Open Access Journal

Journal of Power Technologies 95 (1) (2015) 34–39

journal homepage:papers.itc.pw.edu.pl

Real power-system economic dispatch using a variable weights linear programming method

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Abstract

This paper presents an efficient method to solve an economic load dispatch problem with load flow type network security constraints: the active generation limits and line active power flows limits. A new form of linear programming method is proposed: a variable weights linear programming. The nonlinear dispatch problem is transformed into a linear one through variable weights linear programming. To achieve good linear, representation the nonlinear cost functions between the active generation limits are approximated by the sum of products of cost values multiplied by the variable weights. The nonlinear equality network constraint containing the losses expressed by the specially formulated and calculated function of generated powers is transformed into a linear one, the same manner as above. The set of nonlinear inequality constraints on line active power flows is linearly expressed in terms of active bus generation powers with upper limits on the active power flow to each violating lines. The dispatch problem transformed this manner with or without solving the load flow problem is demonstrated that the proposed method has practical application for real-time control/dispatch.

Keywords: Power system, Economic dispatching, Linear programming

1. Introduction

The determination of optimum generator loadings to meet both a given demand and certain security requirements as well as to give the minimum at the function cost at the heart of any power-scheduling study. Mathematically, it is a complex non linear-dynamic optimization problem and at present, the solution is not attempted in a rigorous way for any scheduling period of interest. The usual method of solution adopted is to treat the dynamic problem as a series of static optimization problems at a number of points in time during the interval considered. Even with this simplification, the static problem remains a nonlinear of great size, accordingly, nonlinear mathematical programming methods require too much time to produce a solution. On-line economic dispatch on the other hand requires solutions procedures witch (i) produce secure loading that is reliable in the sense that a correct answer can be given for the problem posed which (ii) and can be solved by small on-line computers about every 2–3 minutes or every 1% change of system load. To meet these practical requirements certain simplifications must be made. Generally, linearization methods are used and the linear programming method is used in two ways: (i) when solution of the load flow problem is not necessary to obtain the optimal values of economic dispatch with security constraints, and (ii) when solution of the load flow problem is required.

2. Mathematical model

The economic load scheduling problem is one of minimizing certain cost functions subject to a number of constraints. This power dispatch problem [1, 2], is stated in this paper as follows:

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$$\min\left\{\sum_{i=1}^{n_G} C_i(P_{Gi})\right\} \tag{1}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \tag{2}$$

$$P_{ij} \leq P_{ij}^{M} \tag{3}$$

$$\sum P_{Gi} = P_D + P_L \tag{4}$$

where generally $C_i(P_{Gi})$ is quadratic curve:

$$C_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{5}$$

Where a_i, b_i and c_i are the known coefficients.

Constraint (2) is limits of generation, constraint (3) the line power flow limit, and constraint (4) reflects active power balance. This problem will be solved in this paper by the variable weights linear programming method. The first types of solutions considered does not require load flow calculation.

3. Objective

Using the variable weight linear programming method [1], the cost function (1) can be transformed as follows:

$$\sum_{i=1}^{nG}\sum_{s=1}^{ki} C_i(P_{Gi,s}) \cdot x_{i,s} \tag{6}$$

$$\sum_{s=1}^{ki} x_{i,s} = 1 \quad i = 1, ..., n_G \tag{7}$$

$$x_{i,s} \geq 0 \quad s = 1, ..., k_i \tag{8}$$

Following this notation the value of active generation P_{Gi} is obtained by:

$$P_{Gi} = \sum_{s=1}^{ki} P_{Gi,s} \cdot x_{i,s} \tag{9}$$

and the load balance equality constraint is transformed as follows:

$$\sum_{i=1}^{nG} P_{Gi,s} x_{i,s} = P_D + P_L \tag{10}$$

4. Generation constraints

The generation outputs are limited by the generation characteristics and should be in the region expressed by (2). In the variable coefficient programming method the constraint (2) takes the following form:

$$P_{Gi}^{m} \leq \sum_{s=1}^{ki} P_{Gi,s} \cdot x_{i,s} \leq P_{Gi}^{M} \tag{11}$$

5. Line power constraints

To take account of constraints (3) in the optimization problem, it is necessary to express the line power flow as a function of generation powers. The bus active injection powers Pp [1–5] are given by the following formula:

$$\begin{aligned} P_p = \textstyle\sum_{q=1}^{n} \Big\{ & e_p \left(e_q \cdot G_{pq} + f_q \cdot B_{pq}\right) \\ & + f_p \left(f_q \cdot G_{pq} - e_q \cdot B_{pq}\right)\Big\} \end{aligned} \tag{12}$$

and the line active power is obtained by the formula:

$$P_{ij} = \left(e_i^2 - e_i e_j + f_i^2 - f_i f_j\right) G_{ij} + \left(e_j f_i - e_i f_j\right) B_{ij} \tag{13}$$

It expresses (13) as a function of bus power injection at the generation bus in the following manner:

$$\Delta P_{ij} = \sum_{k=1}^{nG} \frac{\partial P_{ij}}{\partial P_k} \Delta P_k \tag{14}$$

where:

$$\frac{\partial P_{ij}}{\partial P_k} = \frac{\partial P_{ij}}{\partial e_k}\left(\frac{\partial P_k}{\partial e_k}\right)^{-1} + \frac{\partial P_{ij}}{\partial f_k}\left(\frac{\partial P_k}{\partial f_k}\right)^{-1} \tag{15}$$

In many practical investigations the practical derivative $\frac{\partial P_{ij}}{\partial f_k}\left(\frac{\partial P_{k P_{ij}}}{\partial f_k}\right)^{-1}$ is neglected so

$$\frac{\partial P_{ij}}{\partial P_k} = \frac{\partial P_{ij}}{\partial e_k}\left(\frac{\partial P_k}{\partial e_k}\right)^{-1} \tag{16}$$

The practical derivative in (15) or ways (16) are easily to calculate, and even for each Pij there are no more than four derivatives $\frac{\partial P_{ij}}{\partial e_k}$ and $\frac{\partial P_{ij}}{\partial f_k}$ together.

These derivatives can be calculated by several means but if the Newton-Raphson load flow method is used the derivatives $\frac{\partial P_k}{\partial e_k}$, $\frac{\partial P_k}{\partial f_k}$ are the same as those in the Jacobean matrix. When angular coordinates are used, another formula is used:

$$\frac{\partial P_{ij}}{\partial P_k} = \sum_{k=1}^{nG}\left\{\frac{\partial P_{ij}}{\partial \delta_k}\left(\frac{\partial P_k}{\partial \delta_k}\right)^{-1} + \frac{\partial P_{ij}}{\partial |E_k|}\left(\frac{\partial P_k}{\partial |E_k|}\right)^{-1}\right\} \tag{17}$$

Bus injection power is expressed by:

$$P_p = P_{Gp} - P_{Dp}$$

where P_{Dp} during each investigation is fixed. It follows from the above that formula (14) takes the changed form:

$$\sum_{k=1}^{nG} \frac{\partial P_{ij}}{\partial P_{Gk}} \Delta P_{Gk} \leq P_{ij}^{M} \tag{18}$$

In the notation of this paper it is possible to state as follows: Let a line connect buses i and j which cannot be overloaded. Thus the active line flow P_{ij} must be restricted as follows:

$$\sum_{k=1}^{nG} \frac{\partial P_{ij}}{\partial P_{Gk}} \sum_{s=1}^{ki} P_{Gk,s} \cdot x_{k,s} \le P_{ij}^{M} \quad (19)$$

This method can be used without calculation of the load flow problem. With the notation of the paper it is expressed as:

$$\sum_{p=1}^{nG} B_{(ij)p} \sum_{s=1}^{ki} P_{Gp,s} \cdot x_{p,s} \le P_{ij}^{M} \quad (20)$$

Shown here are: generation bus p and the violating line with its initial and terminal nodes i and j respectively. $B_{(ij)p}$ are coefficients of the P_{ij} function.

6. The power balance equation and transmission losses

The power balance equation is given by formula (4). To take account of the transmission losses we assume that the transmission loss is a function of power generation [6]. The formula for transmission losses is called B-matrix and it is expressed as follows:

$$P_L = K'_{L0} + \sum_{i=1}^{nG} B'_{i0} \cdot P_{Gi} + \sum_{i=1}^{nG} B'_{ii} \cdot P_{Gi}^2 + \sum_{\substack{i=1 \\ i \ne j}}^{nG} \sum_{\substack{j=1 \\ j \ne i}}^{nG} P_{Gi} \cdot B'_{ij} \cdot P_{Gj} \quad (21)$$

The coefficients K'_{L0}, B'_{i0}, B'_{ii} and B'_{ij} can be determined by least squares or weighted squares method [7]. In this paper this method is investigated with application to the A.E.P / I.E.E.E 14 bus system [8]. In particular, it assumed the following formula by neglecting the fourth term in formula (21):

$$P_L = K_{L0} + \sum_{i=1}^{nG} B_{i0} \cdot P_{Gi} + \sum_{i=1}^{nG} B_{ii} \cdot P_{Gi}^2 \quad (22)$$

So constraint (4) is expressed in the new variables in the following manner:

$$\begin{aligned} &\textstyle\sum_{i=1}^{nG} (1 - B_{i0}) \sum_{s=1}^{ki} P_{Gi,s} \cdot x_{i,s} \\ &- \textstyle\sum_{i=1}^{nG} B_{ii} \sum_{s=1}^{ki} P_{Gi,s} \cdot x_{i,s} = P_D + K_{L0} \end{aligned} \quad (23)$$

This form of B-matrix can be applied without solving load flow and allows the minimization of transmission losses.

7. Digital simulation results

The A.E.P/I.E.E.E 14 bus system [8] was used to test the method described above

The following problem was solved:

$$\min \{C_1 (P_{G1}) + C_2 (P_{G2})\}$$

under the following constraints:

- $135 \le P_{G1} \le 195$ (MW)

Table 1:

k-m	line impedance	line charging admittance
1-2	0.01938+j0.05917	j0.0264
1-5	0.05403+j0.22304	j0.0246
2- 3	0.04699+j0.19797	j0.0219
2-4	0.05811+j0.17632	j0.0187
2-5	0.05695+j0.17388	j0. 0170
3-4	0.06701+j0.17103	j0.0173
4-5	0.01335+j0.04211	j0.0064
4-7	0.00000+j0.02091	j0.0000
4-9	0.00000+j0.55618	j0.0000
5-6	0.00000+j0.25202	j0.0000
6-11	0.09498+j0.19890	j0.0000
6-12	0.12291+j0.25581	j0.0000
6-13	0.06615+j0.13027	j0.0000
7-8	0.00000+j0.17615	j0.0000
7-9	0.00000+j0.11001	j0.0000
9-10	0.03181+j0.08450	j0.0000
9-14	0.12711+j0.27038	j0.0000
10-11	0.08205+j0.19207	j0.0000
12-13	0.22092+j0.19988	j0.0000
13-14	0.17093+j0.34802	j0.0000

- $70 \le P_{G2} \le 145$ (MW)
- $P_D = 259$ (MW)
- $P_{12} \le 120$ (MW)
- $P_{24} \le 75$ (MW)
- $P_{G1} + P_{G2} = 259 + P_L$

where:

- $C_1(P_{G1}) = 120 + 1.9P_{G1} + 0.008P_{G1}^2$
- $C_2(P_{G2}) = 130 + 2.1P_{G2} + 0.009P_{G2}^2$

1. Two formulas (21), (22) of the B-matrix method were tested: the results are presented in table 3. These investigations demonstrate that the approximation proposed in this paper (22) is not much less than (21). The accuracy of approximation is quite acceptable. These results should be compared article [8]. They are quite similar. The values of the B-matrix need to be corrected from time to time or when the load changes significantly. This is the method

Table 2:

N°	bus type	real power	reactive power	bus voltage scheduled
1	slack bus	0.000	0.000	1.06 + j 0.00
2	generation bus	0.183	-0.127	1.00 + j 0.00
3	load bus	-0.942	-0.190	1.00 + j 0.00
4	"	-0.478	0.039	1.00 + j 0.00
5	"	-0.076	-0.016	1.00 + j 0.00
6	"	-0.112	-0.075	1.00 + j 0.00
7	"	0.000	0.000	1.00 + j 0.00
8	"	0.000	0.000	1.00 + j 0.00
9	"	-0.295	-0.166	1.00 + j 0.00
10	"	-0.090	-0.058	1.00 + j 0.00
11	"	-0.035	-0.018	1.00 + j 0.00
12	"	-0.061	-0.016	1.00 + j 0.00
13	"	-0.135	-0.058	1.00 + j 0.00
14	"	-0.149	-0.050	1.00 + j 0.00

to use when seeking to avoid calculation of the load flow with each iteration.

A.E.P 14 test system B-matrix model:

- $P_L = K_{L0} + B_{10}P_{G1} + B_{20}P_{G2} + B_{11}P_{G1}^2 + B_{22}P_{G2}^2$
- $K_{L0} = -76.16605$
- $B_{10} = 0.33079$
- $B_{20} = 1.1944$
- $B_{11} = -0.0011$
- $B_{22} = -0.00544$

A.E.P 14 test system B'- matrix model:

- $P_L = K'_{L0} + B'_{10}P_{G1} + B'_{20}P_{G2} + B'_{11}P_{G1}^2 + B'_{22}P_{G2}^2 + 2B'_{12}P_{G1}P_{G2}$
- $K'_{L0} = -106.2569$
- $B'_{10} = 0.5388$
- $B'_{20} = 1.4572$
- $B'_{11} = -0.00084$
- $B'_{22} = -0.0049$
- $2B'_{12} = -0.00256$

Table 3: B—Matrix method

P_{G1}, MW	P_{G2}, MW	P_L (G-S), MW	P_{L21} (m=6), MW	P_{L2} (m=5), MW	cost, $/h	time, s
145.0	127.0	13.23	13.00	12.78	985.51	0
150.0	123.0	13.37	13.25	13.21	984.46	0
155.0	118.0	13.55	13.70	13.78	979.76	0
157.0	116.0	13.63	13.83	13.91	978.09	0
158.5	114.5	13.68	13.92	13.98	976.92	0
159.5	113.5	13.72	13.96	14.01	976.18	0
161.0	112.0	13.78	14.02	14.02	975.12	0
162.5	110.5	13.84	14.07	14.01	974.12	0
164.0	109.0	13.89	14.10	13.97	973.20	0
165.0	108.0	13.94	14.12	13.93	972.62	0
166.0	107.0	13.98	14.12	13.87	972.77	0
168.0	105.0	14.06	14.12	13.72	971.06	0
170.0	103.5	14.12	14.09	13.55	972.16	0
175.0	100.0	14.27	13.97	13.00	976.25	0
180.0	95.00	14.49	13.70	12.04	975.12	0

2. The results of experiments made with load flow calculations. Using the Gauss-Seidel method [2] for calculation of the transmission losses, we find:

- $P_L = 14.72$ MW

2.1 The task mentioned above was solved by (6), (7), (11), (20) and the following results were obtained:

- P_{G1} = 153.72 MW; P_{G2} = 120 MW; Cost= 955.54 $/hr; Time= 0.22 s

2.2 The same task was solved with the same conditions by another method called the linearization method around the selected point and the following results were obtained:

- $P_{G1} = 160$ MW; $P_{G2} = 123.72$ MW;

- Cost= 961.08 \$/hr;
- Time= 0.05 s

The results of experiments made without load flow calculations by (6), (7), (11), (20), (23) are:

- $P_{G1} = 175$ MW; $P_{G2} = 93.065$ MW;
- Cost= 942.06 \$/hr;
- $P_L = 9.07$ MW; Time= 0.21 s

8. Conclusion

The variable weights linear programming method was applied for the first time to security constrained economic dispatch. Two main approaches should be emphasized. The first is where the solution of load flow is not needed each time we determine the optimal dispatch. The second approach needs the solution of load flow when determining optimal dispatch. The first approach is very much faster, but with lower accuracy than the second one and minimizes the transmission losses (from 14.72 MW to 9.07 MW). The modified formula of transmission losses was proposed and tested with the A.E.P 14 bus model. It achieves similar accuracy to that commonly used. The weights linear programming method can be used for solving the nonlinear and linear dispatch problem. During the transformation of the nonlinear problem to a linear one, this method does not need the determining of derivatives and this is its great advantage.

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Nomenclature

ΔP_{ij}	P_{ij} (scheduled) – P_{ij} (actual)
ΔP_k	P_k (scheduled) – P_k (actual)
B_{ij}	bus susceptance matrix
$C_i(P_{Gi})$	cost function of ith generator
e_i	active part of bus voltage
f_i	reactive part of bus voltage
G_{ij}	bus matrix conductance
n	number of bus
n_G	number of generation
P_D	total active load of network
P_{Gi}	active generation power of the ith generator

P_{Gi}^m, P_{Gi}^M minimum and maximum limit values respectively of P_{Gi}

P_{ij} active line flow power

P_{ij}^M maximum limit value of P_{ij}

P_L active power losses

P_p active bus injection power

Q_p reactive bus injection power