( \frac{n}{\sqrt{T_1}} \right)}{\left( \frac{n}{\sqrt{T_1}} \right)_{nom}} \quad (3)$$

where:  $n$ – rotational speed of the machine, rpm.

The generalized map of the compressor operating in the gas turbine unit is shown in Fig. 2. Corrected rotational speed lines  $N_{corr}$  present in the generalized map of the compressor were developed for a range from 81% to 106.2% of the corrected speed value in ISO conditions ( $T_{ambient} = 15^\circ\text{C}$ ).

The generalized map of an axial compressor includes [2, 5]:

- the flow characteristics of the compressor, which describe the relationship between the corrected air mass flow rate  $\dot{m}_{corr}$  the pressure ratio  $PR$  the corrected speed  $N_{corr}$  and the auxiliary variable  $\beta$  (the lower part of the drawing),
- the characteristics of the internal efficiency showing the relationship between the internal efficiency of the compressor  $\eta_C$  the corrected air mass flow rate  $\dot{m}_{corr}$  the corrected speed  $N_{corr}$  and the auxiliary variable  $\beta$  (the upper part of the drawing).

### 2.2. The flow characteristics of the compressor

The generalized compressor map which is presented in Fig. 2 cannot be directly used in the simulation program for two main reasons [3]:

- in the area of high corrected speed, the speed lines  $N_{corr}$  are almost vertical, which prevents precise determination of the compressor operating point,
- at low rotational speeds, these lines are almost horizontal.

Therefore, the additional, variable  $\beta$  is introduced. The  $\beta$  lines are parallel to the surge line and vary from 0 to 1. The line  $\beta = 0$  indicates the maximum capacity of the compressor, and the line  $\beta = 1$  fits in with the surge line. Fig. 3 shows the course of  $\beta$  lines on the flow characteristic.

An analogous approach was introduced into the efficiency characteristics.

The modeled compressor is also equipped with IGVs (inlet guide vanes), which are designed to regulate the compressed air mass flow rate into the combustion chambers at partial loads of the gas turbine unit [3]. Varying the angle of the IGVs modifies the fluid-dynamic behavior of the compressor. This leads to changes in the flow characteristics [2, 6].

The compressor map presented in Fig. 2 was developed at the maximum IGV angle. The factor describing the influence of IGV angle variation on air mass flow rate is called VACF (Vane Angle Correction Factor) [5]. This parameter determines the percentage change of air mass flow rate when inlet guide vanes are opened (or closed) by about  $1^\circ$  [7].

In order to calculate the value of VACF for the analyzed compressor, the characteristics of air volume stream as a function of actual value of IGV angle was elaborated. The characteristics presented in Fig. 4 were developed on the basis of operating data.

On the basis of the characteristics shown in Fig. 4 it was calculated that the value of VACF for the analyzed compressor is 0.81%.

The compressed air mass flow rate, as a function of the angle of IGVs, can be calculated from the following equation [5]:

$$\dot{m}_1 = \dot{m}_1^{\max} \cdot [1 - VACF \cdot (IGV^{\max} - IGV)] \quad (4)$$

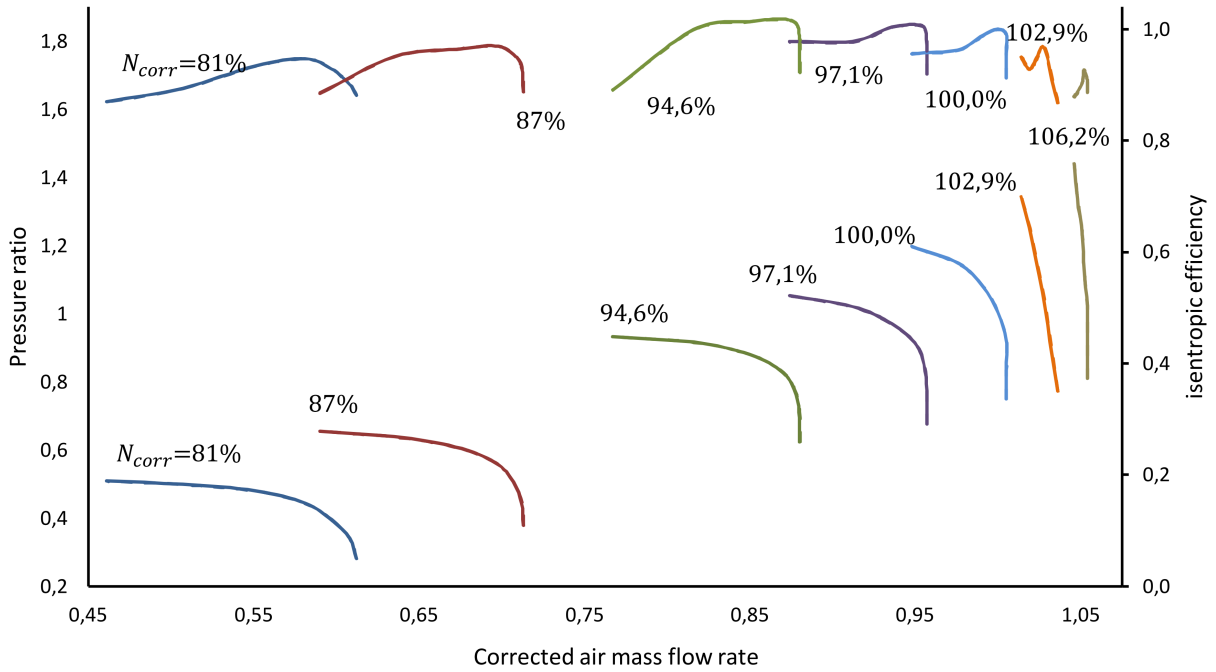


Figure 2: Generalized map of the analyzed axial compressor

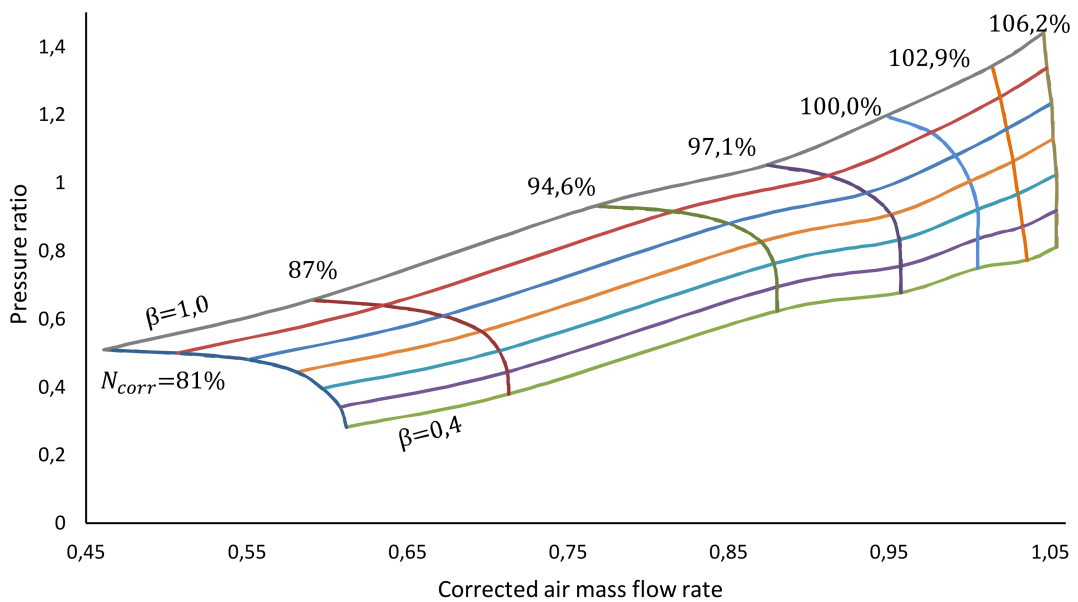


Figure 3: Course of  $\beta$  line on flow characteristics

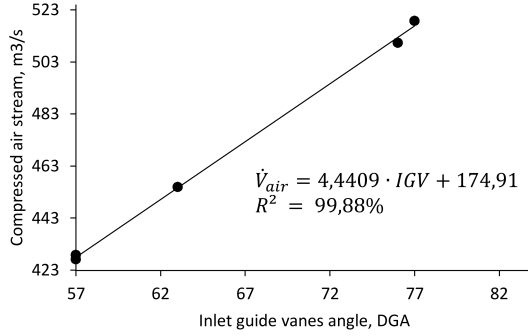


Figure 4: Air volume stream variation as a function of IGV angle

where:  $\dot{m}_1$  – compressed air flow depending on the angle of IGVs, kg/s;  $\dot{m}_1^{\max}$  – compressed air mass flow read from the generalized characteristics of the compressor for the maximum IGVs angle, kg/s; VACF – Vane Angle Correction Factor which determines the effect of a variation in the angle of the inlet guide vanes on the compressed air mass flow, VACF=0.81%;  $IGV^{\max}$  – the maximum value of opening the inlet guide vanes,  $IGV^{\max} = 84^\circ$ .

The generalized flow characteristics of the compressor presented in Fig. 2 were developed at the maximum IGV angle. For the analyzed compressor, this angle is  $84^\circ$ . The variation of IGV angle does not change the shape of the lines  $N_{corr}$ , but they shift horizontally to the left when the IGVs are being closed and to the right when the IGV are being opened. The shape of the compressor surge line and the line  $\beta$  remain unchanged [2, 3, 5]. This is illustrated in Fig.5. The continuous lines relate to compressor operation with the inlet guide vanes closed at an angle of  $IGV=60^\circ$ .

The effect of shifting the line  $N_{corr}$  after changing the IGV angle is included in the calculation algorithm.

### 2.3. Equation for the internal compressor efficiency

Calculation of the compressed air temperature on the basis of internal efficiency characteristics produced highly erroneous results. The mean square error of the compressed air temperature was  $\delta = 4.8$  K. Therefore, it was decided to develop an empirical relationship for the internal efficiency of the compressor based on the measurement data [8].

$$\begin{aligned} \eta_C = & b_0 + b_1 \cdot \dot{m}_{corr} + b_2 \cdot \dot{m}_{corr}^2 + \\ & + b_3 \cdot N_{corr} + b_4 \cdot N_{corr}^2 + b_5 \cdot \dot{m}_{corr} \cdot \\ & \cdot N_{corr} + b_6 \cdot IGV + b_7 \cdot IGV^2 + \\ & + b_8 \cdot \dot{m}_{corr} \cdot IGV + b_9 \cdot N_{corr} \cdot IGV \end{aligned} \quad (5)$$

The unknown values of empirical coefficients were estimated, on the basis of measurements, using the least squares method [5, 9]. The criterion of estimation takes the following form [1]:

$$\sum_{i=1}^{38} (T_{2i}^{mod} - T_{2i}^{meas})^2 \rightarrow \min \quad (6)$$

 Table 1: Values of the estimated coefficients  $b_i$  present in the empirical equation for the internal efficiency of the compressor

Coefficient	Value
$b_0$	32.967
$b_1$	97.489
$b_2$	-28.028
$b_3$	-91.749
$b_4$	55.686
$b_5$	-78.199
$b_6$	-0.854
$b_7$	-0.001
$b_8$	0.422
$b_9$	0.726
$R^2$	99.5%
$\delta$	0.6 K

where: the superscript mod relates to the value obtained from the model and the superscript meas relates to the value based on measurement data.

Table 1 shows estimates of the coefficients  $b_i$ . The quality of model prediction was evaluated on the basis of determination coefficient  $R^2$  and the mean square error  $\delta$  [1].

### 2.4. Algorithm for the calculation of the pressure drop in the air filter

Before the air enters the compressor, it must be cleaned of any contaminants, e.g. dust in the air filter, [2, 10]. The empirical equation which describes the pressure drop in the air filter  $\Delta p_{af}$ , as a function of corrected air mass flow rate takes the following form:

$$\Delta p_{af} = 0.032 \cdot \dot{m}_{corr1} - 0.908 \quad (7)$$

The equation (7) is characterized by the high determination coefficient  $R^2=98.70\%$ .

## 3. Numerical calculation results

The presented generalized compressor map, together with empirical equations describing internal efficiency of compressor (equation (5)) and pressure drop in the air filter (equation (7)) create a simulation model of the axial compressor.

Simulation calculations were performed for 38 sets of measurement data.

The calculation results are presented in the form of diagrams. The dashed line represents the calculation results obtained from the model, and the continuous line the results of operational measurements. The values of the determination coefficient  $R^2$  and the mean square error  $\delta$  are also included [1, 9].

Fig. 6 and 7 confirm the very good agreement between the calculation results and the measurement results.

The determination factor for prediction of compressed air temperature is 99.5%. The good prediction quality is gained by using an empirical equation which describes internal efficiency of the compressor as a function of corrected air mass

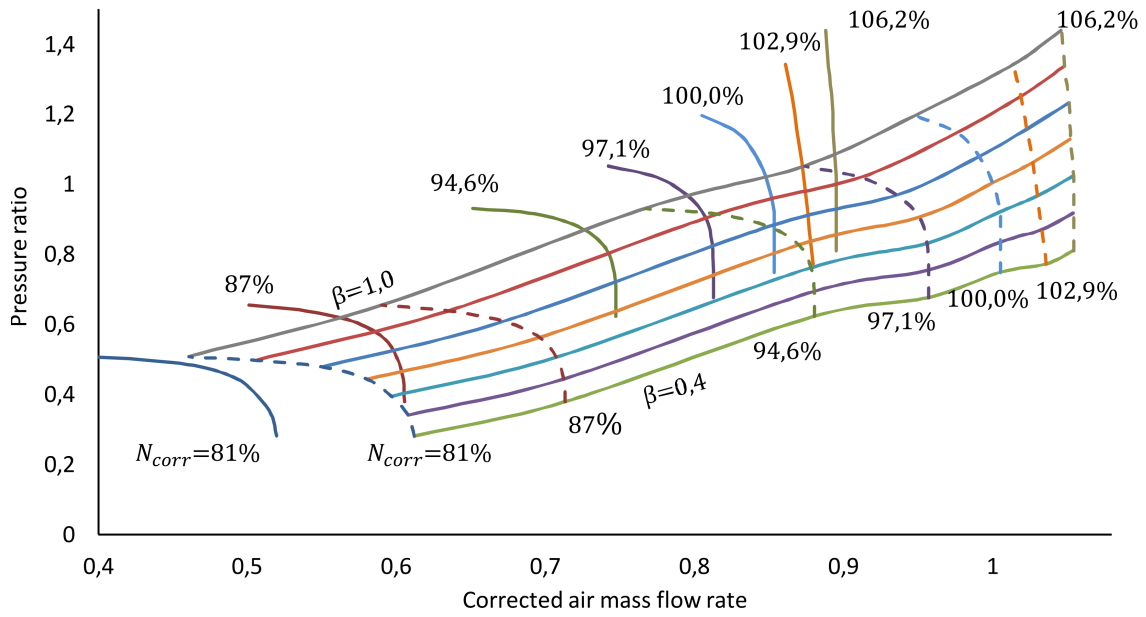


Figure 5: Effect of change in the IGV angle on the compressor flow characteristics

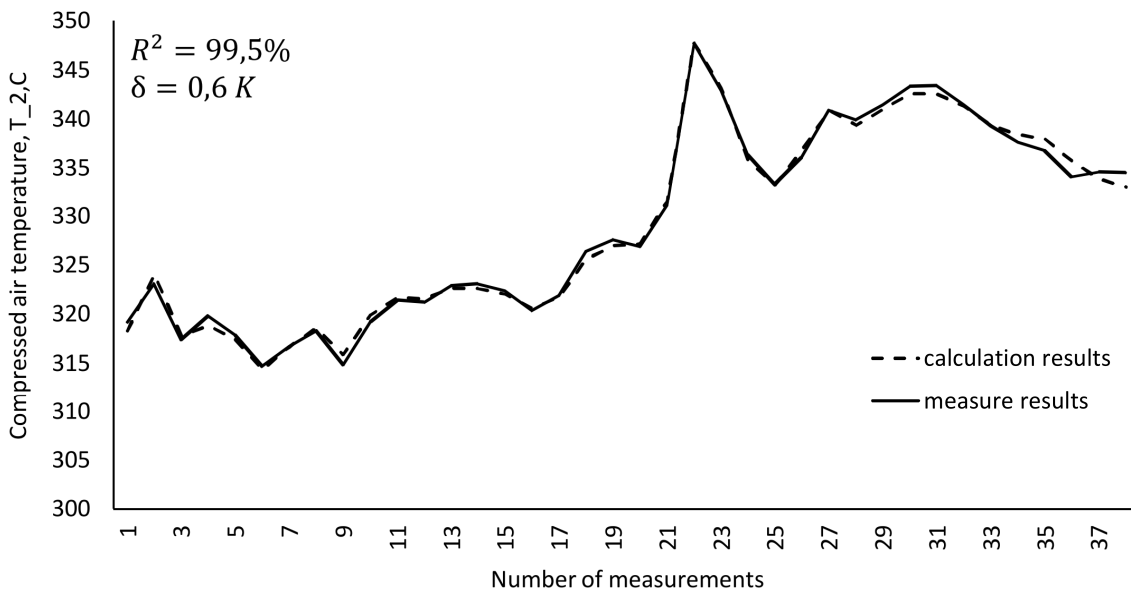


Figure 6: Comparison of the compressed air temperature, calculated from the measurement data and from the model

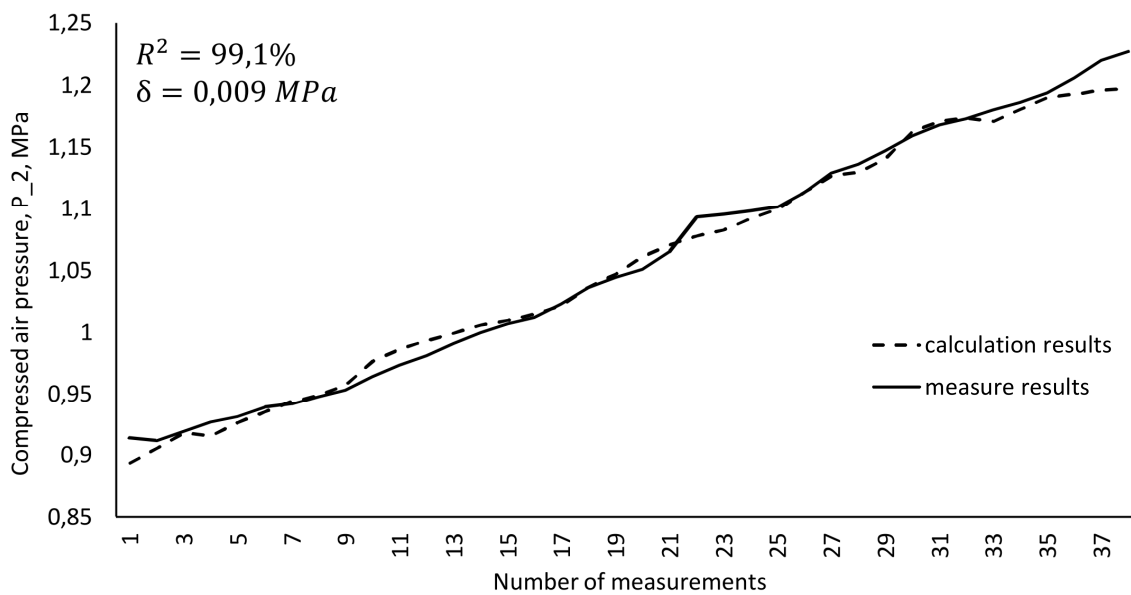


Figure 7: Comparison of the compressed air pressure, calculated from the measurement data and from the model

flow rate, corrected rotational speed and actual value of inlet guide vane angle.

The prediction quality of compressed air pressure is also very good. The determination factor is high ( $R^2 = 99.1\%$ ) and the mean square error is low at 0.009 MPa.

#### 4. Summary

This article presents a mathematical model of the axial compressor, which forms part of a gas turbine unit with a rated output of 125.4 MW. The analyzed axial compressor is additionally equipped with inlet guide vanes.

The developed simulation model encompasses the flow characteristics of the compressor and empirical functions describing the internal efficiency of the compressor and pressure drop in the air filter. The model was calibrated on the basis of measurement results with the use of recorded measure results. The prediction quality of compressed air thermal parameters is satisfying. This is proven by the high values of the determination coefficient  $R^2$  and the low values of the mean square error  $\delta$ .

Owing to its simple structure and short computing time, the presented simulation model of the compressor can be used in prediction control models, hierarchical intelligent control systems OCL and systems for thermal diagnostics of power plants.

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