A mathematical model of the steam condenser in the changed conditions

Rafał Marcin Laskowski *

Institute of Heat Engineering, Warsaw University of Technology
21/25 Nowowiejska Street, 00-665 Warsaw, Poland

Abstract

The paper presents a mathematical model of a steam condenser in the changed conditions. A list of independent parameters on which the water temperature at the outlet from the steam condenser depends was selected and by means of the Buckingham Π theorem a functional relation between two dimensionless quantities was obtained. The exact form of the function was determined on the basis of data from the characteristics of steam condenser for a 200 MW power plant and actual measurement data for different condenser operating in a power plant of 200 MW. A linear relation between two dimensionless quantities was obtained. The correctness of the proposed relation was examined by comparing the measured temperature and calculated temperature from the proposed relation.

1. Introduction

The steam condenser is a heat exchanger where condensation of steam occurs. Wet steam (close to saturation condition) is directed from the turbine to the steam condenser, where it flows through the outer side of the tubes and gives off heat (condenses) the cooling water flowing inside the tubes. The cooling water is usually drawn from the natural sources of water (river, sea, lake). The location of the steam condenser in a power plant of 200 MW with the accepted symbols for heat transfer fluids is presented in Fig. 1.

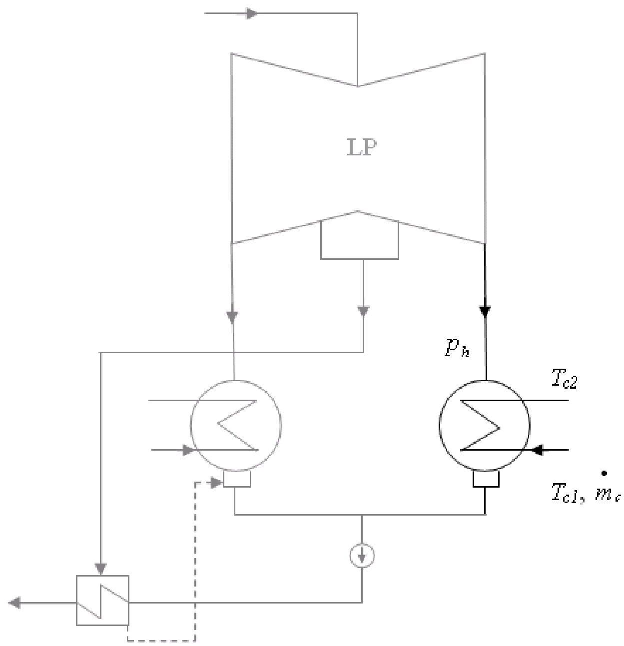

Figure 1: Location of the analyzed steam condenser in a 200 MW power plant with accepted symbols

Condenser as a “lower source of heat” plays a special role in power plant, because the parameters of his work have a significant impact on the efficiency of the installation. Therefore, it is important to know both the condenser operating parameters during the design as well as during operation. For this purpose, mathematical models describing the work of the condenser in the changed conditions are created.

Most popular mathematical model in changed conditions for steam condenser is based on energy

*Corresponding author
Email address: rlask@itc.pw.edu.pl (Rafał Marcin Laskowski)

balance equation and Peclet's law [1, 2], which is completed with the overall heat transfer coefficient as constant or as a function of heat transfer coefficients for both heat transfer fluids

$$\dot{Q} = \dot{m}_h r = \dot{C}_c \left(T_{c2} - T_{c1}\right) \tag{1}$$

$$\dot{Q} = kA\triangle T_{ln} \tag{2}$$

Heat transfer coefficients dependent on dimensionless quantities like the Reynolds Number and the Prandtl Number. Approximation relations for heat transfer coefficients are used within relevant ranges of parameters' changes [3] and are not always accurate [4].

By rearranging (1, 2) equations the work in the changed conditions of the steam condenser can be described by using the heat transfer effectiveness [5, 6]

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = 1 - exp\left(-\frac{kA}{\dot{C}_c}\right) \tag{3}$$

Beckman analysing the parameters from which the heat transfer coefficient depend in relation (3) proposed approximate formula for heat transfer effectiveness for the steam condenser with reference conditions in the following form [7, 8]

$$\frac{\varepsilon}{\varepsilon_o} = \left(\frac{T_{h1}}{T_{h1o}}\right)^{\alpha_1} \left(\frac{T_{c1}}{T_{c1o}}\right)^{\alpha_2} \left(\frac{\dot{m}_h}{\dot{m}_{ho}}\right)^{\alpha_3} \left(\frac{\dot{m}_c}{\dot{m}_{co}}\right)^{\alpha_4} \tag{4}$$

The $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ coefficients are calculated based on the measured data. Approximated relation proposed by Beckman works well in the changed conditions for the steam condenser when the parameters do not change over a wide range. The aim of this paper is to create a simpler approximate relation describing the work of the steam condenser in the changed conditions based on measured parameters. For this purpose the Buckingham Π theorem was used.

2. A mathematical model

The paper analyzes the steam condenser for a 200 MW power plant. In the analyzed steam condenser the following parameters were measured: water temperature at the inlet (T_{c1}) and at the outlet (T_{c2}) of the steam condenser, the cooling water mass flow ($\dot{m}_c$) and vapour pressure (p_h). The paper attempts to obtain the functional relation describing the work of the steam condenser in the changed conditions only using the measured parameters.

The analysis of the (1, 2) relations shows that the water temperature at the outlet of the steam condensor (T_{c2}) depends on temperatures at the inlet of both fluids (T_{h1}, T_{c1}), the heat capacity of cooling water $\left(\dot{C}_c\right)$, the overall heat transfer coefficient (k) and the heat transfer surface area (A)

$$T_{c2} = f\left(T_{h1},\, T_{c1},\, \dot{C}_c,\, k,\, A\right) \tag{5}$$

The heat capacity of the cooling water is equal to the product of specific heat at constant pressure and the mass flow. Assuming that physical properties of the cooling water, in the analized range of changes of parameters, are constant, the heat capacity is only the function of the mass flow

$$\dot{C}_c = c_{pc}\dot{m}_c = f\left(\dot{m}_c\right) \tag{6}$$

Let's look at parameters of which depends the overall heat transfer coefficient in our case. Overall heat transfer coefficient (k) is a function of heat transfer coefficients for both fluids (α_1, α_2) and wall thickness (δ) and wall thermal conductivity (λ). In order to simplify the analysis, overall heat transfer coefficient for flat wall was adopted [9]

$$\frac{1}{k} = \frac{1}{\alpha_h} + \frac{\delta}{\lambda} + \frac{1}{\alpha_c} \tag{7}$$

Heat transfer coefficient from the water side is a function of cooling water mass flow and can be written as follows [3, 10, 11]

$$\alpha_c = \frac{Nu_c\lambda_c}{d_i} = \frac{\lambda_c}{d_i}0.021Re_c^{0.8}Pr_c^{0.43} = C_c\dot{m}_c^{0.8} = f\left(\dot{m}_c\right) \tag{8}$$

where constant C_c is equal

$$C_c = \frac{\lambda_c}{d_i}0.021\frac{d_i}{\eta_c}Pr^{0.43} \tag{9}$$

Heat transfer coefficient from the steam side can be written as [6, 11]

$$\alpha_h = \left[\frac{(\rho_{con} - \rho_v)\, g \lambda_{con}^3 r}{n v_{con} (T_s - T_{wcon})\, d_o}\right]^{\frac{1}{4}} \tag{10}$$

The temperature of saturation (T_s) which is in the formula (10) is a function of the measured steam pressure $T_s = f(p_h)$. Likewise latent heat (r) is the steam pressure function $r = f(p_h)$. Assuming that the other physical parameters of the steam are changing in a small range, heat transfer coefficient from the steam side can be written as

$$\alpha_h = f(p_h, g) \tag{11}$$

So the overall heat transfer coefficient can be written as a function of following parameters

$$k = f(\dot{m}_c, p_h, g) \tag{12}$$

Finally, the water temperature at the outlet of the steam condenser can be written as

$$T_{c2} - T_{c1} = f(T_{h1} - T_{c1}, \dot{m}_c, p_h, A, g) \tag{13}$$

A difference of temperatures was assumed, since if this is the case it is of no importance in which units the temperature is expressed.

For the so selected independent parameters a dimensional analysis may be used. According to the dimensional analysis there can be written

$$T_{c2} - T_{c1} = C (T_{h1} - T_{c1})^a (\dot{m}_c)^b p_h^c A^d g^e \tag{14}$$

The relation (14) is true when units on the left are equal to the ones on the right [10]

$$K^1 = C \cdot K^a \left(kg \cdot s^{-1}\right)^b \left(kg \cdot s^{-2} \cdot m^{-1}\right)^c \left(m^2\right)^d \left(m \cdot s^{-2}\right)^e \tag{15}$$

Comparison of the exponents at the appropriate units gives the following system of equations

[K] $1 = a$

[kg] $0 = b + c$

[s] $0 = -b - 2c - 2e$

[m] $0 = -c + 2d + e$

In the analyzed case we have six independent variables ($n = 6$) and four equations ($r = 4$). According to the Buckingham's Π theorem, the number of dimensionless quantities is equal to two ($k = 2$).

The solution of the equation gives

$$a = 1,\ c = -b,\ d = -\frac{3}{4}b,\ e = \frac{b}{2}$$

The (14) solution takes the following form

$$T_{c2} - T_{c1} = C (T_{h1} - T_{c1})^1 (\dot{m}_c)^b (p_h)^{-b} A^{-\frac{3}{4}b} g^{\frac{b}{2}} \tag{16}$$

Arrangement of the expressions with the same exponents gives

$$\frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = C \left(\frac{\dot{m}_c}{p_h} \frac{\sqrt{g}}{A^{\frac{3}{4}}}\right)^b \tag{17}$$

After introduction of the two dimensionless quantities

$$\Pi_1 = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \tag{18}$$

$$\Pi_2 = \frac{\dot{m}_c}{p_h} \frac{\sqrt{g}}{A^{\frac{3}{4}}} \tag{19}$$

the (14) relation may be eventually written as follows

$$\Pi_1 = f(\Pi_2) \tag{20}$$

On the basis of the conducted analysis a relation between two dimensionless quantities in which there are following measured parameters: water temperature at the inlet to the steam condenser (T_{c1}), cooling water mass flow ($\dot{m}_c$) and steam pressure (p_h) was received. The exact form of the function is determined on the basis of the characteristics and measured data for the steam condenser.

3. Results

At first, the relation (20) between two dimensionless quantities was checked using data obtained from the characteristics of the steam conderser for 200 MW power plant. In Table 1 the values of parameters (steam mass flow, steam pressure, water inlet temperature, diffrence between saturation temperature and water outlet temperature, cooling water mass flow) obtained from characteristics of the steam condenser are presented.

Table 1: Values of parameters from characteristics of the steam condenser

Number	$\dot{m}_h$ $[kg/s]$	p_h $[kPa]$	T_{c1} $[C]$	$T_s(p_h) - T_{c2}$ $[C]$	$\dot{m}_c$ $[kg/s]$
1	60	3	4	2.5	3200
2	80	2.5	4	3	3200
3	110	3.5	4	3.5	3200
4	60	6.3	26	1	3200
5	120	16.4	35	1.5	3200
6	60	6.3	26	1	3200
7	110	2.8	4	4	4000
8	70	1.9	4	3	4000
9	120	5.8	17	2.5	4000
10	90	4.5	17	2	4000
11	120	9	26	2	4000
12	90	7.1	26	1.5	4000
13	60	5.6	26	1	4000
14	110	12.9	35	1.5	4000

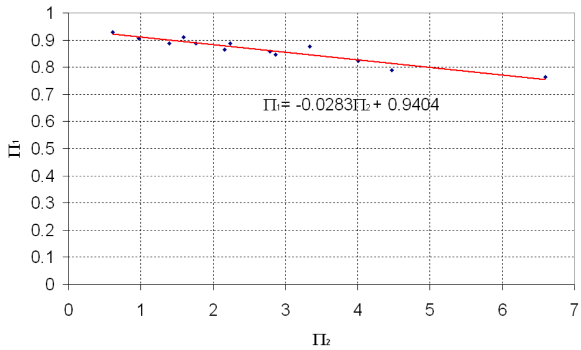

Figure 2: A comparison between two dimensionless quantities (data from characteristics of the steam condenser)

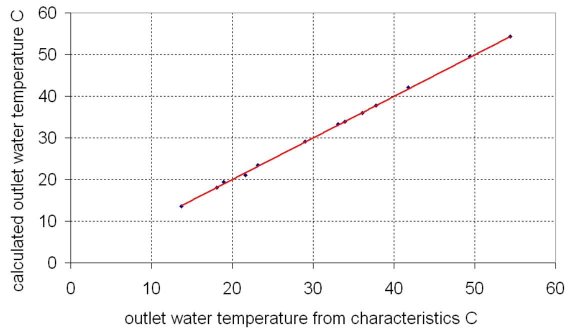

Figure 3: A comparison between water temperature at the outlet of the steam condenser, from characteristics and calculated (21)

Based on data contained in Table 1 two dimensionless quantities was determined and their values are presented in Fig.2. On the basis of the data obtained a linear relation between dimensionless quantities can be assumed with a good approximation.

The coefficients in the straight line has been determined by the least squares method and obtained followning values: for the directional coefficient $a = -0.0283$ and for the intercept coefficient $b = 0.9404$.

The relation between the dimensionless quantities can be written as

$$\Pi_1 = -0.0283 \cdot \Pi_2 + 0.9404 \qquad (21)$$

A comparison between the water temperature at the outlet of the steam condenser obtained from characteristics and calculated from (21) for 14 values is presented in Fig. 3.

The very good correlation between these two temperatures can be observed.

In Fig. 4 the change of water temperature at the outlet of the steam condenser from characteristics and calculated for 14 values is presented. Very good correspondence may be observed between these two temperatures in Fig. 4.

After determining the coefficients in (21) relation, correctness of this formula was checked for the next 11 different values. In Table 2 the next 11 parameters

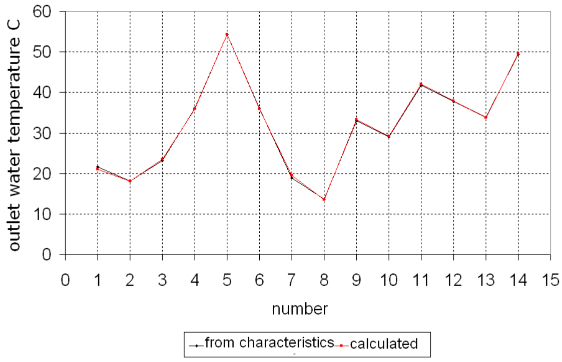

Figure 4: Change of water temperature at the outlet of the steam condenser, from characteristics and calculated

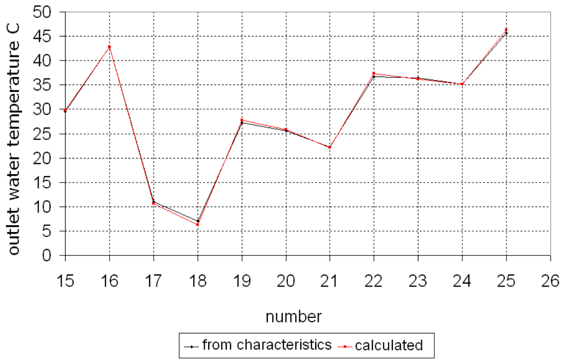

Figure 5: Change of water temperature at the outlet of the steam condenser, from characteristics and calculated for the next 11 values

from the characteristics of the steam condenser are presented.

Change of water temperature at the outlet of the steam condenser, from characteristics and calculated for the next 11 values is presented in Fig. 5. There is very good correlation between these two temperatures.

Correctness of 20 relation was also checked on the basis of measured data for the steam condenser working in different power plants with an electrical power of 200MW. Within the steam condenser the following measurments of parameters: water temperature at the inlet and outlet of the steam condenser, steam pressure and total mass of cooling water flowing through the two parts of the steam condenser were made. Mass of cooling water flowing through a one part of steam condenser obtained by dividing total mass flow of cooling water by two. The data in the control system were recorded every one hour

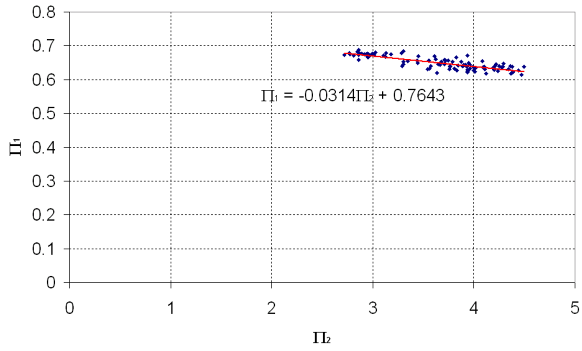

Figure 6: A comparison between two dimensionless quantities (measured data)

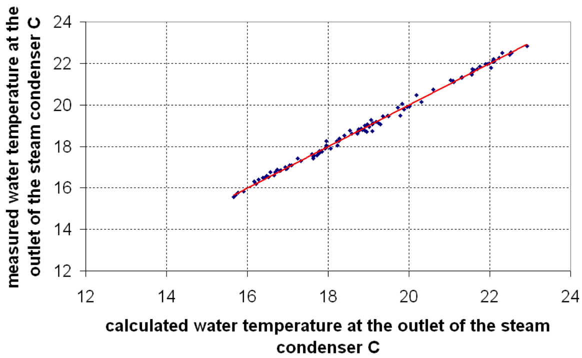

Figure 7: A comparison between water temperature at the outlet of the steam condenser, measured and calculated

during operation of power plant (average value for one hour). Two dimensionless quantities based on 100 values at the beginning of the year were calculeted. The comparison between these two dimensionless quantities is presented in Fig. 6. On the basis of the data obtained a linear relation between these two dimensionless quantities can be assumed with a good approximation. The coefficients in the straight line was determined by the least squares method for 100 measured values and obtained followning values: for the directional coefficient $a = -0.0314$ and for the intercept coefficient $b = 0.7643$.

The relation between the dimensionless quantities can be written as

$$\Pi_1 = -0.0314 \cdot \Pi_2 + 0.7643 \qquad (22)$$

A comparison between the water temperature at the outlet of the steam condenser measured and calculated from (22) is presented in Fig. 7.

The very good correlation between measured and

Table 2: Values of parameters from characteristics of the steam condenser

Number	$\dot{m}_h$ $[kg/s]$	p_h $[kPa]$	T_{c1} $[C]$	$T_s(p_h) - T_{c2}$ $[C]$	$\dot{m}_c$ $[kg/s]$
15	110	4.75	17	2.5	4800
16	70	9	35	1	4800
17	80	1.7	4	4	6400
18	40	1.15	4	2	6400
19	120	4.3	17	3	6400
20	100	3.8	17	2.5	6400
21	60	2.95	17	1.5	6400
22	130	7.05	26	2.5	6400
23	80	6.6	26	1.5	6400
24	60	6	30	1	6400
25	130	11	35	2	6400

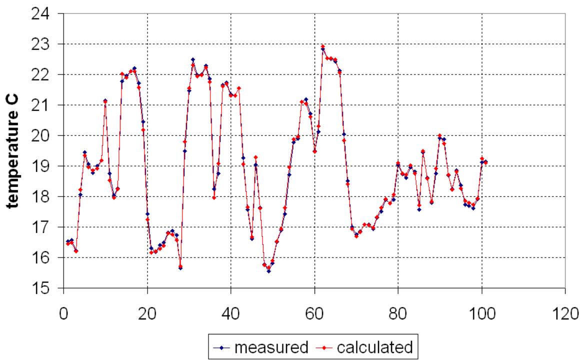

Figure 8: Change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the beginning of the year

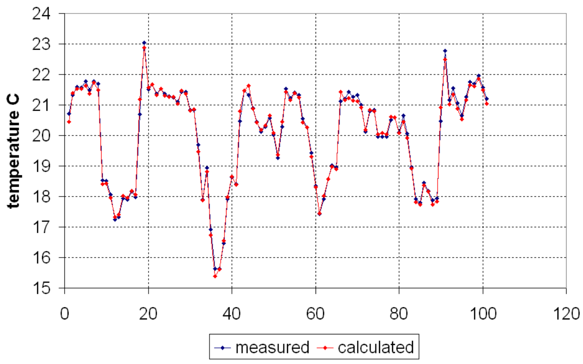

Figure 9: Change of water temperature at the outlet of the steam condenser, measured and calculated for the next 100 values

calculated temperature can be observed. The change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the beginning of the year is presented in Fig. 8.

After determining the coefficients in (22) relation, measured and calculated temperature at outlet of the steam condenser for the next 100 values was compared (Fig. 9).

Change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values in the middle of the year is presented in Fig. 10.

Change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the end of the year is presented in Fig. 11.

Based on performed analysis, it appears that the proposed linear relation for the steam condenser in

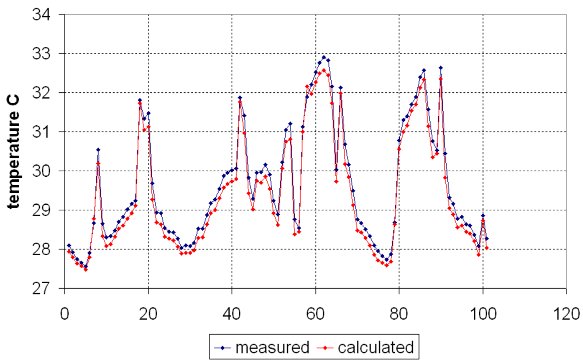

Figure 10: Change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values in the middle of the year

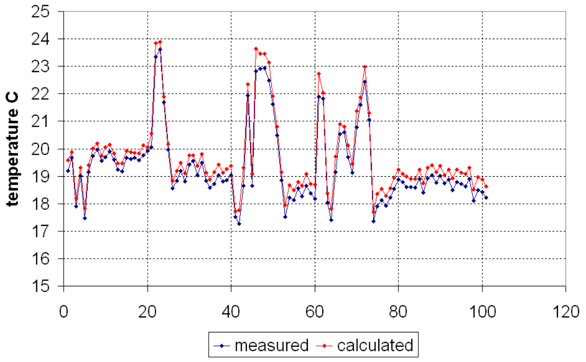

Figure 11: Change of water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the end of the year

changed conditions is correct. Introducing the concept of heat transfer effectiveness, and knowing constant value of the condenser heat transfer surface and a gravity acceleration the relation which describes the work of the steam condenser in the changed conditions can be written as a linear function with two constant

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = D \cdot \frac{\dot{m}_c}{p_h} + E \tag{23}$$

4. Conclusion

This article presents a mathematical model of the steam condenser in the changed conditions. A list of independent parameters was selected and by means of the Buckingham Π theorem two dimensionless quantities was obtained. Based on the data from the characteristics as well as the actual measurement data for the steam condenser in 200 MW power plant a linear relation between two dimensionless quantities was obtained. Simple linear relation was obtained which allows to determine the work of the steam condenser in the changed conditions. To calculate the water outlet temperature from the steam condenser in the changed conditions, two coefficients in the linear relation must be determined and basic measured parameters such as water temperature at the inlet to the steam condenser, cooling water mass flow and steam pressure must be known. A comparison between measured and calculated from the proposed linear relation of the water temperature at the outlet of the steam condenser for the data obtained from characteristics as well as for measured data have been performed. A very good correspondence between these two temperatures for both the data from the characteristics of the steam condenser as well as for measured data was obtained. The observed differences between the measured and calculated water temperature at the outlet of the steam condenser from the proposed relation (in the middle and at the end of the year) may result from a deterioration in working conditions of the steam condenser. Data from the characteristics and actual measurement data are for two different condensers working in different power plants with a electrical power of 200 MW.

References

[1] D. Laudyn, M. Pawlik, F. Strzelczyk, Power plants (in Polish), WNT Warszawa, 1995.

[2] L. Nehrebecki, Heat power plants (in Polish), WNT Warszawa, 1974.

[3] W. Gogół, Heat transfer tables and graphs (in Polish), WPW Warszawa, 1984.

[4] A. I. Elfeituri, The influence of heat transfer conditions in feedwater heaters on the exergy losses and the economical effects of a steam power station, Ph.D. thesis, Warsaw University of Technology (1996).

[5] E. Kostowski, Heat Transfer (in Polish), WPŚ Gliwice, 2000.

[6] T. S. Wiśniewski, Heat Transfer (in Polish), WNT Warszawa, 1997.

[7] G. Beckman, G. Heil, Mathematische modelle für die beurteilung von kraftwerksprozessen, EKM Mitteillungen 10.

[8] P. Bogusz, O. Kopczyński, J. Lewandowski, Simplified model of heat exchanger, XVIII Zjazd Termodynamików, 2002.

[9] B. Staniszewski, Heat Transfer (in Polish), PWN Warszawa, 1979.

[10] J. Szargut, Thermodynamics (in Polish), PWN Warszawa, 1974.

[11] A. Grzebielec, A. Rusowicz, Thermal resistance of steam condensation in horizontal tube bundles, Journal of Power Technologies 91 (1) (2011) 41–48.

Nomenclature

A – heat transfer surface area, m^2

$\dot{C}$ – fluid heat capacity, W/K

C – constat, dimensionless

C_c – constat, dimensionless

c_p – specific heat at constat pressure, J/kg/K
d_i – inside diameter, m
d_o – outside diameter, m
D – constat, 1/(m s)

a, b, c, d, e – constats, dimensionless

E – constat, dimensionless
g – gravity acceleration, m/s^2

i – enthalpy, J/kg

k – overall heat transfer coefficient, W/(m^{2}K)

$\dot{m}$ – mass flow. kg/s

n – number of tubes in a row, dimensionless
Nu – Nusselt number, dimensionless
Pr – Prandtl number, dimensionless
r – latent heat, J/kg
Re – Reynolds number, dimensionless

T_s – saturation temperature, K

T_{con} – condensate temperature, K
$\dot{Q}$ – heat flow, W

α – heat transfer coefficient, W/m^2/K

α_1, α_2, α_3, α_4 – coefficients of power series

δ – wall thickness, m

ε – heat transfer effectiveness, dimensionless
η – dynamic viscosity, kg/m/s

λ – thermal conductivity, W/m/K

λ_{con} – thermal conductivity of condensate, W/m/K

ν_{con} – kinematic viscosity of condensate, m^2/s

ρ_v – vapour density, kg/m^3
ρ_{con} – condensate density, kg/m^3

$\triangle T_{ln}$ – logarithmic mean temperature difference, K

Π_i – $(i = 1, 2)$ number of similarities, dimensionless

Index

c – cold fluid

h – hot fluid

o – reference state

1 – inlet to the heat exchanger

2 – outlet of the heat exchanger