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Simplified and approximated relations of heat transfer effectiveness for a steam condenser

Rafał Laskowski*, Janusz Lewandowski

Institute of Heat Engineering, Warsaw University of Technology
21/25 Nowowiejska Street, 00-665 Warsaw, Poland

Abstract

This paper presents two simplified and approximated relations of heat transfer effectiveness for a steam condenser. The proposed relations of heat transfer effectiveness are the function of the measured and reference parameters. The correctness of relations was verified on the basis of data from the characteristics and measured data for the 200 MW steam condenser. A comparison of the outlet cooling water temperature calculated from the proposed relations and Beckman formula was performed. The formula proposed by Beckman for the heat transfer effectiveness of a steam condenser, in a simplified form, is the function of the saturation temperature, mass flow rate of cooling water and the reference parameter. It contains two constant exponents, which must be determined on the basis of the measured data. To assess the accuracy of the relations, the average value and standard deviation were used. The proposed relations have simpler forms but are slightly less accurate than the Beckman relation.

Keywords: heat transfer effectiveness, steam condenser, approximated relation

1. Introduction

This paper presents two simplified and approximated relations of heat transfer effectiveness for a steam condenser. The proposed relations of heat transfer effectiveness are the function of the measured and reference parameters. The correctness of relations was verified on the basis of data from the characteristics and measured data for the 200 MW steam condenser. A comparison of the outlet cooling water temperature calculated from the proposed relations and Beckman formula was performed. The formula proposed by Beckman for the heat transfer effectiveness of a steam condenser, in a simplified form, is the function of the saturation temperature, mass flow rate of cooling water and the reference parameter. It contains two constant exponents, which must be determined on the basis of the measured data. To assess the accuracy of the relations, the average value and standard deviation were used. The proposed relations have simpler forms but are slightly less accurate than the Beckman relation.

The intensity of heat flux in a heat exchanger in the changed conditions can be represented in the form of heat transfer effectiveness in the function of two dimensionless parameters [1, 2, 3, 4]

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = f\left(\frac{\dot{C}_{min}}{\dot{C}_{max}}, \frac{UA}{\dot{C}_{min}}\right) \quad (1)$$

The function contains the heat capacity for both fluids, overall heat transfer coefficient and heat trans-

*Corresponding author

Email addresses: rlask@itc.pw.edu.pl (Rafał Laskowski*), jlew@itc.pw.edu.pl (Janusz Lewandowski)

fer surface area. Heat capacity is the product of the mass flow rate and the specific heat at constant pressure.

$$\dot{C} = c_p \dot{m} \tag{2}$$

$\dot{C}_{min}$ means lower and $\dot{C}_{max}$ means higher heat capacity between $\dot{C}_h$, $\dot{C}_c$. Overall heat transfer coefficient (U) is a function of heat transfer coefficients for both fluids (α_1, α_2) and wall thickness (δ) and wall thermal conductivity (λ). Heat transfer coefficients are a function of Nusselt, Reynolds and Prandtl numbers. To accurately determine the overall heat transfer coefficient, the temperatures at the outlets of the heat exchanger must be known, so the process needs to be made by iteration. Diffrent relations for the overall heat transfer for different flow conditions and mathematical relations describing heat transfer effectiveness for different types of heat exchangers can be found in the literature [5, 6].

Another approach to describe the intensity of heat flux for a heat exchanger was suggested by Beckman in the following form [7, 8, 9]

$$\varepsilon = C\,(T_{h1})^{\alpha_1}\,(T_{c1})^{\alpha_2}\,(\dot{m}_h)^{\alpha_3}\,(\dot{m}_c)^{\alpha_4} \tag{3}$$

In this approach it is assumed that heat transfer effectiveness is a function of input parameters to the heat exchanger: mass flow rates and temperatures of both fluids. In relation we have four constant exponents and a constant C. The α_1, α_2, α_3, α_4 coefficients are calculated, based on the measured data. The parameter C can be eliminated by introducing a reference state. The relation then takes the form

$$\frac{\varepsilon}{\varepsilon_o} = \left(\frac{T_{h1}}{T_{h1o}}\right)^{\alpha_1}\left(\frac{T_{c1}}{T_{c1o}}\right)^{\alpha_2}\left(\frac{\dot{m}_h}{\dot{m}_{ho}}\right)^{\alpha_3}\left(\frac{\dot{m}_c}{\dot{m}_{co}}\right)^{\alpha_4} \tag{4}$$

In the case of the phase change of one fluid, for the steam condenser the heat capacity of the steam is much greater than the heat capacity of cooling water $\dot{C}_{max} = \dot{C}_h \geqslant \dot{C}_{min} = \dot{C}_c$. The relation (1) simplifies to the form

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = f\left(\frac{UA}{\dot{C}_{min}}\right) \tag{5}$$

Beckman showed that for steam condenser α_2, α_3 the exponents are close to zero and can be omitted [7, 8].

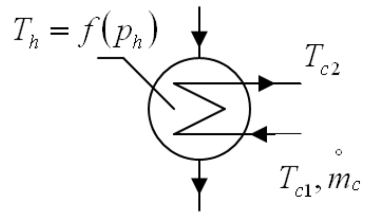

Figure 1: Scheme of a 200 MW steam condenser with measured parameters

The heat transfer effectiveness for the steam condenser ($T_{h1} = T_{h2} = T_h$) can be written in the form

$$\frac{\varepsilon}{\varepsilon_o} = \left(\frac{T_h}{T_{ho}}\right)^{\alpha_1}\left(\frac{\dot{m}_c}{\dot{m}_{co}}\right)^{\alpha_4} \tag{6}$$

Even in a simplified approximated form of the heat transfer effectiveness for a steam condenser, there are two constant exponents that must be determined based on measured data. Szapajko and Rusinowski [10, 11] proposed a linear relation of the heat transfer effectiveness for a steam condenser and feed water heat exchanger as a function of the mass flow rate, assuming that the steam pressure is constant

$$\varepsilon = D \cdot \dot{m}_c + E \tag{7}$$

In this case we also have two coefficients D and E, which are calculated based on the measured data. Linear relations of the heat transfer effectiveness for a feed water heat exchanger and counterflow and crossflow heat exchanger were proposed in the papers [12, 13], too.

The purpose of this paper is to propose the approximated relations of heat transfer effectiveness for the steam condenser using measured input parameters to the heat exchanger, parameters in reference state, without additional coefficients that need to be set based on measured data.

2. Mathematical models

2.1. First mathematical model

Steam condensers for a 200 MW power plant were analyzed. In Fig. 1 a scheme of the steam condenser with measured parameters is presented. Within the steam condenser the mass flow rate of cooling water, cooling water temperature at the inlet and outlet of the steam condenser and the pressure in the steam condenser were measured.

Cooling water outlet temperature can be expressed as a function of the following parameters

$$T_{c2} - T_{c1} = f\,(T_h - T_{c1},\, \dot{m}_c,\, p_h,\, A,\, g) \qquad (8)$$

Using Buckingham Π theorem, the above relation can be converted to a form in which there is heat transfer effectiveness

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_h - T_{c1}} = C\left(\frac{\dot{m}_c}{p_h}\frac{\sqrt{g}}{A^{\frac{3}{4}}}\right)^b \qquad (9)$$

After introduction of the two following dimensionless parameters

$$\Pi_1 = \frac{T_{c2} - T_{c1}}{T_h - T_{c1}} \qquad (10)$$

$$\Pi_2 = \frac{\dot{m}_c}{p_h}\frac{\sqrt{g}}{A^{\frac{3}{4}}} \qquad (11)$$

The relation takes the form

$$\Pi_1 = C\,(\Pi_2)^b \qquad (12)$$

After applying logarithms of both sides, we obtain the equation

$$ln\,(\Pi_1) = lnC + b \cdot ln\,(\Pi_2) \qquad (13)$$

The function of the logarithm of the x variable can be expanded in a series [14].

$$lnx = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - ..... \qquad (14)$$

Considering only the first term in the expansion of the logarithm functions we obtain

$$\Pi_1 - 1 = C - 1 + b \cdot (\Pi_2 - 1) \qquad (15)$$

After arranging

$$\Pi_1 = C + b \cdot \Pi_2 - b \qquad (16)$$

When the cooling water mass flow rate is decreased, the outlet temperature of water can be heated to a higher value. Thus, when the cooling water flow rate tends to zero, heat transfer effectiveness tends to one. From this condition we obtain

$$b = C - 1 \qquad (17)$$

After inserting the above relation and the transformations we obtain

$$\Pi_1 - 1 = (C - 1) \cdot \Pi_2 \qquad (18)$$

For the reference state we can write

$$\Pi_{1o} - 1 = (C - 1) \cdot \Pi_{2o} \qquad (19)$$

Dividing one equation by the second one we get

$$\frac{\Pi_1 - 1}{\Pi_{1o} - 1} = \frac{\Pi_2}{\Pi_{2o}} \qquad (20)$$

After entering the formulae for dimensionless parameters we have

$$\frac{\varepsilon - 1}{\varepsilon_o - 1} = \frac{\dot{m}_c}{p_h}\frac{p_{ho}}{\dot{m}_{co}} \qquad (21)$$

We received a relation of heat transfer effectiveness depending on the cooling water mass flow rate, the pressure in the steam condenser and the parameters in reference state without additional coefficients or exponents. After the transformation, heat transfer effectiveness for the steam condenser in the changed conditions, can be written as

$$\varepsilon = 1 - \frac{\dot{m}_c}{p_h}\frac{p_{ho}}{\dot{m}_{co}}\,(1 - \varepsilon_o) \qquad (22)$$

2.2. *Second mathematical model*

An example of temperature distribution along the heat transfer surface area for the steam condenser is shown in the Fig. 2.

Outlet cooling water temperature can be presented as

$$T_{c2} = T_h - \triangle T \qquad (23)$$

The above relation can be transformed into the following form

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_h - T_{c1}} = 1 - \frac{\triangle T}{T_h - T_{c1}} \qquad (24)$$

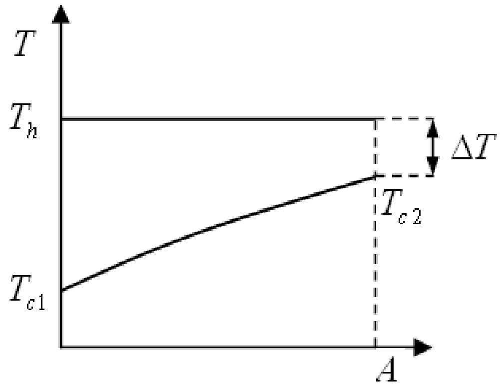

Figure 2: Temperature distribution along the heat transfer surface area for the steam condenser

Assuming that approximately $\triangle T$ is a linear function of cooling water mass flow rate

$$\triangle T \cong a \cdot \dot{m}_c \tag{25}$$

Heat transfer effectiveness can be written as

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_h - T_{c1}} = 1 - \frac{a \cdot \dot{m}_c}{T_h - T_{c1}} \tag{26}$$

Having a reference state it can be written as

$$\varepsilon = 1 - \frac{\dot{m}_c}{T_h - T_{c1}} \frac{T_{ho} - T_{c1o}}{\dot{m}_{co}} (1 - \varepsilon_o) \tag{27}$$

We determined a relation between heat transfer effectiveness depending on cooling water mass flow rate, the difference between temperature in the steam condenser and inlet outlet cooling temperature and the parameters in the reference state without additional coefficients or exponents.

3. Results

3.1. Results for the first mathematical model

The correctness of the first proposed relation was checked for data from the characteristics as well as on the basis of the measured data for a 200 MW power plant steam condenser. Fig. 3 shows a comparison between the outlet water temperature from the characteristics and as calculated from the first proposed relation for 25 values.

The comparison of outlet temperatures shows that the change in the calculated outlet temperature from

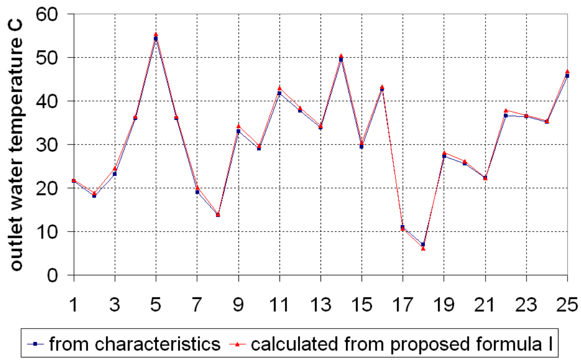

Figure 3: Change in water temperature at the outlet of the steam condenser, from characteristics and calculated from the first proposed formula for 25 values

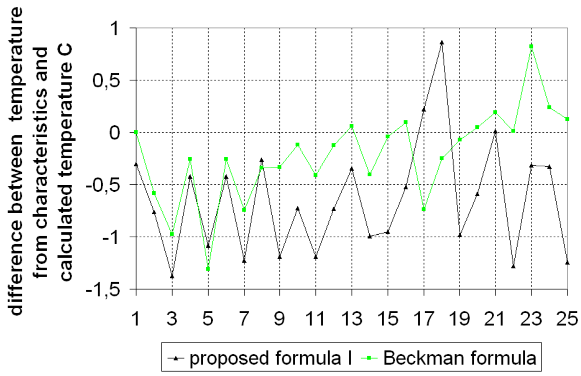

Figure 4: Difference between the temperature from characteristics and the calculated temperature for 25 values, green line—temperature calculated from the Beckman formula, black line—temperature calculated from the first proposed formula

the first proposed relation and from the characteristics has a similar character. In order to investigate the accuracy of the model the difference between the temperature from the characteristics of the steam condenser and the calculated temperature from the first proposed relation was determined. Fig. 4 shows the difference between the measured water temperature at the outlet of the steam condenser and the calculated temperature (the green line determines the difference between the outlet temperature from the characteristics and the temperature from the Beckman relation, the black line determines the difference between the outlet temperature from the characteristics and the temperature from the first proposed relation). The comparison was made for 25 values.

To assess the accuracy of two relations the average value and standard deviation were used. For the first proposed relation an average value of -0.54 and

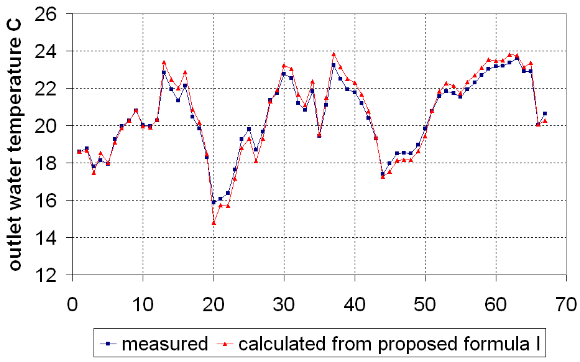

Figure 5: Change of water temperature at the outlet of the steam condenser, measured and calculated from the first proposed formula for 67 values

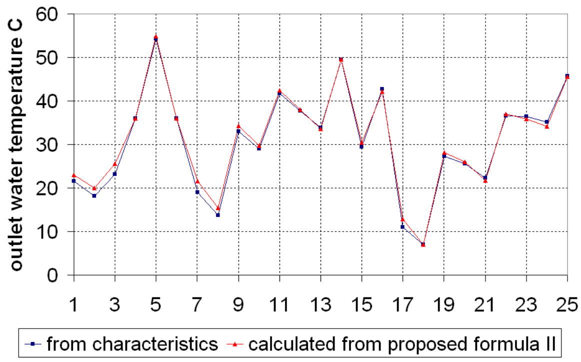

Figure 7: Change in water temperature at the outlet of the steam condenser, from characteristics and calculated from the second proposed formula for 25 values

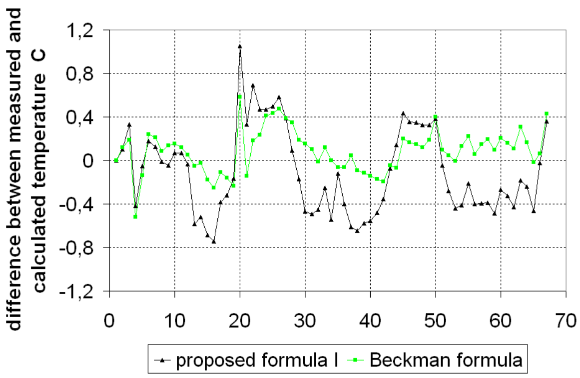

Figure 6: Difference between measured and calculated temperature for 67 values, green line—temperature calculated from the Beckman formula, black line—temperature calculated from teh first proposed formula

the standard deviation of 0.6 were obtained. For the Beckman relation an average value of -0.21 and the standard deviation of 0.44 were obtained. The average value for the Beckman relation is closer to zero, and the standard deviation that shows how data are spread from the average value is closer to zero too. On the basis of the obtained data, we can observe that the outlet water temperature which is determined from the Beckman relation is slightly more accurate. The error designation of the cooling water temperature for both relations is in the range $\pm 1.5°C$. A similar comparison was made for the measured data. A comparison between measured and calculated from the first proposed relation water temperature at the outlet of the steam condenser is shown in Fig. 5 for 67 values.

For the real data, too, the outlet water temperature calculated from the first proposed relation has a similar character of change as the measured temperature. Fig. 6 shows the difference between the measured water temperature at the outlet of the steam condenser and the calculated temperature (the green line determines the difference between the measured outlet temperature and the temperature from Beckman relation, the black line determines the difference between the measured outlet temperature and the temperature from the first proposed relation). The comparison was made for 67 values.

Based on measured data obtained for the first proposed relation the average value is -0.10 and the standard deviation is 0.39. For the Beckman relation an average value of 0.09 and the standard deviation of 0.20 were obtained. The average value for the Beckman relation is closer to zero and standard deviation is also closer to zero. For those data the outlet water temperature determined from the Beckman relation is slightly more accurate. The error designation of the cooling outlet water temperature according to the first proposed relation is in the range $\pm 1°C$ and for the Beckman relation is in the range $\pm 0.5°C$. Compared to data from the characteristics, in this case the error of cooling water is smaller for both relations, because the range of changes in input parameters to the steam condenser was smaller.

3.2. Results for the second mathematical model

The correctness of the second proposed relation was checked for data from the characteristics as well as on the basis of the measured data for a 200 MW power plant steam condenser. Fig. 7 shows a comparison between the outlet water temperatures from the

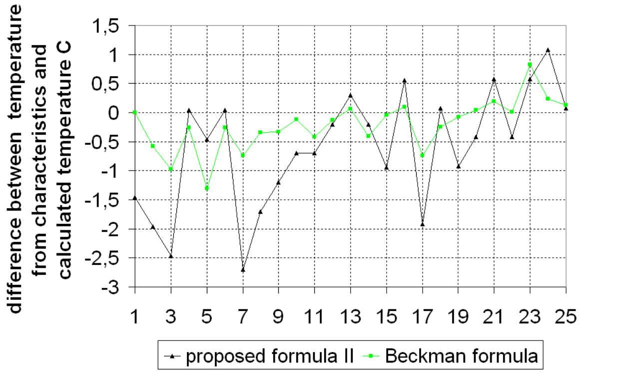

Figure 8: Difference between temperature from characteristics and calculated temperature for 25 values, green line—temperature calculated from the Beckman formula, black line—temperature calculated from the second proposed formula

characteristics and calculated from the second proposed relation for 25 values.

It results from the shown comparison of outlet temperatures that the change in calculated outlet temperature from the second proposed relation and from the characteristics has a similar character. To investigate the accuracy of the model, the difference between the temperature from the characteristics of the steam condenser and the calculated temperature from the second proposed relation was determined. Fig. 8 shows the difference between the measured water temperature at the outlet of the steam condenser and the calculated temperature (the green line determines the difference between the outlet temperature from the characteristics and the temperature from the Beckman relation, the black line determines the difference between the outlet temperature from the characteristics and the temperature from the second proposed relation). The comparison was made for 25 values.

To assess the accuracy of two relations the average value and standard deviation were used. For the second proposed relation an average value of -0.6 and the standard deviation of 1.0 were obtained. For the Beckman relation an average value is -0.21 and the standard deviation is 0.44. The average value for the Beckman relation is closer to zero, and standard deviation that shows how data are spread from the average value is closer to zero too. On the basis of the obtained data, we can observe that the outlet water temperature determined from the Beckman relation

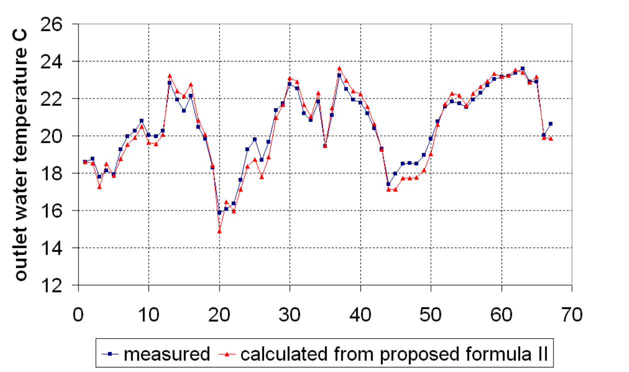

Figure 9: Change in water temperature at the outlet of the steam condenser, measured and calculated from the second proposed formula for 67 values

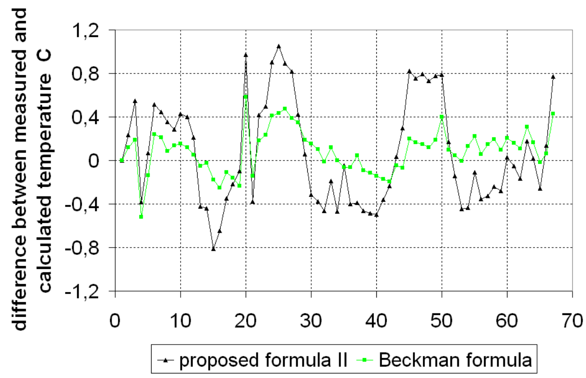

Figure 10: Difference between measured and calculated temperature for 67 values, green line—temperature calculated from Beckman formula, black line—temperature calculated from the second proposed formula

is slightly more accurate. The error designation of the cooling outlet water temperature according to the second proposed relation is in the range $\pm 2.5°C$ and for the Beckman relation is in the range $\pm 1.5°C$. A similar comparison was made for the measured data. A comparison between water temperature at the outlet of the steam condenser as measured and as calculated from the second proposed relation is shown in Fig. 9 for 67 values.

For the real data, too, outlet water temperature calculated from the second proposed relation has a similar character of change as the measured temperature. Fig. 10 shows the difference between the measured water temperature at the outlet of the steam condenser and the calculated temperature (the green line determines the difference between the measured outlet temperature and the temperature from the Beckman relation, the black line determines the difference between the measured outlet temperature and the temperature from the second proposed relation). The comparison was made for 67 values.

Based on measured data obtained for the second proposed relation the average value is 0.07 and the standard deviation 0.48. For the Beckman relation obtained the average value is 0.09 and the standard deviation 0.20. The standard deviation for the Beckman relation is closer to zero. For those data, too, the outlet water temperature determined from the Beckman relation is slightly more accurate. The error designation of the cooling outlet water temperature according to the second proposed relation is in the range $\pm 1^{\circ}C$ and for the Beckman relation it is in the range $\pm 0.5^{\circ}C$. Compared to data from the characteristics in this case the error regarding cooling water is smaller for both relations, because the range of changes in input parameters to the steam condenser was smaller.

4. Conclusion

The paper presents two simplified and approximated relations of heat transfer effectiveness for a steam condenser. The first proposed relation of heat transfer effectiveness is a function of the cooling water mass flow rate, pressure in the steam condenser and reference parameters. The second one is a function of cooling water mass flow rate, the difference between temperature in the steam condenser and inlet outlet cooling temperature and the parameters in the reference state. Compared to the approximated relation of heat transfer effectiveness for the steam condenser, given by Beckman, the two proposed relations have simpler forms, because they do not contain two constant exponents, which must be determined on the basis of the measured data.

The correctness of relations was verified on the basis of data from the characteristics of the condenser manufacturer and measured data for the 200 MW steam condenser. A comparison between Beckman and proposed relations was made for data from the characteristics and measured data. In both cases, the heat transfer effectiveness, proposed by Beckman, is slightly more accurate. To assess the accuracy of both relations, the average value and standard deviation was used. The error designation of the outlet cooling water temperature for the two proposed relations is slightly larger than for the Beckman relation: for data from the characteristics it is in the range $\pm 1.5^{\circ}C$ for the first model and $\pm 2.5^{\circ}C$ range for the second model, and for measured data it is in the range $\pm 1^{\circ}C$ for both models, which corresponds to the measured error of temperature. The larger error for the data from the characteristics is the result of a wider change in inlet parameters to the steam condenser.

It seems that the obtained accuracy allows the use of the two proposed formulae to be used for determining outlet water cooling temperature in changed conditions. The two proposed models have a similar degree of accuracy.

The purpose of this article is to present the relations (22, 27) of heat transfer efficiency for the steam condenser as a function of the measured parameters only inlet to the condenser. From these relations (22, 27) the temperature of the outlet cooling water can be determined and then the mass flow rate of the steam from the energy balance can be calculated.

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Nomenclature

a—constant

A—heat transfer surface area, m^2

$\dot{C}$—fluid heat capacity, W/K

C—constant, dimensionless

c_p—specific heat at constant pressure, J/kg/K

b—constants, dimensionless

D, E—empirical coefficients

g—gravity acceleration, m/s^2

$\dot{m}$—mass flow, kg/s

p—pressure, Pa

U—overall heat transfer coefficient, $W/(m^2K)$

$\dot{Q}$—heat flow, W

$\dot{Q}_{max}$—maximum heat flow, W

α_1, α_2, α_3, α_4—coefficients of power series

ε—heat transfer effectiveness, dimensionless

Π_i—($i = 1, 2$) number of similarities, dimensionless

Index

c—cold fluid

h—hot fluid

o—reference state

1—inlet to the heat exchanger

2—outlet of the heat exchanger