

# A multi-slack Optimization Model for Scheduling Energy Hubs in Smart Grids

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## Abstract

This paper provides a multi-slack optimization model in order to manage the operation of an energy hub in smart grids. This model is centralized on a multi-slack one in which the proposed slack variables are in line with actual energy providers. Both electrical and thermal loads are considered in this model. An external grid and boilers are respectively used for slack generation units for satisfying electrical and thermal loads. In order to reduce the penalty factors in the optimization model, we addressed fair and suitable slack variables in the optimization model. In a real power system, energy storage devices could effect optimal operation in short-term planning. The main role of such devices in smart grids is to reduce the operating costs because of their state of charge (SOC) in peak, medium and base loads. Such devices could also handle load and generation uncertainties in the real world. In this model, we implement this feature to handle the uncertainties in the random variable generation sector of optimization algorithm. The proposed method could handle this challenge by discharging the stored energy if the slack unit is unable to satisfy the demanded load and vice versa. In order to evaluate the effectiveness of the proposed method, a benchmark is provided in this paper. The hourly electrical and thermal demands were extracted from DesignBuilder® for a commercial building. The simulation results show that the presented method is both satisfactory and consistent with expectations.

**Keywords:** Energy Hub; Multi-Slack Optimization Model; State of Charge

## 1. Introduction

### 1.1. Energy Hub Background

An energy hub is a multi-generation system where multiple energy carriers inputs to the hub are converted, stored and distributed in order to satisfy energy demands. The solution to the energy hub operation problem determines the energy carriers to be purchased and stored in order to satisfy energy requests while minimizing the cost function [1]. The energy hub includes both electrical and thermal energy generation and storage units. The main reason for introducing such energy hubs is to integrate energy carriers in smart grids to reduce the operating cost. The energy crisis combined with environmental concerns also open some research windows [2]. The incorporation of energy production, conversion and storage technologies in a modern power grid could increase the overall benefits for generation companies, system operator and end users. In such systems,

Distributed Energy Resources (DERs) contribute by reducing the total costs of operation and being more environment-friendly than centralized power systems. A real energy hub contains the full range of energy generation, conversion and storage units. However, some of them may be omitted due to (i) limitations at end user locations and (ii) environmental issues. Moreover, some economic constraints affect the operation of such units in the operational horizon. Combined Heat and Power (CHP) units are one of the most beneficial technologies and their performance will be explored in detail in this paper. The main advantage of a CHP unit is its ability to generate both power and heat simultaneously. Accordingly, whole system efficiency is improved by using waste heat to satisfy heating demand. Considering some boilers in line with CHPs could provide desirable thermal energy delivery to the customers. The electrical energy supply chain includes both local generation with DERs and purchase of energy from the grid. Integration of electrical storage units in this model maintains energy delivery and may influence total cost in the planning horizon.

### 1.2. Literature Review

As the energy hub concept has already been considered in numerous studies, this section presents an overview of key

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research in this area. The concept was first initiated at ETH Zurich [3]. As defined in [4], an energy hub is an integrated system with multiple energy carriers at its input where energy production, conversion and storage technologies such as CCHP, renewable energy resources and batteries are deployed in order to supply certain required services such as electricity, heating and cooling at its output [2]. Adoption of composite multi-generation systems in an energy hub may lead to significant benefits in terms of higher energy efficiency, reduced CO<sub>2</sub> emissions and enhanced economy. In this light, the scientific community is taking a broadbased approach to the analysis and planning of DERs, factoring in technical, environmental, economic and social issues [5] (see [6] for an exhaustive review). A considerable number of recent studies deal with characterization, planning, evaluation and optimization in energy hubs, which can be considered functional units where multiple energy carriers are converted, stored, and dissipated [7] and [8]. Researchers are seeking to achieve an optimal mix of energy hub components, combinations and connections [9]. Power flow studies analyze couplings and interactions between hubs and other energy infrastructures [10]. Energy hub operational scheduling addresses the optimal energy carriers' purchase and storage utilizations over the operational horizon [9]. From the modeling point of view, the optimization problem is frequently set up to minimize the total energy cost in the system, within a deterministic framework of load demands, prices, efficiencies and constraints [3, 10, 11, 12]. Many classes of solution algorithms aimed at addressing this problem have been proposed in the literature [6]. They include linear algorithms, which are based on the linearization of both the objective function and the problem constraints, and nonlinear programming techniques, which deal with problems involving nonlinear objective and/or constraint functions. These solution methods represent a useful tool only from a user perspective, since they allow the analyst to effectively optimize the operation of a single energy hub without considering its impact on the multi-carrier energy network operation. Consequently, research for alternative techniques aimed at optimizing the operation of interconnected and distributed energy hubs through ensuring effective and reliable operation of the multi-carrier energy network is dealing with a still unresolved issue and further investigation is required. Original research was proposed by [9] to address a nonlinear formulation for energy hub operation. However, their proposed power flow model was represented in a linear programming model [13]. Also, an economic load dispatch model was extended by them to reach the optimal working point in CHP engines, gas furnaces and heat exchangers [14]. Different energy sources and novel technologies such as CHP in the residential energy hub model draw attention to the optimal selecting of energy sources and the manner of energy flow. Despite the extensive research in the field of the multi-carrier energy system and energy hub, only a few works such as [15] and [16] have addressed the modeling and operation of a residential energy hub. Ref. [15] concentrated on the elaboration of a methodology that is able to model and

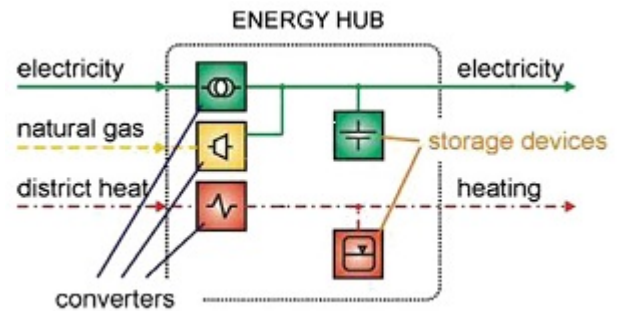


Figure 1: General scheme of an energy hub: a multi-source multi-product system [5]

optimize the coupling between energy demand and energy supplies in a building at the design concept stage, taking into account all the constraints that arise in real building design. Some of the operating issues of residential hubs are addressed in [16], where optimal dispatch of the residential hub is proposed [17].

### 1.3. Contributions

The main contributions of this paper are threefold:

1. To introduce a new framework for modeling the dynamic nature of charging, discharging and idle mode of storage units, especially electrical storage devices;
2. To provide a fair thermal and electrical demands forecast using DesignBuilder® platform;
3. To solve this large-scale, dynamic and mixed-integer optimization problem by introducing a mapping procedure.

## 2. Energy Hub Model

This paper focuses on the modeling and operation scheduling of an energy hub, such as a commercial building, hospital or a college. From a system point of view, an energy hub is a unit supplied by multiple energy carriers at its input ports and provides required energy services (i.e. electricity, heating, cooling, compressed air), also referred to as energy hub loads, at the output ports [3] and [10]. Energy hubs comprise the following basic elements: direct connections, converters, storage devices. Direct connections provide in output an input carrier without converting it into another form (e.g. electric cables, pipelines).

Fig. 2 illustrates an example of an energy hub exchanging electricity, natural gas and heat through three converters; output energy carriers (electricity and heat) could be stored in two devices. Examples of facilities that can be modeled using this concept are: conventional power plants (pumped storage hydropower, thermal with community heat extraction), industrial plants (steel production, petrochemical plants) and big buildings (airports, hospitals). The basic operational questions in the model are: (i) How much of each energy carrier should the hubs consume? and (ii) How

should they be converted in order to meet the loads at their outputs? The proposed model in this paper consists of two energy carriers.

The energy hub is fed by electrical energy from the grid and natural gas from a main natural gas network. The output ports are heating and electrical consumption nodes. The proposed energy hub in this paper consists of:

1. Electrical Energy Storage (EES): Electrical energy could be stored in a commercial battery for use at a particular time to reduce energy consumption from the grid;
2. Combined Heat and Power (CHP): The output carriers of CHP are heat and electricity. This system is fed by natural gas. The co-generation type is adopted in this paper due to its simple generation technology and flexible operation mode;
3. Electrical Heater (EH): This unit generates thermal energy. EH is supplied by electrical energy and produces thermal energy. This system could produce heat energy at local load points without fossil-fired generating systems;
4. Auxiliary Boiler: Natural gas is used as the input to this facility and it gives heat energy as its output. The Auxiliary Boiler could be used when CHP and EH are unable to meet heat demand.

The main challenge in the short term operation of an energy hub is utilization of all available basic elements, especially operation of the energy storage system. The State of Charging (SOC) of such devices should be handled by implementing dynamic programming in order to keep the energy at predefined levels. The energy storage devices have three states: Charging Mode, Discharging Mode and Idle Mode. In Charging Mode, the energy storage unit absorbs energy from the supply source while in Discharging Mode the energy storage unit feeds the load. In Idle Mode, the energy storage unit is disconnected and there is no energy transaction either from the supply side or to the demand side. In the other word, in Idle Mode, the amount of stored energy in the unit remains fixed.

The characteristic operating area of a simple back-pressure CHP plant is often convex. However, the characteristic may be non-convex for more advanced cogeneration technologies, such as back-pressure plants with condensing and auxiliary cooling options, gas turbines, and combined gas and steam cycles. A CHP plant can also have a number of alternative operating modes that shift some or all of the characteristic points. This makes the characteristic non-continuous (and thus non-convex). The shutdown state also typically makes the characteristic non-continuous. However, a non-convex CHP plant model can be divided into multiple convex sub-models, which can be encoded as alternative model components [18].

In the EH unit, thermal energy is generated by a conventional heater. Recently, electrical heater pumps have been proposed. An electrical heat pump transforms the solar energy stored in air, water and earth to thermal energy for cooling and heating purposes. The electrical energy required

for this equipment is only used for extracting heat from air, water and earth and the heat is not generated by electricity. In fact, heat pumps take as their energy source: hydrothermal [19], geothermal [20] and aero thermal [21] energy. This equipment normally delivers several units of heating energy, receiving only one unit of electrical energy. Therefore, the efficiency of this equipment is normally more than 100%, which means it generates more heat or cold than it consumes electrical energy. Using this equipment is one of the efficient ways of supplying thermal loads using electrical energy [22]. A fair model of an electrical heat pump also needs to factor in fluid mechanic issues, which lies outside the scope of this paper. For simplicity, we consider electrical heaters which convert electrical energy to heat energy. If thermal demand is greater than the thermal energy produced by CHP and EH, the thermal energy gap should be generated by an Auxiliary Boiler. In such case, the Auxiliary Boiler plays the role of thermal slack unit in the proposed model. The Auxiliary Boiler transforms natural gas to steam and then the heat extraction is done by high pressure pipes at the load center. In this paper, both electrical and thermal slack buses are considered with a view to handling the absence of energy carriers. The grid could inject and/or deliver electrical energy to/from energy hub, while the thermal slack (Auxiliary Boiler) only generates the thermal energy required. If the amount of thermal energy generated by CHP and EH is greater than the demand, the overproduction will be wasted. In order to reduce waste energy, the operator should manage the production level of each thermal generation unit. On the other hand, because of the volatility of load forecasting for both electrical and thermal loads, the operator would be faced with an energy gap or waste the overforecast energy at the operation time. These challenges could be met by using energy storage devices in the energy hub and enhancing the forecasting through combining information from each sector in a smart grid. In a smart grid, a comprehensive model is needed to deliver an acceptable relationship among energy carriers, converters, energy storage units and consumer demand. A fair modeling of these sectors would be considered in the operation horizon model. As the energy prices are different for each time period in the short term planning horizon, this issue should be considered by operators. In this area bidirectional power flow between the grid and energy hub could only handle an electrical energy mismatch. The communication system in a smart grid provides information transfer infrastructure and the operator could manage the dispatch of electrical energy. However, in the thermal section the consumption threat is more tractable than electrical loads. So, it would be a more straightforward task for the system operator to manage thermal loads and satisfy this form of energy than with the electrical ones.

### 3. Energy Hub Operation Model

The proposed energy hub operation model is based on maximizing the total benefits of short-term operation of an energy hub in terms of the following sections:

- Total Hub to Grid (H2G) Benefits.
- Total Grid to Hub (G2H) Costs.
- Total CHP operation Costs.
- Total Auxiliary Boiler.

Costs. The cost function of these sections is integrated in Z as follows:

$$\begin{aligned} \text{Max } Z = & \sum H2G \text{ Income}(t) - \sum G2H \text{ Cost}(t) \\ & - \sum \text{CHP Cost}(t) - \sum \text{Boiler Cost}(t) \end{aligned} \quad (1)$$

in which we seek to maximize system benefits by minimizing system costs. H2G consists of the hourly benefit of selling electrical energy to the grid. In order to calculate the hourly benefit, the amount of energy sold is multiplied by the hourly price of electrical energy.

$$H2G \text{ Income}(t) = E^{\text{Hub to Grid}}(t) \cdot \lambda^{\text{Sale}}(t) \quad (2)$$

On the other hand, G2H is provided to show the grid to hub costs. G2H consists of the hourly cost of purchasing electrical energy from the grid. Also, to calculate the hourly cost, the amount of purchased energy is multiplied by the hourly price of electrical energy.

$$G2H \text{ Cost}(t) = E^{\text{Grid to Hub}}(t) \cdot \lambda^{\text{Purchase}}(t) \quad (3)$$

As the Auxiliary Boiler and CHP consume natural gas, the hourly operating costs of these assets could be calculated by multiplying the price of natural gas by the amount of fuel consumed. For the Auxiliary Boiler we could consider a linear model such as:

$$\text{Boiler Cost}(t) = \text{Fuel}^{\text{Boiler}}(t) \cdot \lambda^{\text{Natural Gas}}(t) \quad (4)$$

In this model, we consider the ideal technology for the Auxiliary Boiler, meaning the efficiency of the boiler is 100%. For CHP, based on the operating point at each hour, the fuel consumption follows a technological non-linear model:

$$\text{CHP Cost}(t) = \text{Fuel}^{\text{CHP}}(t) \cdot \lambda^{\text{Natural Gas}}(t) \quad (5)$$

$$\begin{aligned} \text{Fuel}^{\text{CHP}}(t) = & \alpha \cdot (E^{\text{CHP}}(t))^2 + \beta \cdot E^{\text{CHP}}(t) + \gamma \cdot (H^{\text{CHP}}(t))^2 \\ & + \zeta \cdot H^{\text{CHP}}(t) + \kappa \cdot E^{\text{CHP}}(t) \cdot H^{\text{CHP}}(t) + \rho \end{aligned} \quad (6)$$

in which, the technological model is a function of both electrical and thermal generation levels. The cost coefficients of CHP are  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\zeta$ ,  $\kappa$ , and  $\rho$  which could be attained from the historical data for a typical CHP.

### 3.1. Electrical Constraints

The electrical constraints are as follows:

- Grid Constraints

Associated grid constraints involve the decomposition of hub to grid electrical energy delivery. The total electrical energy sold consists of the hourly surplus of EES and CHP. As (i) the energy purchased from the grid is more reliable than local energy generation, and (ii) the average cost of purchasing energy from the grid is lower than generating energy in CHP and EES, the surplus of these assets could be transmitted to the grid.

$$E^{\text{Hub to Grid}}(t) = E^{\text{EES to Grid}}(t) + E^{\text{CHP to Grid}}(t) \quad (7)$$

In the other hand, the grid to hub electrical energy transfer consists of three parts. Grid to residential, grid to electrical storage and grid to electrical heater are associated loads from the grid point of view.

$$E^{\text{Grid to Hub}}(t) = E^{\text{Grid to EL}}(t) + E^{\text{Grid to EES}}(t) + E^{\text{Grid to EH}}(t) \quad (8)$$

At the same time, energy transfer from grid to hub and vice versa is limited by transmission line capacity. Also, the transmission line is a bidirectional asset so we could apply a binary decision variable to avoid the simultaneous sale and purchase of electrical energy through the transmission line. The mentioned binary variables are  $M(t)$  and  $N(t)$ .

$$0 \leq E^{\text{Hub to Grid}}(t) \leq \text{Line Capacity} \cdot M(t) \quad (9)$$

$$0 \leq E^{\text{Grid to Hub}}(t) \leq \text{Line Capacity} \cdot N(t) \quad (10)$$

$$M(t) + N(t) \leq 1 \quad (11)$$

- CHP Constraints

Electrical energy generation in CHP could be consumed by residential electrical demand, EES, EH and transferred to the grid. The electrical generation equivalent in CHP is as follows:

$$E^{\text{CHP}}(t) = E^{\text{CHP to EL}}(t) + E^{\text{CHP to Grid}}(t) + E^{\text{CHP to EH}}(t) + E^{\text{CHP to EES}}(t) \quad (12)$$

- EES Constraints

Transferring energy to EES and from EES models are provided in (13) and (14), respectively. For EES, charging and discharging efficiency affect the stored energy in EES. As charging efficiency may be different from the associated efficiency for the discharging mode, two coefficients have been considered for this issue:

$$E^{\text{EES}}(t) = (E^{\text{EES to EL}}(t) + E^{\text{EES to Grid}}(t) + E^{\text{EES to EH}}(t)) \cdot \eta_{\text{EES}}^{\text{Out}} \quad (13)$$

$$E_{EES}(t) = \frac{E^{CHP \to EES}(t) + E^{Grid \to EES}(t)}{\eta_{EES}^n} \quad (14)$$

The stored energy in EES at time (t) is a function of total stored energy at time (t-1) and the amount of transferred or delivered energy by EES at each time.

$$Eng^{EES}(t) = Eng^{EES}(t-1) + E^{EES}(t) - E_{EES}(t) \quad (15)$$

The amount of stored energy in EES is limited by the maximum and minimum level of energy for EES. The initial and final stored energy are considered to be same, to save the amount of energy for the next planning horizon. This constraint could be extended for weekly operation if so required.

$$Eng_{min}^{EES} \leq Eng^{EES}(t) \leq Eng_{max}^{EES} \quad (16)$$

$$Eng^{EES}(1) = Eng^{EES}(24) \quad (17)$$

Based on hourly scheduling of the energy hub, EES could not be at different states at the same time. So, two binary variables are considered here to avoid such situations. These binary variables are X(t) and Y(t). The X(t) is reserved for charging state while the discharging state is pointed by Y(t). If both of them are at "0", it means that EES is in idle mode.

#### 4. Multi-Slack Framework to Solve Energy Hub Management Problem

In this paper, a multi-slack framework is proposed to manage the operation of an energy hub. As mentioned above, the energy hub could import electrical energy from the grid and could also export to the grid electrical energy that is surplus to requirements. The bidirectional power flow model is adopted for electrical energy as the first energy carrier in this model. However, the thermal energy flow should be consumed at the local centers. In other words, the thermal energy excess would be wasted by relaxing in the air. In such circumstances, the operator misspends the system's fortunes. In the proposed model, the management of both electrical and thermal energy carriers would be attained by implementing the multi-slack framework. The proposed model could also handle unforeseen load volatility through factoring in slack units for both energy carriers. The presence of energy storage units in the system could assist the hub operator in energy transition management. The multi-slack framework has three phases, at the first stage, the operation point of CHP should be determined. At the second phase, the status of the energy storage would be determined. At this stage, the amount of charging and discharging energy is set. At the end phase, the output of the previous stages

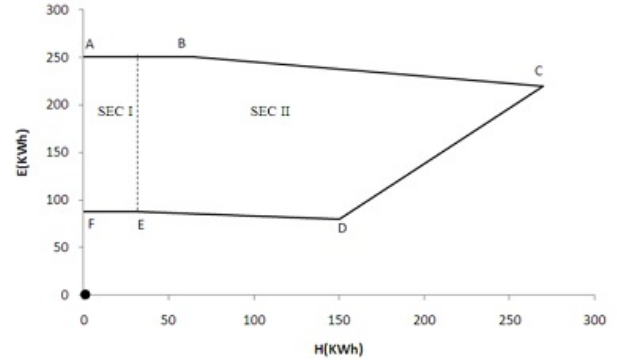


Figure 2: Typical co-generation feasible operation region

is combined to determine the operation conditions of the energy hub. The last stage is more complicated than the first and second stages. The details of these phases are provided in the subsections that follow. Then, these phases are incorporated in an integrated model.

##### 4.1. Phase 1: CHP Model in Energy Hub Operation

The central core of the proposed energy hub is the CHP system. The operating point of CHP means how much electrical and thermal energy could be obtained by burning natural gas. In CHP electricity and heat are not generated independently—the generation of one of them affects the other. This dependency is represented by using a feasible region for each CHP. In fact, the operation point of CHP determines the feasible amount of both electrical and thermal energy. Fig. 2 shows the heat-power feasible operation region (FOR) of a cogeneration unit used in this model. Points A-F are the corners of the operational area where working points of the CHP system occur. Since this area has an angle of more than 180 degrees, FOR in Fig. 2 is regarded as non-convex. Specific linear equations were proposed by [23] to describe this area and they are used in this paper. The non-convex operational area of the CHP system is divided into two convex areas, section I and section II.

As energy generation in CHP depends on the working point, it is essential to implement a mapping procedure to illustrate the relationship between generated random variables as decision variables in the optimization technique and the working point.

##### 4.2. Phase2: EES Model in Energy Hub Operation

After determining the working point of CHP for each time interval in the planning horizon (24 hours in this study), the operation of EES should be managed. As mentioned above, the state of charging (SOC) for EES would be determined in the second phase.

SOC means how much energy would be generated or consumed by energy storage. It is also instructive in determining the charging, discharging and idling states of energy storage. The energy balance equation at time (t) depends on

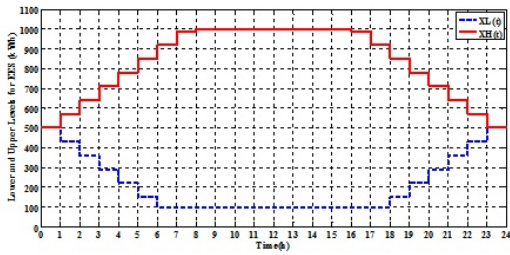


Figure 3: The inherent upper and lower levels of energy in EES

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X(1)=500; XH(1)=570; XL(1)=430;
for p=2:23
    X(p)=U(p)*[XH(p)-XL(p)]+XL(p); % U(p) is the Decision Variable
    XH(p+1)=min(XH(p),XH(p))+1.5*Pe; % Updating Upper Level of EES based on Dynamic Model
    XL(p+1)=max(XL(p),XL(p))-1.5*Pe; % Updating Lower Level of EES based on Dynamic Model
    A(p)=X(p)-X(p-1); % Net Energy Calculation to Provide SOC
    % Conditional Statement for SOC States Determination
    if A(p)>Pe
        A(p)=0;
    elseif A(p)<-Pe
        A(p)=0;
    end
    X(p)=X(p-1)+A(p); % Updating Stored Energy at SOC
end
X(24)=500;
    
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Figure 4: The proposed pseudo code for TMP.

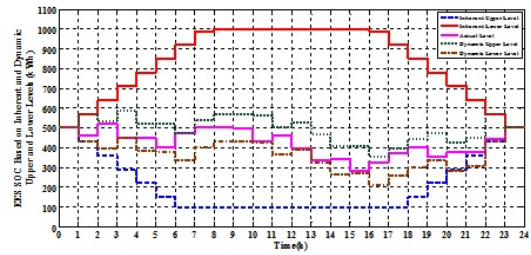


Figure 6: Reduced upper and lower levels of energy in EES.

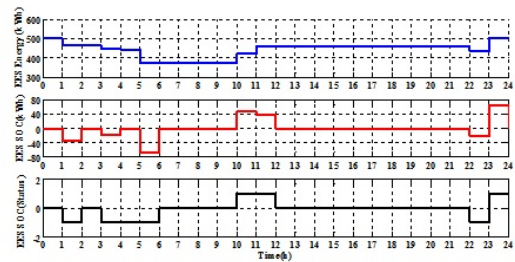


Figure 7: Fig. 7. Hourly level of energy for EES

the amount of stored energy at time (t-1) plus charged energy in the battery at time (t) minus discharged energy from battery at time (t).

Inherent lower and upper levels for EES in this study are shown in Fig. 3. The inherent upper and lower levels of energy in EES can be easily determined by considering the fact that the level of energy at each time interval would be  $Eng_{EES}(t) = Eng_{EES}(t-1) \pm Pe_{Max, EES}(t)$ . As the  $Pe_{Max, EES}$  is set at 70 kW for each time period in this study, it is clear that the inherent levels for this EES are same as Fig. 3.

In order to reduce the acceptable band for energy storage, this study adopts a dynamic procedure. Due to predefined levels of energy at the start and end of the operation period and considering the  $\pm 70$  kW for charging and discharging states, a backward-forward dynamic procedure could be implemented to attain this goal. The mapping procedure in this phase is based on extracting three states for the energy storage device. As the random variable is between 0 and 1, it is necessary to apply this band to a Three-State Map (TMP). The proposed pseudo code for TMP is as follows:

Figure 4 illustrates the pseudo code for TMP, while Fig. 5 shows the proposed mapping procedure for determining when EES is in idle, charging or discharging modes.

A typical SOC for EES is shown in Figure 6. As is evident from this figure, the restriction on the upper and lower

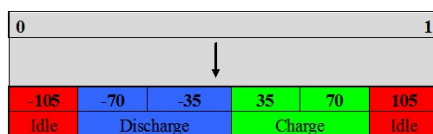


Figure 5: Mapping technique for SOC determination based on associated decision variable for EES.

levels narrows the SOC margin. The actual level of EES is extracted based on typical decision variables.

In Figure 7 the hourly level of energy for EES is illustrated in line with the amount of hourly generation and consumption by EES. The status of EES is also provided in this figure. Status “1”, “-1” and “0” denote “Charging”, “Discharging” and “Idle” modes, respectively. In this figure it is obvious that the energy transfer by EES follows the boundary conditions (the energy at the start and end of the planning horizon should be same).

At the end of second phase, SOC would be available for incorporation by the energy hub manager. SOC contains both the amount of generation or consumption energy and the status of charging, discharging and idling modes. In this figure, the top subfigure (marked blue), shows the SOC of EES, mid subfigure (marked red), shows the Charging, Discharging and Idle mode of EES and the bottom subfigure (marked black), illustrates the status of EES and are extracted from the mid subfigure.

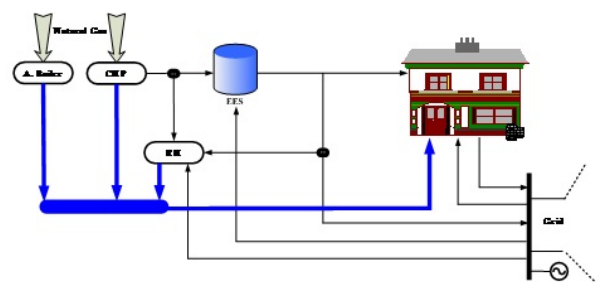


Figure 8: The energy flow in the proposed energy hub

Table 1: Electrical and Thermal Loads of Consumers in the Energy Hub

Parameter	Description	Value
$p^{EES,max}$	Maximum power of EES (kW)	70
$p^{Battery,max}$	Maximum heating power of A.boiler (kW)	400
$p^{Battery,min}$	Minimum heating power of A.boiler (kW)	20
$p^{EH,max}$	Maximum heating power of EH (kW)	300
Line Capacity	Grid Connection cable capacity(kW)	300
$\eta_{cur}^{EES}, \eta_{in}^{EES}$	Charging/Discharging efficiency of EES (%)	100
$\eta_{Boiler}^{EH}$	Efficiency of A.boiler (%)	100
$\eta^{EH}$	Efficiency of EH (%)	100
$E^{EES,max}$	Maximum state of charge of EES(kWh)	1000
$E^{EES,min}$	Minimum state of charge of EES(kWh)	100
$E^{EES,init}(1)$	Initial level of stored energy in EES(kWh)	500
$E_{min}^{EES}(24)$	Natural Gas Price	20

Table 2: Electrical and Thermal Loads of Consumers in the Energy Hub

Hour	Electrical Load (kW)	Thermal Load (kW)
1	32.5	80.0
2	39.0	86.5
3	45.5	99.5
4	58.5	105.5
5	65.0	159.5
6	78.0	153.0
7	110.5	140.0
8	136.5	166.0
9	143.0	210.0
10	136.5	218.0
11	91.0	205.0
12	84.5	216.5
13	78.0	255.5
14	84.5	225.0
15	91.0	242.5
16	104.0	185.0
17	110.5	150.0
18	130.0	125.0
19	142.5	119.0
20	123.5	112.5
21	104.0	101.0
22	78.0	94.5
23	58.5	68.0
24	32.5	86.5

### 4.3. Incorporation of CHP and EES Outputs in the Energy Hub

After the energy delivery determination procedure in CHP and EES, the slacking technique would be applicable. In this stage, based on the operator decision making preferences, the surplus of electrical energy could be transmitted through the grid and the operator would obtain the hourly price from the market and vice versa. In such circumstances, if the total available thermal energy from CHP and EH is insufficient, the auxiliary boiler makes up for the lack of thermal energy for the thermal load. This stage consists of rigorous functions to provide a fair condition for techno-economical operation. Figure 8 illustrates the energy flow in the proposed energy hub in this study.

## 5. Case Study

A case study for operation of an energy hub based on a multi-slack optimization model is provided in this section. This case study consists of both electrical and thermal loads, and all mentioned elements of supply infrastructure were considered. For simplicity, the efficiency coefficients for EES and Auxiliary Boiler are considered to be 100%. As a result, in EES this assumption means that the total injected energy from EES to the hub would be equal to the amount of charging energy delivered to EES, because the energy level at time 0 is the same as the energy level at time 24. The transmission capacity is considered to be 300 kW. Table 1 provides the techno-economical parameters for the system.

The target in this paper is to maintain the forecast demands (both electrical and thermal) for a commercial building. In this study, the demanded loads were forecast based on seasonal intervals. To do so, we modeled the aforementioned hub in DesignBuilder® software. The hourly electrical and thermal loads are provided in Table 2.

The provided loads are associated with a working day in winter. For other seasons, the thermal and electrical loads would be different. As the scope of this paper is to manage available assets, long-term issues such as capacity determination of assets lie outside the remit of this study. The prices of electricity for each hour in the operation horizon are listed in Table 3. In this table, both purchasing and selling prices are provided.

Table 3: Hourly Prices for Electrical Energy in the Energy Hub

Hour	Purchasing Price (\$/kWh)	Selling Price (\$/kWh)
1	40	40
2	40	40
3	40	40
4	40	40
5	40	40
6	40	60
7	40	60
8	40	60
9	40	60
10	60	60
11	100	80
12	100	80
13	100	80
14	100	80
15	60	60
16	60	60
17	60	60
18	60	60
19	100	80
20	100	80
21	100	80
22	60	60
23	60	60
24	40	40

The decision variables of this model are 2\*24 variables for CHP to determine the operating points and 1\*24 for EES in order to determine the operation modes of EES. In the operation horizon, incorporation of multi-slack variables could satisfy the load balance for both electrical and thermal demands. In the proposed model, the ideal EES, Auxiliary Boiler and EH were considered for the sake of simplicity. However, this model could provide acceptable results for real world conditions. The best result is \$110890 per day for given demands.

Figure 9 provides the hourly level of stored energy in EES. From this figure it is evident that EES would be discharged at peak time duration and in the off-peak time horizon the charging mode will be activated. As the initial energy was set at 500 kWh, at the end of the planning horizon the stored energy should be equal to the initial energy.

Figure 10 illustrates the SOC (Battery) charging, discharg-

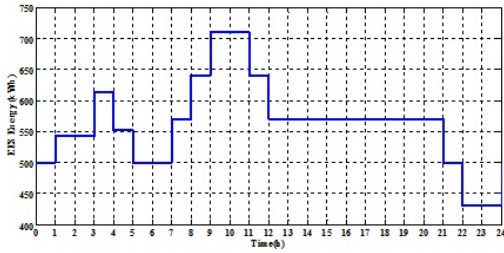


Figure 9: Energy level at EES for daily operation

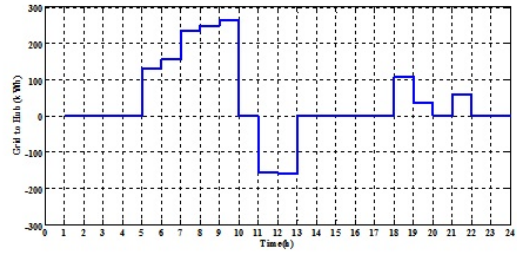


Figure 12: Daily Grid to Hub electrical energy transaction.

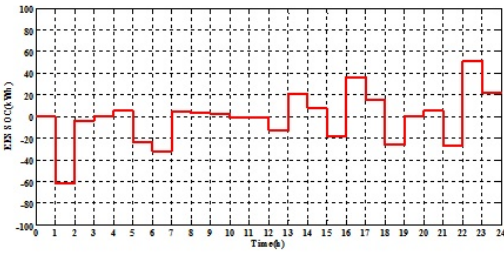


Figure 10: Fig. 10. The SOC of EES for daily operation.

ing and idle states in line with the transition states of EES during operation. This figure confirms that the hourly charging and discharging energy does not exceed the predefined levels. The predefined level in this study was set at EES capacity (70kW). Figure 11 provides the loading of the auxiliary boiler for the operating horizon.

The daily energy transaction between the hub and grid is illustrated in Figure 12. Simulation results confirm that the flowing powers in the off-peak hours are from grid to hub while the transactions for peak loads are vice versa. Also, the power purchased from the grid would be consumed by EES, to be charged at off-peak hours and the stored energy is to be released at peak hours.

## 6. Conclusion

The main aim of this paper is to implement a multi-slack optimization model to solve the short-term planning horizon problem in the operation of an energy hub. The main

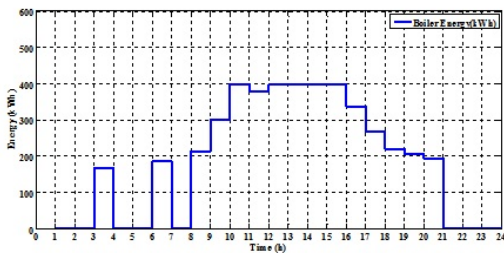


Figure 11: Fig. 11. Hourly operation of auxiliary boiler

novelty of this paper is that it recasts—in an integrated model—energy hub operation through a multi-slack optimization model based on a non-linear and mixed-integer optimization framework and determination of operating points of CHP, EES, EH and A. Boiler. The energy hub operation problem was broken down to a master and slave problem. In the master problem, the operating points of CHP and EES were determined. Then the implementation of auxiliary assets, such as EH, A. Boiler and the power grid were managed. The simulation results show that the proposed model could handle the large amount of infeasibilities through incorporating slack variables in the optimization model.

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