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The concept of a new approximate relation for exchanger heat transfer effectiveness for a cross-flow heat exchanger with unmixed fluids

Rafał Laskowski*

Institute of Heat Engineering, Warsaw University of Technology
21/25 Nowowiejska Street, 00-665 Warsaw, Poland

Abstract

This paper presents an approximate relation for the heat transfer effectiveness for a counter-flow heat exchanger, which was compared with the exact solution. Based on the obtained approximate relation for a counter-flow heat exchanger the approximate heat transfer effectiveness for a cross-flow heat exchanger in which both fluids do not mix is proposed. This approximate exchanger heat transfer effectiveness was compared with the exact solution proposed by Mason, the most well-known relation. A comparison between the most frequently used approximate formula and the exact solution proposed by Mason was made, too. The exchanger heat transfer effectiveness was analyzed for the ratio of the heat capacity rate of fluids C in the range from 0 to 1 and the number of transfer units NTU from the most common range 0–5.

Keywords: cross-flow heat exchangers, counter-flow heat exchangers, exchanger heat transfer effectiveness

1. Introduction

The exact analytical solution for the heat transfer effectiveness for a cross-flow heat exchanger was given by Nusselt in 1911 [1, 2]. In the literature some accurate relations for the heat transfer effectiveness for a cross-flow heat exchanger can be found [3–7]. Some authors divide them into two groups: the first group of the formula includes dual development of infinite nested series such as for instance Smith's formula [4] and Mason's formula [3, 5], which is undoubtedly the best known. The second group of formulas uses single development of infinite series that includes modified Bessel functions such as for instance Binnie and Poole [6] formula and Baclic [7] formula. The use of infinite series of functions as well as the Bessel function is quite difficult and therefore a new approximate formula has been sought.

The most commonly used approximate formula for a cross-flow heat exchanger has the form [3, 8–10]

$$\varepsilon = 1 - \exp\left\{\frac{NTU^{0.22}}{C}\left[e^{-C\cdot NTU^{0.78}} - 1\right]\right\} \quad (1)$$

The authorship of this relationship is not certain and is attributed to R. M. Drake [11]. The relationship consists of two constant parameters (0.22, 0.78) and the number of NTU transfer units cannot be simply determined from this formula. The second approximate formula was proposed by Alain Triboix [12] and consists of two functions with five constant parameters. The advantage of this approximate relation is the ability to designate the number of transfer units in a direct way. In this article an attempt

*Corresponding author

Email address: rlask@itc.pw.edu.pl (Rafał Laskowski)

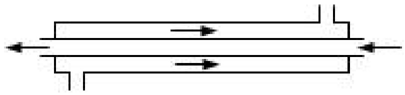

Figure 1: Diagram of a counter-flow heat exchanger with the direction of fluids along the wall

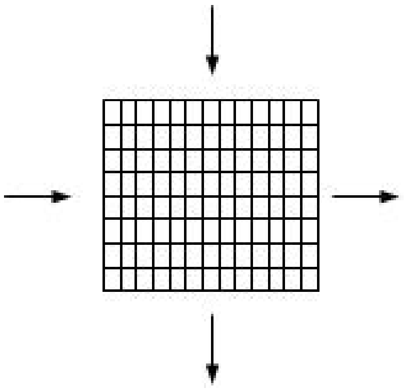

Figure 2: Diagram of a cross-flow heat exchanger with the direction of fluids along the wall

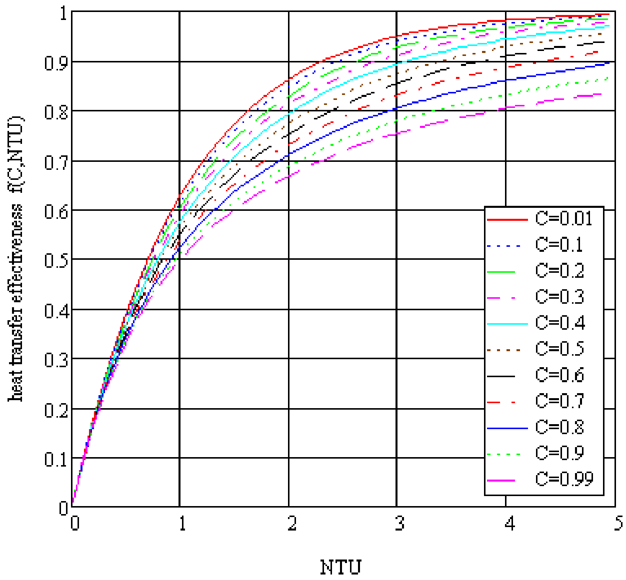

Figure 3: Heat transfer effectiveness for a counter-flow heat exchanger in the *NTU* function

has been made at formulating new approximate heat transfer effectiveness for a cross-flow heat exchanger in the form of one function with one constant parameter.

2. The mathematical model of the proposed approximate exchanger heat transfer effectiveness

2.1. Approximate heat transfer effectiveness of a counter-flow heat exchanger

Two types of heat exchangers: a counter-flow and a cross-flow heat exchangers are analyzed. In a counter-flow heat exchanger the direction of fluids along the wall is opposite. Diagrammatically a counter-flow heat exchanger with the direction of fluids along the wall is presented in Fig. 1.

In a cross-flow heat exchanger the direction of fluids along the wall is perpendicular. Diagrammatically a cross-flow heat exchanger with the direction of fluids along the wall is presented in Fig. 2.

To describe the performance (effectiveness) of a heat exchanger a function called exchanger heat transfer effectiveness is used.

The exchanger heat transfer effectiveness is the ratio of the current heat flow transferred to the maximum heat flow that could be transferred at the same temperatures at the inlet and is in the range from 0 to 1 [8, 9, 13]

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} \tag{2}$$

For the considered exchangers (recuperators) heat transfer effectiveness can be expressed as a function of two parameters: the heat capacity rate ratio of two fluids $C = \frac{\dot{C}_1}{\dot{C}_2}$ and the parameter called the Number of Transfer Units $NTU = \frac{kA}{\dot{C}_1}$

$$\varepsilon = f(C, NTU) \tag{3}$$

The heat transfer effectiveness for a counter-flow heat exchanger is determined by the relation [8, 14, 15]

$$\varepsilon = \frac{1 - e^{(C-1)\cdot NTU}}{1 - C \cdot e^{(C-1)\cdot NTU}} \tag{4}$$

Considerations based on the approximate heat transfer effectiveness for a cross-flow heat exchanger were started first from the study for heat transfer effectiveness for a counter-flow heat exchanger (4).

The dependence on the heat transfer effectiveness for a counter-flow heat exchanger is usually presented on two graphs in the coordinates of heat transfer effectiveness as a function of *NTU* (ε-*NTU*, Fig. 3) and heat transfer effectiveness as a function of

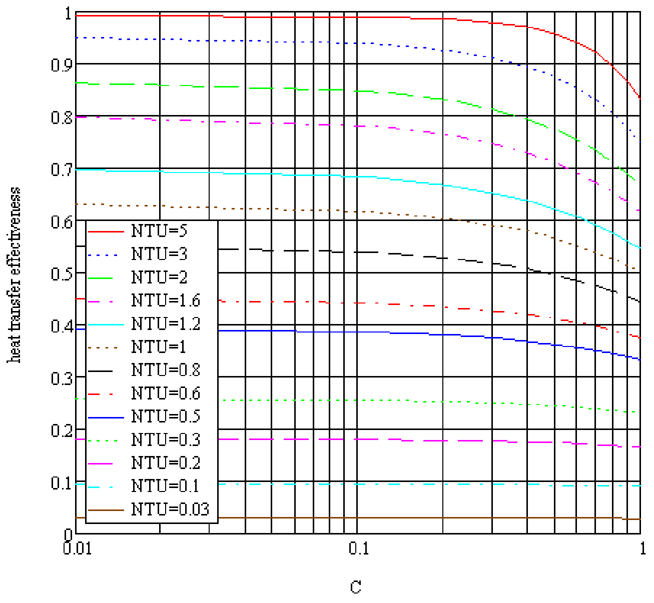

Figure 4: Heat transfer effectiveness for a counter-flow heat exchanger (logarithmic scale on the horizontal axis)

heat capacity rate ratio (ε-C, Fig. 4) where a logarithmic scale finds itself on the horizontal axis.

Equation (4) meets the following conditions:

– when the ratio of heat capacity rate tends to zero the exchanger heat transfer effectiveness takes the form

$$\lim_{C\to 0}\varepsilon = \lim_{C\to 0}\frac{1-e^{(C-1)\cdot NTU}}{1-C\cdot e^{(C-1)\cdot NTU}} = 1-e^{-NTU} \tag{5}$$

– derivative of the exchanger heat transfer effectiveness of the NTU (for $NTU = 0$) is equal (Fig. 5)

$$\frac{d\varepsilon}{dNTU}\Big|_{NTU=0} = \frac{d}{dNTU}\left(\frac{1-e^{(C-1)\cdot NTU}}{1-C\cdot e^{(C-1)\cdot NTU}}\right)\Big|_{NTU=0} = 1 \tag{6}$$

– when the ratio of heat capacity rate tends to one the exchanger heat transfer effectiveness has the following form

$$\lim_{C\to 1}\varepsilon = \begin{Bmatrix}0\\0\end{Bmatrix} \underset{=}{H} \frac{NTU}{1+NTU} \tag{7}$$

– value of the exchanger heat transfer effectiveness for $NTU = 0$

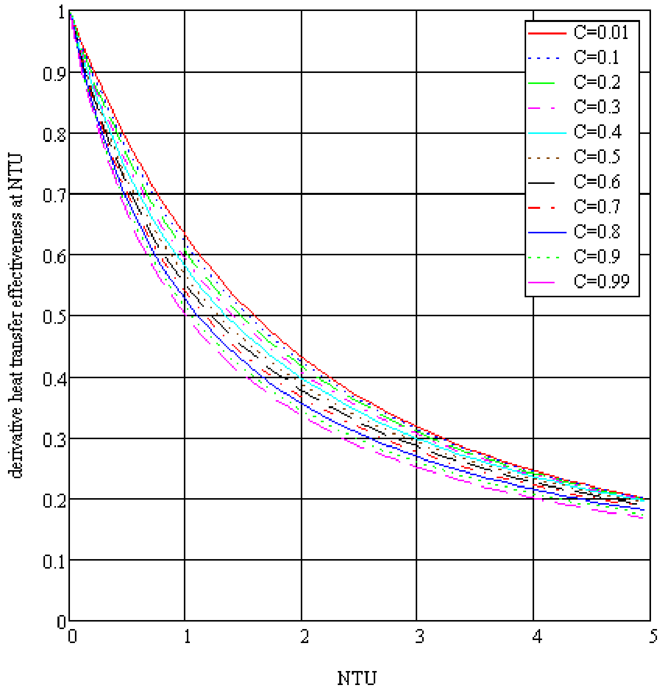

Figure 5: Derivative exchanger heat transfer effectiveness at *NTU*

$$\varepsilon(NTU=0) = \frac{1-e^{(C-1)\cdot NTU}}{1-C\cdot e^{(C-1)\cdot NTU}} = 0 \tag{8}$$

The same values of exchanger heat transfer effectiveness in linear coordinates for NTU constant values are found along straight lines (Fig. 6). Only for the value of $NTU > 3$ do we see a greater deviation from a straight line and that is why the following linear relation is proposed for describing exchanger heat transfer effectiveness

$$\varepsilon = a\cdot C + b \tag{9}$$

From the conditions for $C = 0$ and $C = 1$ a and b coefficients have been determined.

For $C = 0$ capacity rate ratio the b coefficient is equal to exchanger heat transfer effectiveness for the change in the phase of one of the fluids

$$b = 1-e^{-\frac{kA}{C_1}} = 1-e^{-NTU} \tag{10}$$

For the capacity rate ratio $C = 1$ exchanger heat transfer effectiveness takes the following form

$$\frac{1}{1+\frac{1}{\frac{kA}{C_1}}} = \frac{1}{1+\frac{1}{NTU}} = a+b \tag{11}$$

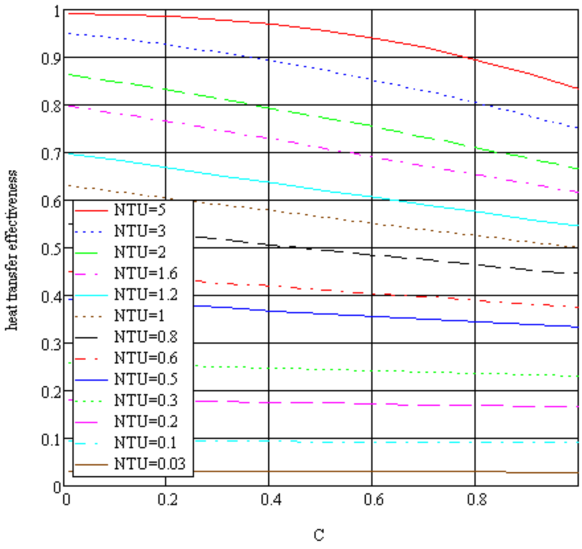

Figure 6: Heat transfer effectiveness for a counter-flow heat exchanger (linear scale on the horizontal axis)

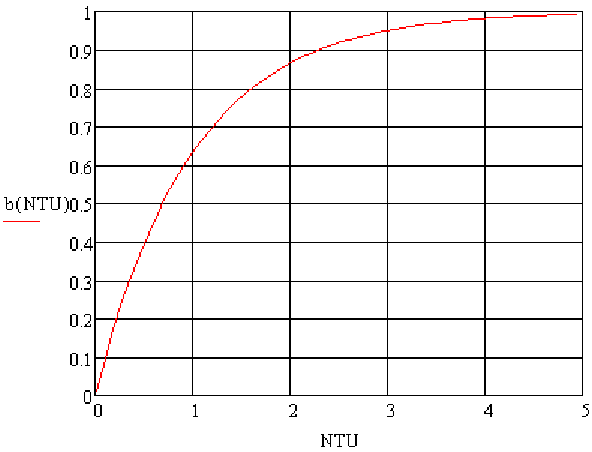

Figure 7: Values of b coefficient

Hence the slope of the straight line may be expressed as

$$a = \frac{1}{1+\frac{1}{NTU}} - \left(1 - e^{-NTU}\right) = \frac{NTU}{1+NTU} - \left(1 - e^{-NTU}\right) \qquad (12)$$

The values of b coefficient (exchanger heat transfer effectiveness for the capacity rate ratio $C = 0$) are presented in Fig. 7. Values of exchanger heat transfer effectiveness for the capacity rate ratio $C = 1$ are presented in Fig. 8. Values of a coefficient are given in Fig. 9.

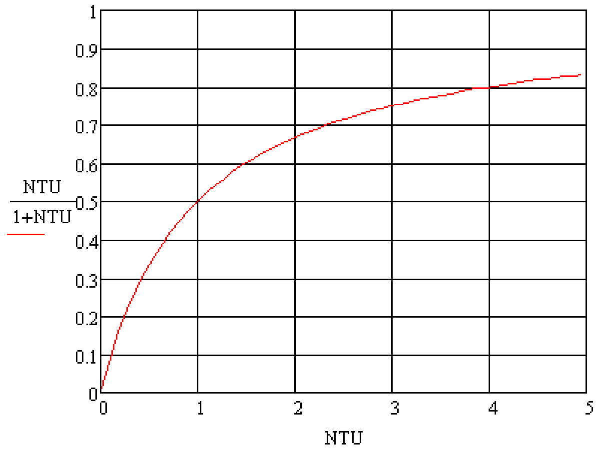

Figure 8: Changes in exchanger heat transfer effectiveness for the $C = 1$ capacity rate ratio

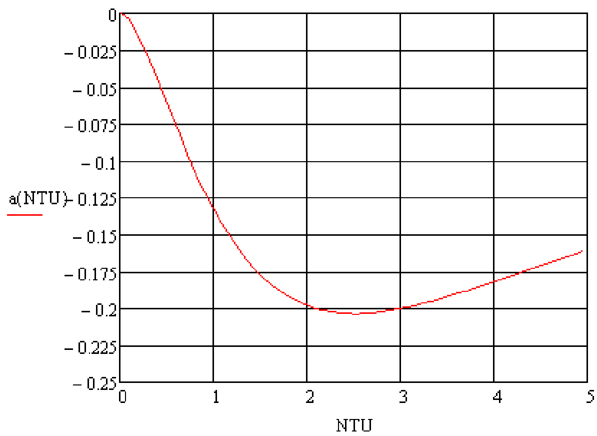

Figure 9: Values of a coefficient

After determining a and b parameters an approximate relation for heat transfer effectiveness for a counter-flow heat exchanger may be expressed as

$$\Gamma = 1 - e^{-NTU} \tag{13}$$

$$\varepsilon = \left[\frac{NTU}{1+NTU} - \Gamma\right] \cdot C + \Gamma \tag{14}$$

This relation meets the same conditions as the exact relationship (4):

$$\lim_{C\to 0} \varepsilon = 1 - e^{-NTU} \tag{15}$$

$$\lim_{NTU\to 0} \frac{\varepsilon}{NTU} = 1 \; or \; \left(\frac{d\varepsilon}{dNTU}\right)_{NTU=0} = 1 \tag{16}$$

$$\lim_{C\to 1} \varepsilon = \frac{NTU}{1+NTU} \tag{17}$$

$$\varepsilon\,(NTU = 0) = 0 \tag{18}$$

2.2. Approximate heat transfer effectiveness of a cross-flow heat exchanger

The following heat transfer effectiveness relation for a cross-flow heat exchanger (both fluids unmixed) proposed by Mason was analyzed similarly [3]

$$L = 1 - e^{-NTU} \cdot \sum_{m=0}^{n} \frac{NTU^m}{m!} \tag{19}$$

$$P = 1 - e^{-C\cdot NTU} \cdot \sum_{m=0}^{n} \frac{(C \cdot NTU)^m}{m!} \tag{20}$$

$$\varepsilon = \frac{1}{C \cdot NTU}\left(\sum_{n=0}^{\infty} L \cdot P\right) \tag{21}$$

The heat transfer effectiveness for a cross-flow heat exchanger as a function of NTU (ε - NTU) is presented in Fig. 10. The exchanger heat transfer effectiveness as a function of heat capacity flow rate ratio C (ε - C) is presented in Fig. 11, where a logarithmic scale finds itself on the horizontal axis.

Equation (21) meets the following conditions:

– when the ratio of heat capacity rate tends to zero the exchanger heat transfer effectiveness has the following form (Fig. 12)

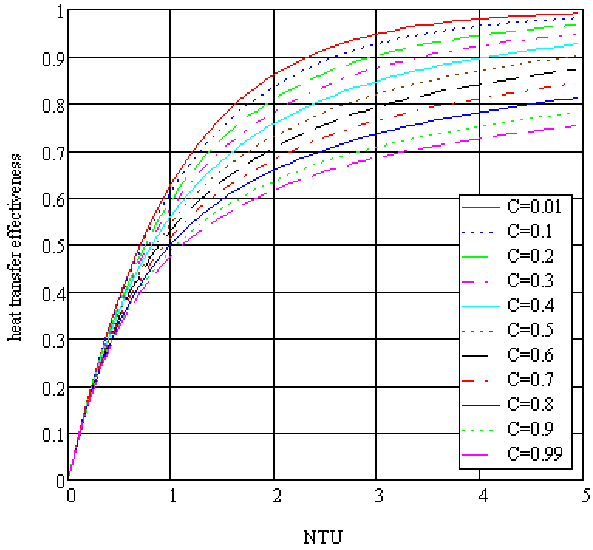

Figure 10: Heat transfer effectiveness for a cross-flow heat exchanger in the *NTU* function

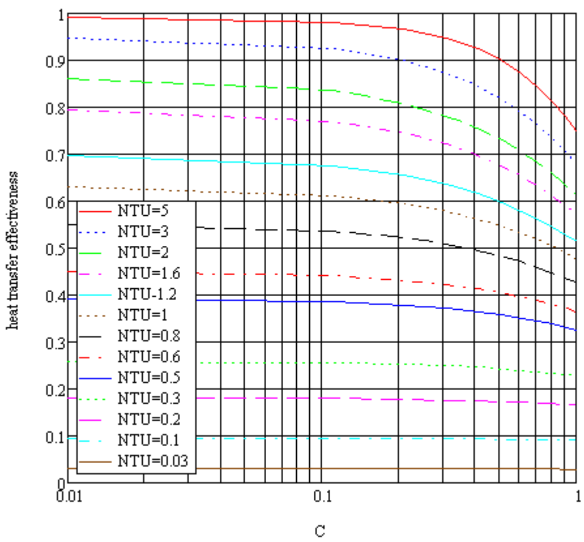

Figure 11: Heat transfer effectiveness for a cross-flow heat exchanger (logarithmic scale on the horizontal axis)

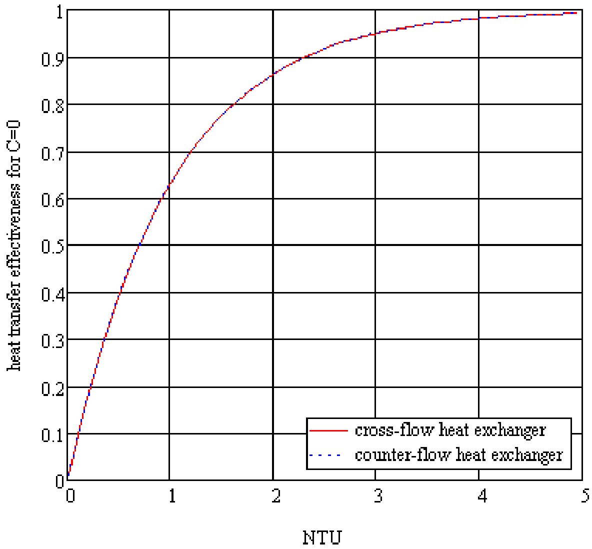

Figure 12: Changes in exchanger heat transfer effectiveness for the C=0 capacity rate ratio

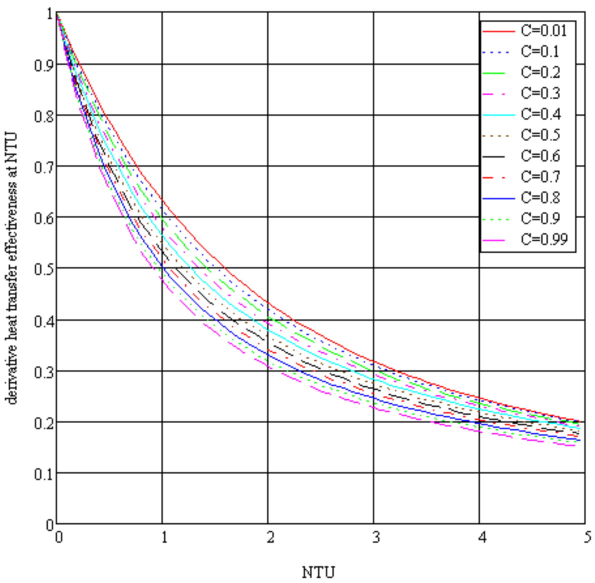

Figure 13: Derivative of exchanger heat transfer effectiveness at NTU

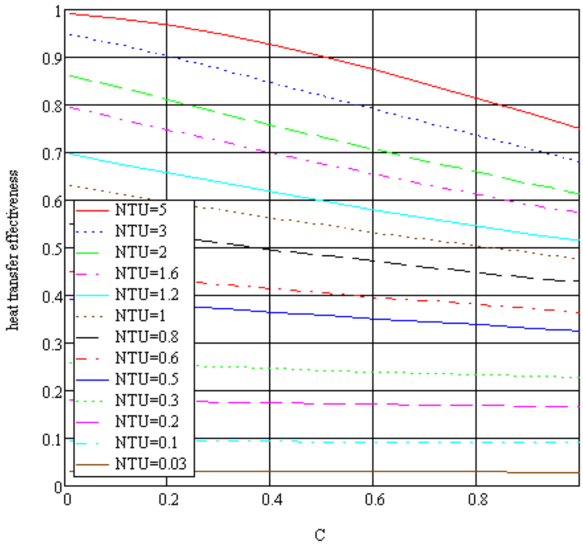

Figure 14: Heat transfer effectiveness for a cross-flow heat exchanger in the function of C capacity rate ratios (linear scale on the horizontal axis)

$$\lim_{C\to 0}\varepsilon = 1 - e^{-NTU} \tag{22}$$

– derivative of the exchanger heat transfer effectiveness of the NTU (for $NTU = 0$) is equal (Fig. 13)

$$\lim_{C\to 0}\frac{\varepsilon}{NTU} = 1 \; or \left(\frac{d\varepsilon}{dNTU}\right)_{NTU=0} = 1 \tag{23}$$

– when the ratio of heat capacity rate tends to one the exchanger heat transfer effectiveness has the following form

$$\lim_{C\to 1}\varepsilon = \frac{1}{NTU}\left(\sum_{n=0}^{\infty}\left(1 - e^{-NTU}\cdot\sum_{m=0}^{n}\frac{NTU^{m}}{m!}\right)^{2}\right) \tag{24}$$

– when NTU tends to zero the exchanger heat transfer effectiveness is equal to zero

$$\lim_{NTU\to 0}\varepsilon = 0 \tag{25}$$

In Fig. 14 changes in exchanger heat transfer effectiveness in C function and NTU constant capacity rate ratios are presented.

As in the case of a counter-flow heat exchanger (Fig. 6) the points lie along straight lines. Therefore

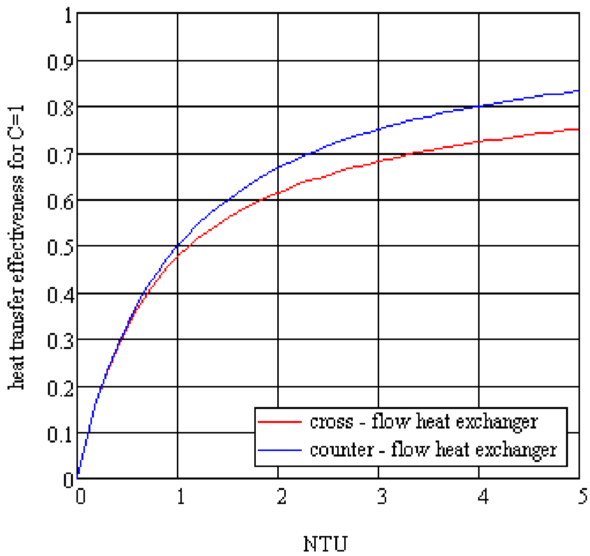

Figure 15: Comparison of heat transfer effectiveness values for a counter-flow heat exchanger and a cross-flow heat exchanger for C=1

it is proposed to approximate heat transfer effectiveness for a cross-flow heat exchanger using a linear relation (9).

In Fig. 15 changes in heat transfer effectiveness for a counter-flow heat exchanger and a cross-flow heat exchanger for the condition of $C = 1$ capacity rate ratio have been presented.

Changes in heat transfer effectiveness for a cross-flow heat exchanger are similar to changes in heat transfer effectiveness for a counter-flow heat exchanger for $C = 1$, but they assume smaller values for relevant NTU values (Fig. 15).

In order to simplify the approximate heat transfer effectiveness for a cross-flow heat exchanger for $C = 1$ (24) it was assumed that this effectiveness is described by relation (7) with a constant parameter m in the form of

$$\frac{NTU}{1 + m \cdot NTU} \tag{26}$$

The place of m parameter was determined on the basis of four conditions (22 – 25). The value of m parameter was determined by means of the least squares method on the basis of the data presented in Fig. 15 and the value of 1.1238 was obtained.

Conclusively an approximate relation for heat transfer effectiveness for a cross-flow heat exchanger may be presented as

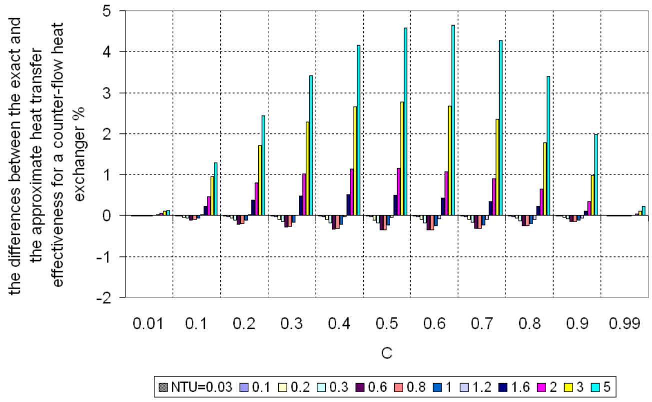

Figure 16: Comparison between the exact (4) and approximate (14) relation for heat transfer effectiveness for a counter-flow heat exchanger

$$\Gamma = 1 - e^{-NTU} \tag{27}$$

$$\varepsilon = \left[\frac{NTU}{1 + m \cdot NTU} - \Gamma\right] \cdot C + \Gamma \tag{28}$$

3. Results

A comparative analysis of the proposed approximate formulas with the exact relations was performed.

The percentage differences between the exact (4) and approximate (14) relation heat transfer effectiveness for a counter-flow heat exchanger are presented in Fig. 16. It is known from the presented Fig. 16 that the differences between relations (4) and (14) grow with the increase in NTU. The biggest difference is around 5 % for $\frac{kA}{\dot{C}_1} = 5$. For $NTU < 2$ the difference is about 1 %.

The percentage differences between the exact (21) and the approximate (28) relations for heat transfer effectiveness for a cross-flow heat exchanger (both fluids unmixed) are presented in Fig. 17.

The biggest difference between relationships (18) and (24) is for $\frac{kA}{\dot{C}_1} = 5$ and is about 3 %.

In Fig. 18 the percentage differences between the exact solution proposed by Mason for heat transfer effectiveness for a cross-flow heat exchanger (both fluids unmixed) and the approximate relation (1).

The comparison between the exact (21) and the proposed approximate (28) relation for heat transfer

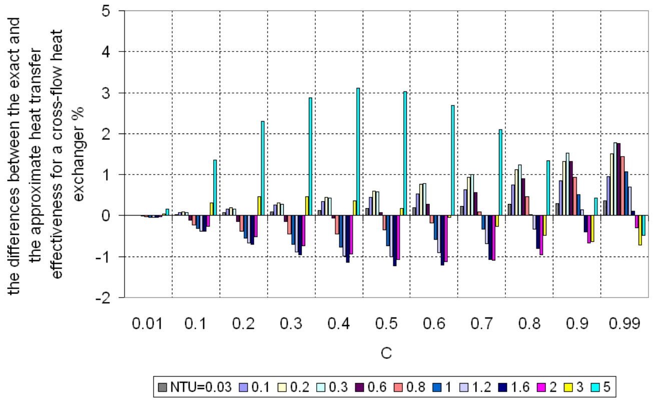

Figure 17: Comparison between the exact (21) and the approximate (28) relation for heat transfer effectiveness for a cross-flow heat exchanger

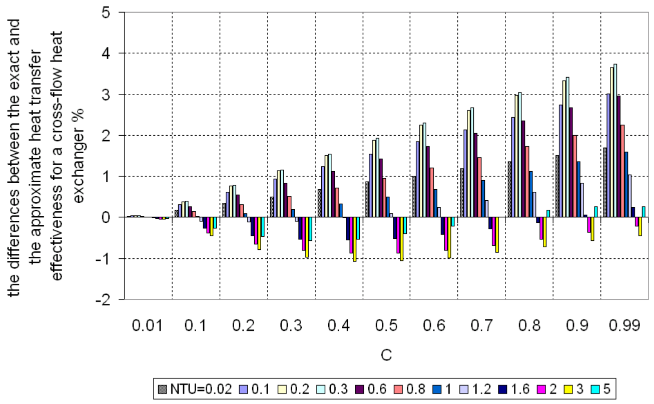

Figure 18: Comparison between the exact (21) and the approximate (1) relation for heat transfer effectiveness for a cross-flow heat exchanger

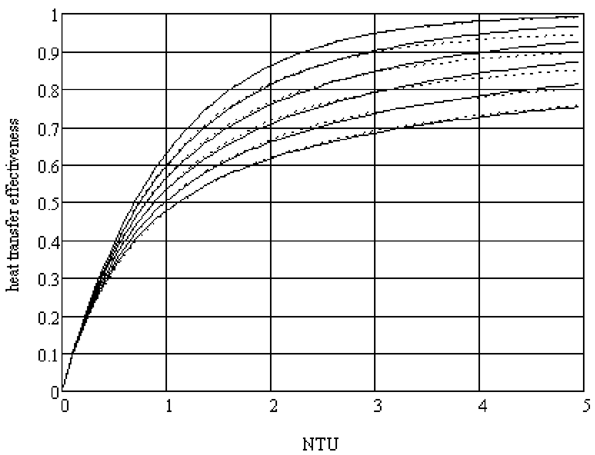

Figure 19: Comparison between the exact (21) and the approximate (28) relation for heat transfer effectiveness for a cross-flow heat exchanger in the *NTU* function, continuous line – exact relation (21), dotted line – the approximate relation (28)

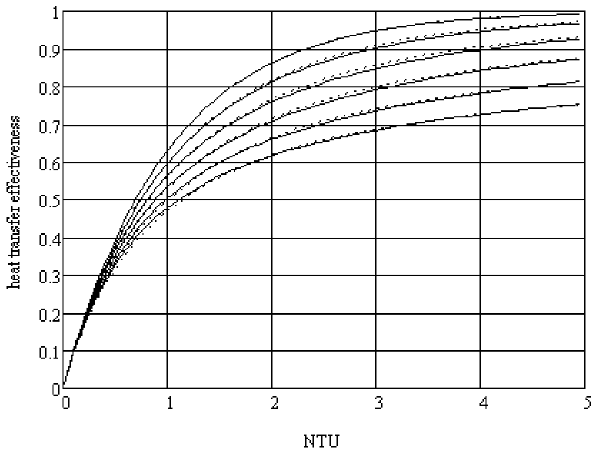

Figure 20: Comparison between the exact (21) and the approximate (1) relation for heat transfer effectiveness for a cross-flow heat exchanger in the *NTU* function, continuous line – exact relation (21), dotted line – the approximate relation (1)

effectiveness for a cross-flow heat exchanger in the *NTU* function is presented in Fig. 19.

The comparison between the exact (21) and the proposed approximate (1) relation for heat transfer effectiveness for a cross-flow heat exchanger in the *NTU* function is presented in Fig. 20.

4. Conclusions

The article presents the approximate formula of heat transfer effectiveness for a counter-flow and cross-flow heat exchanger. The exchanger heat transfer effectiveness was analyzed for the most commonly used range of *NTU* from 0 to 5.

For a counter-flow heat exchanger the biggest difference between the exact (4) and approximate (14) relation for heat transfer effectiveness is about 5 % for $NTU = 5$ (Fig. 16). For $NTU < 2$ the difference is about 1 %. For values of $NTU > 3$ the exchanger heat transfer effectiveness loses its linear character and with the growth of *NTU* parameter the difference increases reaching the maximum value of about 5 % for $NTU = 5$ (Fig. 6, 16). The proposed approximate relation (14) meets the same conditions (5 – 8) as the exact relation (4).

Based on the approximate relation for heat transfer effectiveness for a counter-flow heat exchanger (14) the approximate relation for heat transfer effectiveness for a cross-flow heat exchanger was created (28) and compared with the exact relation proposed by Mason (21). For this case, the largest difference is about 3 % (Fig. 17). Just like for a counter-flow heat transfer effectiveness (14) the heat transfer effectiveness for a cross-flow heat exchanger (28) loses its linear character for $NTU > 3$ and the difference reaches the maximum value of about 3 % for $NTU = 5$ (Fig. 14, 17). The proposed approximate formula (28) meets the same conditions (22 – 25) as the exact formula (21).

A comparison was also performed between the most frequently used approximate formula (1) and the exact relation (21) for heat transfer effectiveness for a cross-flow heat exchanger. The largest difference is about 4 % for the value of NTU in the range from 0.2 to 0.3. For values of $NTU > 3$ the difference is about 1 % (Fig. 18).

For heat transfer effectiveness for a cross-flow heat exchanger the proposed formula (28) is more accurate than the most commonly used approximate relation (1) for the value of $NTU < 3$. For values of $NTU > 3$ the most often used approximate relation (1) is more accurate (Fig. 17, 18).

In the proposed approximate formula (28) there is one constant parameter $m = 1.1238$, whereas in the approximate formula (1) there are two constant parameters 0.22 and 0.78. The proposed approximate formula (28) is more convenient and faster to use than solutions involving infinite series.

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Nomenclature

A–heat transfer surface area, m^2
C–heat capacity rate ratio, dimensionless
$\dot{C}$–fluid heat capacity (1 – smaller, 2 – greater), W/K
L–parameter, dimensionless
NTU–number of heat transfer units, $NTU = \frac{kA}{\dot{C}_1}$
P–parameter, dimensionless
k–overall heat transfer coefficient, $W/(m^2K)$
ε–exchanger heat transfer effectiveness, dimensionless
Γ– parameter, dimensionless
$\dot{Q}$–heat flow, W
$\dot{Q}_{max}$–maximum heat flow, W