The application of the Buckingham Π theorem to modeling high-pressure regenerative heat exchangers in off-desing operation

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Abstract

The study presents the possibility of applying the Buckingham Π theorem to modeling high-pressure regenerative heat exchangers in changed conditions. A list of independent parameters on which the water temperature at the outlet of the heat exchanger depends was selected; and by means of the Buckingham Π theorem a functional relation between two dimensionless quantities, where there is no overall heat transfer coefficient, was obtained. The exact form of the function was determined on the basis of actual measurement data and a linear relation between two dimensionless quantities has been obtained. The correctness of the proposed relation was examined for two high-pressure regenerative exchangers for a 200 MW power plant.

Keywords: heat exchanger, off-desing, heat recuperation, high pressure

1. Introduction

Regenerative heat exchangers play an important role in power plant systems since they are used to the heat water feeding the boiler, which enhances the system's efficiency [1]. Knowledge of the working parameters of a regenerative exchanger in changed conditions is useful at the stages of design and operation of the system. That is why mathematical models of regenerative exchangers in changed conditions are created. Changed conditions are understood as a steady state different from the design state.

In power systems high- and low-pressure regenerative exchangers can be distinguished [1, 2]. In most cases regenerative exchangers are powered by superheated vapour. The exchangers consist of three zones so that heat can be efficiently transferred from

vapour to water. In the first zone the vapour is cooled, whereas in the second one the vapour is condensed and in the third one the condensate is supercooled.

Due to the presence of three zones as well as the phase change of one of the fluids, modeling regenerative heat exchangers is not simple. In the literature there are two popular mathematical models of the regenerative heat exchanger [1, 3–5]. In the first model the three zones are not distinguished and the model consists of an energy balance equation and Peclet's law [1, 2]. The model is completed with the overall heat transfer coefficient, which is assumed as constant or is a function of heat transfer coefficients dependent on dimensionless quantities like the Reynolds Number and the Prandtl Number [3, 6]. In the second model the exchanger is conventionally divided into three zones and an energy balance and Peclet's law are formulated for each zone [4, 5, 7–9]. This model is completed with overall heat transfer coefficients for each of the zones, which can be

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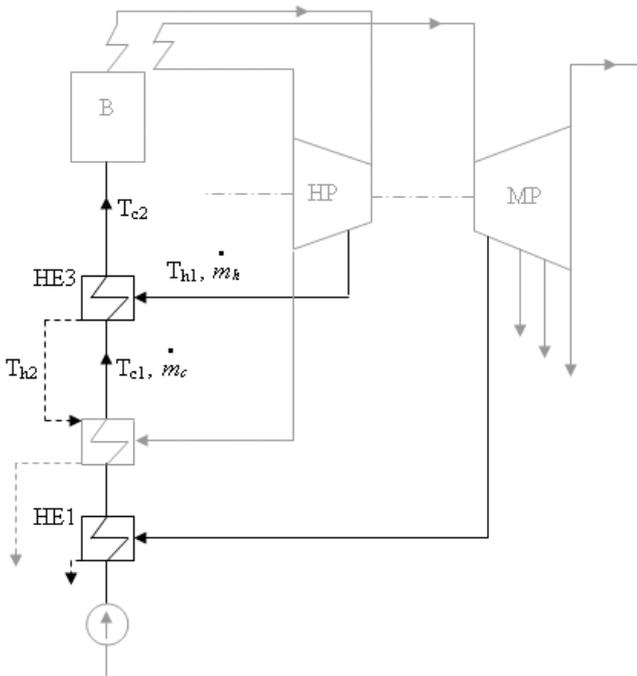
Figure 1: Location of the analyzed regenerative heat exchangers in a 200 MW power plant

accepted as constant or as a function of dimensionless quantities. In particular the second model, where account is taken of the dimensionless relations, is time-consuming and the solution should be achieved through iteration. In addition, approximation relations for heat transfer coefficients are used within relevant ranges of parameter changes [6] and are not always accurate [5]. In the literature, we can find mathematical models in changed conditions for the regenerative heat exchanger divided into six or four zones [10]. But the form of these models is even more complicated. An overall heat transfer coefficient is present in all the models under consideration. An attempt was made to create a simple model of a regenerative heat exchanger in changed conditions, based on measured data, which will not include an overall heat transfer coefficient. For this purpose the Buckingham Π theorem was used.

2. Model of regenerative exchanger in changed conditions

The paper analyzes two high-pressure heat exchangers for a 200 MW power plant. The location of the two analyzed high-pressure exchangers together

with accepted symbols for heat transfer fluids is presented in Fig. 1.

To determine the temperature of heated water at the outlet of the heat exchanger, a dimensional analysis was used (the Buckingham Π theorem). A given physical phenomenon can be presented in the form of a function of the independent parameters that influence this phenomenon [11–13]

$$f(Q_1, Q_2, Q_3, ..., Q_n) = 0 \qquad (1)$$

After a complete list of independent variables on the basis of correspondence of units is drawn up, dimensionless Π quantities are determined. The number of dimensionless quantities (k) is equal to the difference between the independent variables (n) and the number of equations created on the basis of correspondence of units (r)

$$k = n - r \qquad (2)$$

The (1) relation can be written as follows

$$f(\Pi_1, \Pi_2, ..., \Pi_{n-r}) = 0 \qquad (3)$$

or

$$\Pi_1 = f(\Pi_2, ..., \Pi_{n-r}) \qquad (4)$$

The first difficulty in application of the Buckingham theorem is the correct selection of independent parameters that influence the studied phenomenon, since omission of one important independent variable can yield erroneous results. In most cases, using the Buckingham theorem we are unable to determine the function (f) describing a given phenomenon. We usually only know on which dimensionless quantities a given phenomenon depends. The exact form of the function may be determined on the basis of experimental data.

In order to select the list of independent parameters for a high-pressure regenerative heat exchanger, consideration was given to a heat exchanger where heat is transferred between vapour and water (one of the zones of the high-pressure regeneration heat exchanger). The energy balance equation and Peclet's law can be used for describing the exchanger

$$\dot{Q} = \dot{C}_h (T_{h1} - T_{h2}) = \dot{C}_c (T_{c2} - T_{c1}) \qquad (5)$$

$$\dot{Q} = kA\overline{\triangle T} \qquad (6)$$

The mean temperature difference is equal to the product of the correction coefficient and the logarithmic mean temperature difference [6]

$$\overline{\triangle T} = \varepsilon_T \triangle T_{ln} \qquad (7)$$

Analysis of the (5), (6) relations shows that the water temperature at the outlet of the heat exchanger depends on temperatures at the inlet of both fluids (T_{h1}, T_{c1}), the heat capacity of both fluids $\left(\dot{C}_h, \dot{C}_c\right)$, the overall heat transfer coefficient k and the heat transfer surface area A

$$T_{c2} = f\left(T_{h1}, T_{c1}, \dot{C}_h, \dot{C}_c, k, A\right) \qquad (8)$$

The heat capacity of the fluid is equal to the product of specific heat at constant pressure and the mass flow. Given that physical properties of the fluids are constant the heat capacity is only the function of the mass flow. For both fluids we can write

$$\dot{C}_h = c_{ph}\dot{m}_h = f(\dot{m}_h) \qquad (9)$$

$$\dot{C}_c = c_{pc}\dot{m}_c = f(\dot{m}_c) \qquad (10)$$

The overall heat transfer coefficient is a function of temperatures at the inlet to the heat exchanger and mass flows of fluids

$$k = f(T_{h1}, T_{c1}, \dot{m}_h, \dot{m}_c) \qquad (11)$$

So the (8) functional relation for the water temperature at the outlet may now be recorded as a function of temperatures at the inlet to the heat exchanger, mass flows and heat transfer surface area

$$T_{c2} = f(T_{h1}, T_{c1}, \dot{m}_h, \dot{m}_c, A) \qquad (12)$$

A similar analysis can be carried out for the two other heat exchanger zones, so the water temperature at the outlet of the high-pressure regenerative exchanger can be written as a function of the following parameters

$$T_{c2} - T_{c1} = f(T_{h1} - T_{c1}, \dot{m}_h, \dot{m}_c, A) \qquad (13)$$

A difference of temperatures was assumed, since if this is the case it is of no importance in which units the temperature is expressed.

A dimensional analysis may be used for the independent parameters selected in this way. Further to the dimensional analysis it can be written

$$T_{c2} - T_{c1} = C(T_{h1} - T_{c1})^a (\dot{m}_h)^b (\dot{m}_c)^c A^d \qquad (14)$$

The units on the left must be equal to the ones on the right so that the (14) relation is true [14]

$$K^1 = C \cdot K^a \left(kg \cdot s^{-1}\right)^b \left(kg \cdot s^{-1}\right)^c \left(m^2\right)^d \qquad (15)$$

Comparison of the exponents at the relevant units gives the following system of equations

[K] $1 = a$
[kg] $0 = b + c$
[s] $0 = -b - c$
[m] $0 = 2d$

In the analyzed case we have 5 independent variables ($n = 5$) and four equations. One of the equations repeats itself and that is why r is equal to three ($r = 3$). According to the Buckinghama Π theorem the number of dimensionless quantities is equal to two ($k = 2$).

The solution of the equation gives

$$a = 1, \ c = -b, \ d = 0$$

The (14) solution takes the following form

$$T_{c2} - T_{c1} = C(T_{h1} - T_{c1})^1 (\dot{m}_h)^b (\dot{m}_c)^{-b} A^0 \qquad (16)$$

Arrangement of the expressions with the same exponents gives

$$\frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = C\left(\frac{\dot{m}_h}{\dot{m}_c}\right)^b \qquad (17)$$

After introduction of the two dimensionless quantities

$$\Pi_1 = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \qquad (18)$$

$$\Pi_2 = \frac{\dot{m}_h}{\dot{m}_c} \qquad (19)$$
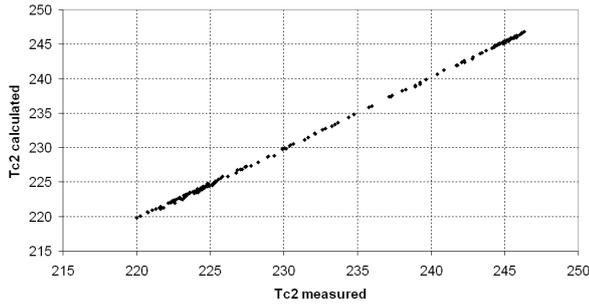
Figure 2: A comparison between water temperature at the outlet of the heat exchanger, measured and calculated (for the HE3 heat exchanger)

the (14) relation may be eventually written as follows

$$\Pi_1 = f(\Pi_2) \qquad (20)$$

On the basis of the conducted analysis a relation was determined between two dimensionless quantities in which there occur: temperatures at the inlet to the heat exchanger (T_{c1}, T_{h1}), mass flows of both fluids ($\dot{m}_c$, $\dot{m}_h$) and water temperature at the outlet of the exchanger (T_{c2}). The exact form of the function is determined for the high-pressure heat exchanger on the basis of actual measurement data.

3. Results

In the analyzed heat exchangers the following parameters were measured: vapour pressure (p_{h1}) and temperature (T_{h1}) in the bleeding, temperature of the condensate (T_{h2}), water temperature at the inlet (T_{c1}) and at the outlet (T_{c2}) of the heat exchanger and the water mass flow ($\dot{m}_c$). The vapour mass flow ($\dot{m}_h$) was determined from the energy balance equation

$$\dot{m}_h = \frac{\dot{m}_c (i_{c2} - i_{c1})}{i_{h1} - i_{h2}} \qquad (21)$$

3.1. Analysis of the HE3 heat exchanger working parameters

The (20) relation between two dimensionless quantities is presented in Fig. 2 for 200 work points for the HE3 regenerative heat exchanger supplied from the first bleeding of the HP part of the turbine (Fig. 1). On the basis of the data obtained, a linear
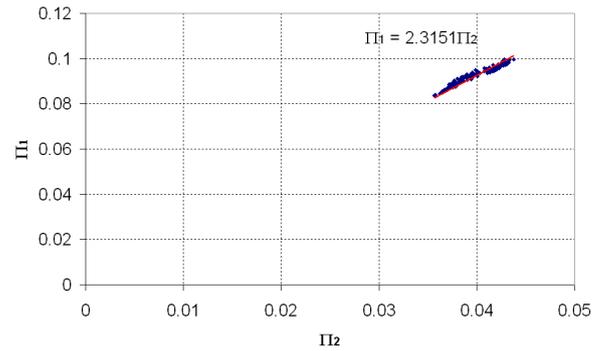
Figure 3: A comparison between two dimensionless quantities (for the HE3 heat exchanger)
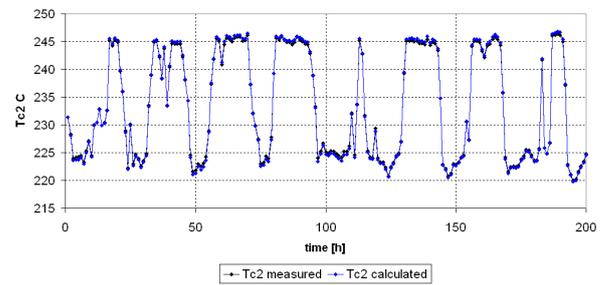
Figure 4: Change of water temperature at the outlet of the heat exchanger, measured and calculated (for the HE3 heat exchanger)

relation between dimensionless quantities can be assumed with a good approximation. The value of the directional coefficient of the straight line was determined by the least squares method and the value of 2.3151 was obtained.

The (20) relation between the dimensionless quantities can be written as

$$\Pi_1 = 2.315 \cdot \Pi_2 \qquad (22)$$

The (22) relation may also be presented in the following form using reference parameters

$$\frac{\Pi_1}{\Pi_{1o}} = \frac{\Pi_2}{\Pi_{2o}} \qquad (23)$$

A comparison between the water temperature at the outlet of the HE3 heat exchanger measured and calculated for 200 values is presented in Fig. 2.

The correlation between the two temperatures is very good.

Fig. 4 presents temperature measured and calculated in time for 200 values. Very good correspon-
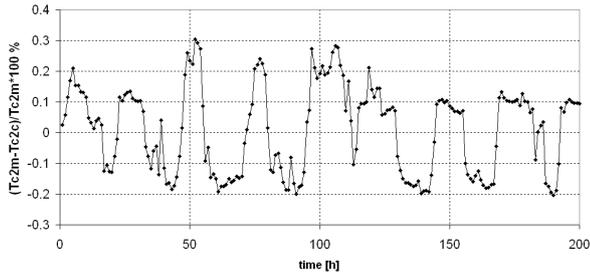
Figure 5: Relative difference between water temperature at the outlet of the heat exchanger, measured (m) and calculated (c) in % (for the HE3 heat exchanger)

Figure 6: Comparison between water temperature at the outlet of the heat exchanger measured and calculated (for the HE3 heat exchanger) for data at the end of the year
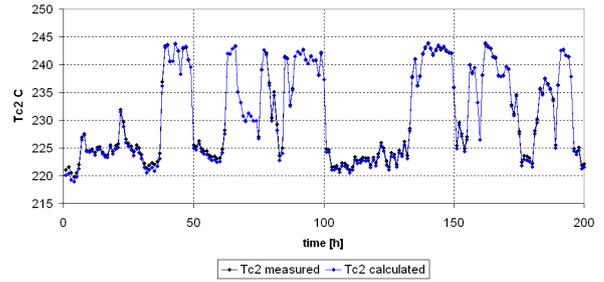
Figure 7: Change of water temperature at the outlet of the exchanger measured and calculated (for the HE3 heat exchanger) for data at the end of the year
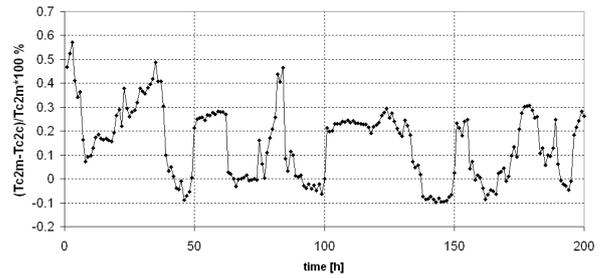
Figure 8: Relative difference between water temperature at the outlet of the heat exchanger, measured (m) and calculated (c) in % (for the HE3 heat exchanger) for data at the end of the year

dence of these two temperatures may be observed in Fig. 4.

The relative error (in percent) between the water temperature at the outlet of the heat exchanger measured and calculated for 200 values is presented in Fig. 5. The value of the error is in the -0.3 % to 0.3 % range, which is considered a minor relative error.

The correctness of the relations was also checked for the HE3 heat exchanger for data at the end of the year for 200 measured values. Fig. 6 presents a comparison between water temperature at the outlet of the heat exchanger, measured and calculated from the (22) relation. There is very good correlation between these two temperatures.

Fig. 7 shows the measured and calculated outlet water temperature at the time for 200 measured values at the end of the year. Very good agreement can be observed between these two temperatures.

Fig. 8 presents the relative error (in percent) between water temperature at the outlet of the heat exchanger measured and calculated for 200 values at the end of the year. The value of the error is in the

-0.1 % to 0.6 % range, which is considered a minor relative error.

3.2. Analysis of working parameters of the HE1 heat exchanger

An analysis similar to that carried out on the HE3 heat exchanger was performed for the HE1 heat exchanger, which is supplied from the first bleeding of the MP part of the turbine (Fig. 1).

Fig. 9 presents the relation between the two dimensionless quantities for 200 work points of the HE1 regenerative heat exchanger. On the basis of the data obtained, a linear relation between dimensionless quantities can be assumed with a good approximation. The value of the directional coefficient of the straight line was determined by the least squares method and the value of 2.0502 was obtained.

The relation between the dimensionless quantities can be written as

$$\Pi_1 = 2.0502 \cdot \Pi_2 \qquad (24)$$

The relation may also be presented using reference parameters (23).
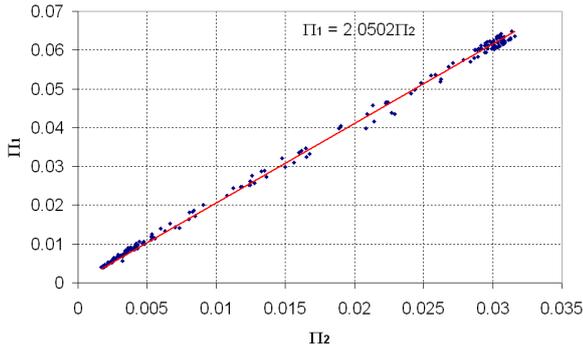
Figure 9: Change of water temperature at the outlet of the heat exchanger, measured and calculated (for the HE1 heat exchanger)
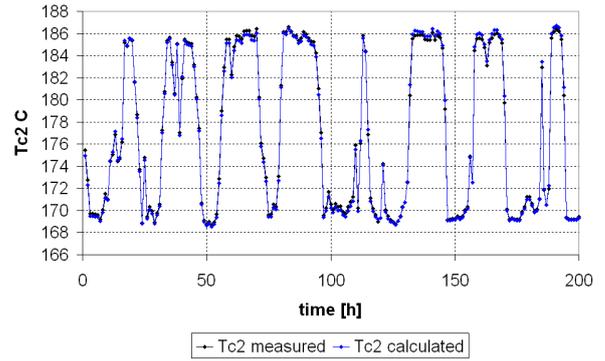
Figure 11: Change of water temperature at the outlet of the heat exchanger, measured and calculated (for the HE1 heat exchanger)
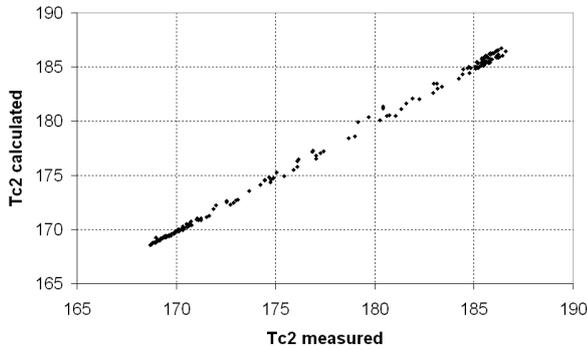
Figure 10: A comparison between water temperature at the outlet of the heat exchanger, measured and calculated (for the HE1 heat exchanger)

Figure 12: Relative difference between water temperature at the outlet of the heat exchanger, measured (m) and calculated (c) in % (for the HE1 heat exchanger)

Fig. 10 presents a comparison between the water temperature at the outlet of the HE1 heat exchanger, measured and calculated for 200 measured values. There is very good correlation between the two temperatures.

Fig. 11 presents temperature measured and calculated in time for 200 values for the HE1 heat exchanger. Fig. 11 shows a very good correspondence between these two temperatures.

Fig. 12 presents the relative error (in percent) between water temperature at the outlet of the HE1 heat exchanger, measured and calculated for 200 values at the beginning of the year. The value of the error is in the -0.6 % do 0.3 % range, which is considered a minor relative error.

The correctness of the (24) relation was checked for the HE1 heat exchanger for data at the end of the year, also for 200 measured values. Fig. 13 presents a comparison between water temperature at the outlet of the exchanger, measured and calculated from the relation (24). There is very good correlation between the two temperatures.

Fig. 14 presents the calculated from (24) relation and measured water temperature at the outlet of the HE1 heat exchanger in time for 200 values at the end of the year. There is very good correspondence between the two temperatures.

Fig. 15 presents the relative error (in per cent) between the water temperature at the outlet of the HE1 heat exchanger measured and calculated for 200 values at the end of the year. The value of the error is in the -0.3 % to 0.4 % range, which is considered a minor relative error.
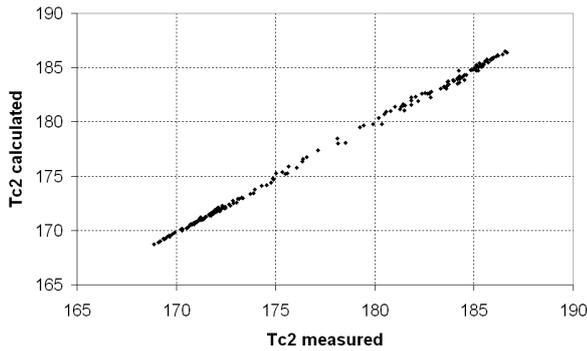
Figure 13: A comparison between water temperature at the outlet of the heat exchanger measured and calculated (for the HE1 heat exchanger) for data at the end of the year
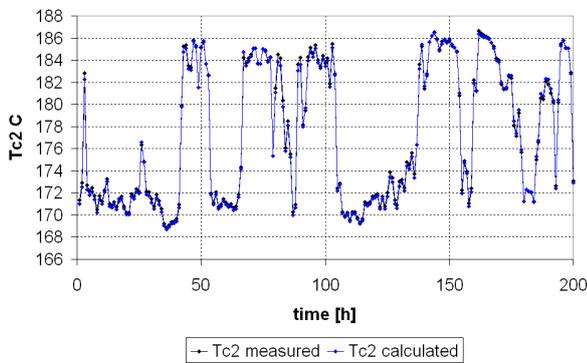
Figure 14: Change of water temperature at the outlet of the heat exchanger, measured and calculated (for the HE1 heat exchanger) for data at the end of the year

4. Conclusion

Modeling high-pressure regenerative heat exchangers in changed conditions is not simple, since in these heat exchangers three zones can be distinguished in which heat is transferred between vapor and water, condensing vapor and water and condensate and water. To receive satisfactory accuracy for the model in changed conditions, the heat exchanger should be divided into three zones and for each zone due account should be taken of overall heat transfer coefficients, which depend on heat transfer coefficients for both fluids. Heat transfer coefficients are functions of dimensionless quantities such as the Reynolds Number and the Prandtl Number. Application of relations for heat transfer coefficients is limited depending on the range of change of the parameters as well as the type of fluid flow (laminar and turbulent flow). This is complicated even further by
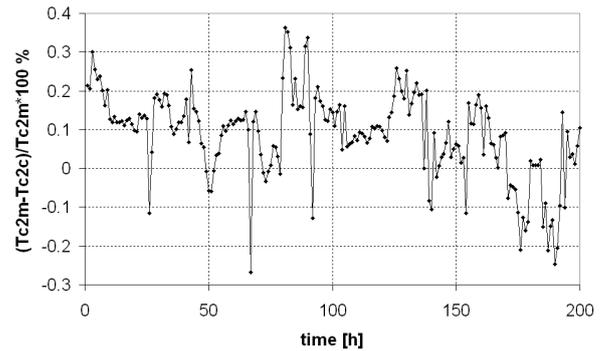
Figure 15: Relative difference between water temperature at the outlet of the heat exchanger, measured (m) and calculated (c) in % (for the HE1 heat exchanger) for data at the end of the year

a model which involves iteration. That is why the study attempts to create a simple model in changed conditions for a high-pressure regenerative heat exchanger based on measured values. For this purpose the Buckingham Π theorem was applied. A list of independent parameters was selected and on the basis of the Buckingham Π theorem a functional relation between two dimensionless quantities, where there is no overall heat transfer coefficient, was used. Using actual measurement data for two high-pressure regenerative heat exchangers, a linear relation between two dimensionless quantities is observed. Thus a simple relation allowing determination of work of a high-pressure heat exchanger in changed conditions was obtained. The following parameters are needed for determining the water temperature at the outlet of the heat exchanger in changed conditions: temperature and vapor mass flow at the inlet to the heat exchanger, temperature and water mass flow at the inlet to the heat exchanger and reference (design) parameters of the above-mentioned parameters completed with the reference temperature of water at the outlet of the heat exchanger. A comparison was performed between the water temperature, measured and calculated from the proposed linear relation at the outlet of the heat exchanger. For the HE3 heat exchanger relative error of the temperature for 200 values from the beginning of the year was in the -0.3 % to 0.3 % range, whereas for 200 values from the end of the year it was in the -0.1 % to 0.6 % range. For the HE1 heat exchanger relative error of the temperature for 200 values from the beginning of the year was in

the -0.6 % to 0.3 % range, whereas for 200 values from the end of the year it was in the -0.3 % to 0.4 % range. A simple exact model of a regenerative heat exchanger in changed conditions based on measured parameters, in which there is no overall heat transfer coefficient, was created. This model was tested for two high-pressure regenerative heat exchangers and it can be used for these types of heat exchanger.

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Nomenclature

$\dot{C}$ fluid heat capacity, W/K

$\dot{m}$ mass flow

$\dot{Q}$ heat flow, W

$\overline{\triangle T}$ mean temperature difference, K

$\overline{\triangle T_{ln}}$ logarithmic mean temperature difference, K

Π_i $(i = 1, 2)$, dimensionless quantities

ε_T correction coefficient, dimensionless

A heat transfer surface area, m^2

a, b, c, d constants, dimensionless

C constant, dimensionless

c cooler fluid

c_p specific heat at constant pressure, J/kg/K

d_i inside diameter, m

i enthalpy, J/kg

k overall heat transfer coefficient, $W/(m^2 K)$

o reference state

Q_i $(i = 1, .., n), independent variables$

T temperature, K

1 inlet to the heat exchanger

2 outlet of the heat exchanger

h hotter fluid