

# FAST TERMINAL SYNERGETIC BASED CONTROL OF COLEMAN -HODGDON HYSTERESIS MODEL

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## Abstract

A new fast terminal synergistic controller for trajectory tracking of a P.E.A. piezoelectric positioning mechanism with a Coleman-Hodgdon hysteresis model is proposed in this manuscript. The control proposed in this paper is based on classical synergistic control, it is considered a powerful robust control design methodology, with a choice of nonlinear macro-variable can often be adjusted appropriately to achieve a fast convergence rate, the system states can reach the equilibrium point in a finite time. The proposed control law is capable of handling disturbances and uncertainties, which allows the system state to converge in finite time. The implementation conditions of the control developed in this work are studied and the closed-loop stability is guaranteed by the Lyapunov synthesis. A comparison between the terminal fast synergetic control and the classical synergetic control shows the efficiency and robustness of the proposed control in terms of overshoot and response time and that the performance errors of the terminal fast control are improved.

**Keywords:** Fast Terminal Synergistic Control, Coleman-Hodgdon Hysteresis Model, Piezoelectric Positioning Mechanism, Finite Time Convergence, Robust Control Design

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## Introduction

Precision motions over extended strokes, on the order of micrometers to nanometers, with the utmost consistency, are the hallmark of piezoelectric positioning devices. Positioning mechanisms are used in a wide range of industrial fields, including research and laboratory technology [1, 2]. This is a significant area of use for our positioning systems, similar to the semiconductor industry [4]. New application opportunities in single cell technology and surface analysis are made possible by atomic force microscopy [5] and microfluidics [6]. Piezo actuator positioning

mechanisms are linked, nonlinear, and their description's parameters change depending on the load [7]. On the other hand, a mathematical model is nothing more than a rough approximation of physical reality, however, we can only use this model to create a law of control. Consequently, according to the system's physics, which we will have to take into account, the control must be robust in the sense that it must ensure low sensitivity to uncertainties on the parameters, to their variations and to disturbances. In reality it is usually challenging to fully understand all the variables involved in a process and to appropriately depict it. As a result, in order to overcome non-linearities or

identification mistakes, the rule controlling it must be strong. [8] As a result, various nonlinear control techniques have been introduced in the literature in recent years [9, 10], including sliding mode control [15, 16], model predictive control [13, 14], and control by reverse [11, 12]. For nonlinear systems, several iterations of these control systems have been created and updated. Because of its straightforward design process, resistance to external perturbations, simplicity of implementation in real-world systems, and other advantages, sliding mode control (SMC), one of the many techniques put forth, has drawn significant attention from both academic and industry circles, and low sensitivity to parameter variations [17, 18]. Traditional sliding mode control has some major drawbacks: theoretically, the equilibrium point is only reached after an infinite time, and chattering phenomena cause a large control amplitude [19]. A nonlinear sliding surface is used in order to achieve finite-time convergence, which we call the terminal sliding mode controller. Convergence performance can be improved by the terminal approach, this is demonstrated by the proposal of the following recent publications [20, 21, 22], with the aim of reducing steady-state errors and having stability and fast and dynamic response. Synergetic control is conceptually similar to sliding mode control without its main drawback: chattering phenomenon. It has only recently emerged as a powerful methodology for a robust control design. Inspired by these ideas, we present a contribution in the elaboration of the synergetic control to the positioning mechanism which offers superior properties such as speed, finite time convergence, and increased accuracy [23]. Nevertheless, a disadvantage of this control remains consisting of the possible presence of a singularity that is overcome by the contribution of a terminal evolution constraint [24, 25]. The proposed approach uses an integral technique of a non-singular terminal attractor which ensures convergence in finite time and provides a control signal without chattering. The principle of this controller is implemented and developed,

and its use is illustrated on a tracking problem of a piezo actuator positioning mechanism which contains a nonlinear C-H model. The stability of the controller is checked using Lyapunov stability theory. To verify the performance and efficiency of the fast terminal synergetic control, a comparative study between the developed control and the synergetic control is presented.

### Modeling and identification

The C-H model of PEA is described by the following equations:

$$x(u) = \begin{cases} \left( C_1 - C_2 e^{-C_3 V_f} \right) u - C_4 \left( 1 - \frac{2e^{-C_5 u}}{e^{-C_5 V_m} + e^{-C_5 V_M}} \right) & \text{si } \dot{u} \geq 0 \\ \left( C_1 - C_2 e^{-C_3 V_f} \right) u - C_4 \left( 1 - \frac{2e^{C_5 u}}{e^{C_5 V_m} + e^{C_5 V_M}} \right) & \text{si } \dot{u} < 0 \end{cases} \quad (1)$$

where  $x$  is the mechanism's output displacement,  $u$  is the control input and its derivative for the PEA,  $V_f = V_M - V_m$  is the voltage range, and  $V_M, V_m$  are the maximum and minimum input voltages, and  $C_1 - C_5$  are constant parameters. The application of the PSO algorithm in the determination of the optimal position has been described in [26]. In [27] the experimental model validates the configuration used for identification and modeling the C-H model PEA. Figure 1 compares the C-H model's tracking performance to that of experiments when the piezoelectric positioning stage is driven by a sinusoidal input. The experimental results match the C-H modeling results very well.

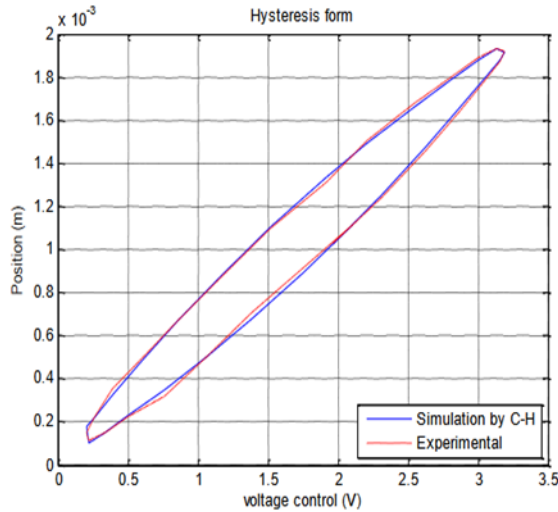


Figure 1: Experimental and PSO-based C-H model hysteresis with a 1 Hz input voltage

### Description system control

The dynamic of the nonlinear system can be expressed in the following state space model:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1, x_2, t) + g(x_1, x_2, t)u + d(t) \end{cases} \quad (2)$$

Where  $x = [x_1, x_2]^T$  the state space vector, the vector of outputs is given by,  $y = [x_1, x_2]^T$   $u$  and  $d(t)$  the control input and disturbances. The dynamic behavior of our system is described by a nonlinear C-H model. This model requires robust control and must be insensitive and stable to the variation of the parameters and disturbances. These expected characteristics of the control can be realized by a synthesis based on the technique of synergistic control [28]. Synergetic control theory is one of the new promoter options in modern and emerging control theory. The purpose of the control is to regulate the tracking of the trajectory of the position of the PEA.

We define the following error:

$$\xi = x_{ref} - x \quad (3)$$

The proposed approach will be designed here so that the states of the system approach exponentially a specified constraint in a finite time according to the following equation:

$$\tau \Gamma^q + \Gamma = 0 \quad (4)$$

$\tau$  is a design parameter that indicates the rate of convergence towards the attractor specified by the macro variable.  $p$  and  $q$  are odd positive constants, such that  $1 < p/q < 2$ .

The macro variable  $\Gamma$  is expressed by:

$$\Gamma = \dot{\xi} + k_0 \xi \quad (5)$$

$k_0$  are positive constants chosen by the designer. The goal of synergetic control is to force the system to evolve in the domain specified by the equation beforehand by the designer (4). Using equations (4) and (5) gives the following results

$$\dot{\Gamma} = \ddot{\xi} + k_0 \dot{\xi} = \left( \frac{-\Gamma}{\tau} \right)^{\frac{p}{q}} \quad (6)$$

Equations (5) and (6) lead to the following control law

$$u = \frac{1}{g(x_1, x_2, t)} \left[ \left( \frac{\Gamma}{\tau} \right)^{\frac{p}{q}} + \ddot{x}_{ref} - f(x_1, x_2, t) + k_0 \dot{\xi} + d(t) \right] \quad (7)$$

### Theorem

The control application law (7) to a nonlinear PEA system second order (1), the error rate asymptotically converges to zero in finite time

and the system errors, the convergence rate depends on  $\tau, p$  and  $q$  parameters.

Proof:

Consider the Lyapunov candidate function as:

$$V = \frac{1}{2} \Gamma(\xi)^2 \tag{8}$$

We derive (8), we get:

$$\dot{V} = \Gamma(\xi)\dot{\Gamma} \tag{9}$$

We replace (9) in (6), we get:

$$\begin{aligned} \dot{V} &= \Gamma(\xi)(-\tau^{-1})^p \\ &\leq (-\tau^{-1})^p \Gamma \frac{q}{p} \\ &\leq -\tau^p \frac{q}{2} \frac{(p+q)}{2p} \frac{1}{2} (\Gamma^2) \frac{(q+p)}{2p} \\ &\leq -\tau_1 V_1(t) \frac{(p+q)}{2p} \end{aligned} \tag{10}$$

Where

$$\tau_1 = \tau \frac{-q}{p} \frac{(p+q)}{2p}$$

from equation (4) we have  $1 < p/q < 2$ ,  $\forall \Gamma$  the singularity problem does not exist for this control

$$\dot{V}(t) = -k_2 V^\delta(t), \forall t > t_0, V(t_0) \geq 0 \tag{11}$$

Lemma:

Assume inequality is obeyed by a continuous, positive, and definite function.

Where  $k_2 > 0, 0 < \delta < 1$  are dependable, next, for each,  $t_0, V(t)$  fulfills the ensuing inequality

$$V^{1-\delta} < V^{1-\delta}(t_0) - k_2(1-\delta)(t-t_0) \tag{12}$$

$t_0 < t < t_1$  And  $V(t) = 0, \forall t \geq t_1$  with  $t_1$  given by

$$t_1 = t_0 + \frac{V^{1-\delta}(t_0)}{k_2(1-\delta)} \tag{13}$$

We can see from the lemma that the synergistic manifold can converge to zero at a finite time  $t_1$  given by

$$t_1 = \frac{(1 - \frac{p+q}{2p})(0)}{\tau_1(1 + \frac{p+q}{2p})} \tag{14}$$

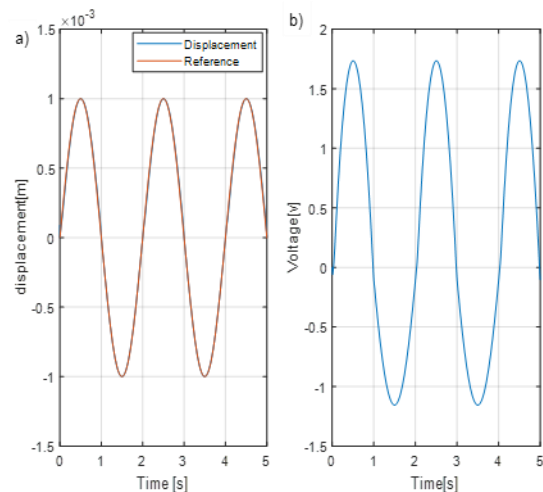


Figure 2: Periodic sinusoidal control results at 1mm, 0.5 Hz for  $p=5, q=7$ : Tracking response (a); control effort (b).

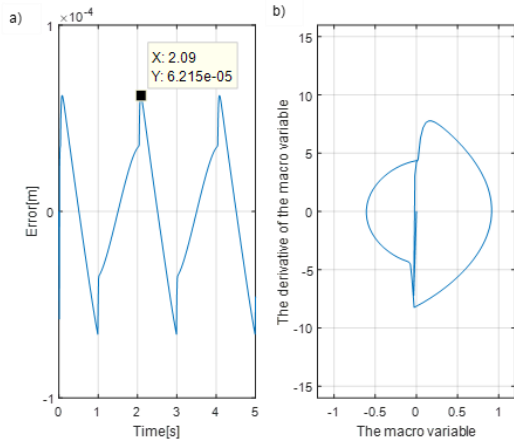


Figure 3: Periodic sinusoidal control results at 1mm, 0.5 Hz for  $p=5$ ,  $q=7$ : a) tracking error b) phase plane ( $r.d\Gamma$ ).

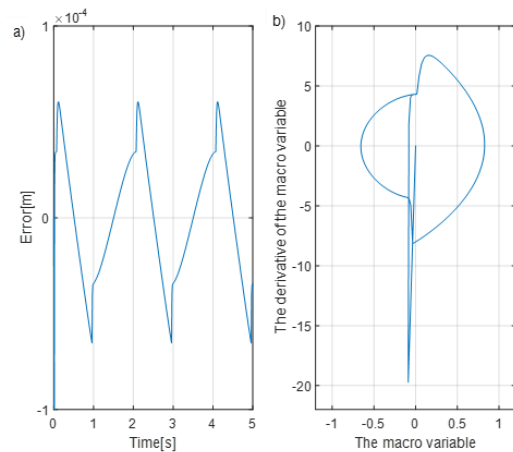


Figure 5: Periodic sinusoidal control results with 10 N load at 1mm, 0.5 Hz for  $p=5$ ,  $q=7$ : tracking error b) phase plane ( $r.d\Gamma$ ).

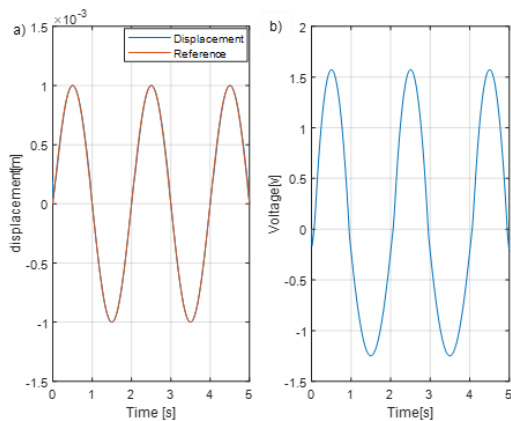


Figure 4: Sinusoidal periodic control results with 10 N load for conditions at 1mm, 0.5 Hz for  $p=5$ ,  $q=7$ : (a) tracking response; (b) control effort.

## Simulation results

The piezo-electric positioning mechanism containing the C-H hysteresis model was simulated using Matlab/Simulation. We apply a sinusoidal reference signal of amplitude 1000-micron meter with a frequency of 0.5 Hz. The displacement tracking performance is shown in figure 2.a, where one can see a fast convergence of the system output to the reference signal. Figure 2.b shows the control input of the system, which is bounded, and of smooth shape. The tracking error is shown in Figure 3.a, it quickly converges to a value of  $6.215 \times 10^{-5}$  m. We have seen in Figure 4.a, the results of the responses of the system by an application of a disturbance of a load of 10 N, and the control input, Figure 4.b. The controller reacts and forces the system to return to the desired trajectory, this is shown by the phase diagram in figure 5.a. Figure 6 shows the results of the classical synergistic control system responses ( $p=q=1$ ).

## Comparison study

In order to highlight the performance of the control law, we are comparing our results of the proposed algorithm with that of the classic synergetic control under the same conditions (frequency, simulation time). We can see that

the best tracking performance is obtained in our work, that all signals in the closed-loop system are bound. This illustrates the effectiveness of the proposed technique. For better understanding of this comparative study, see Figure 7. It can be seen that the proposed controller makes it possible to confer the best performance in terms of tracking errors and control effort.

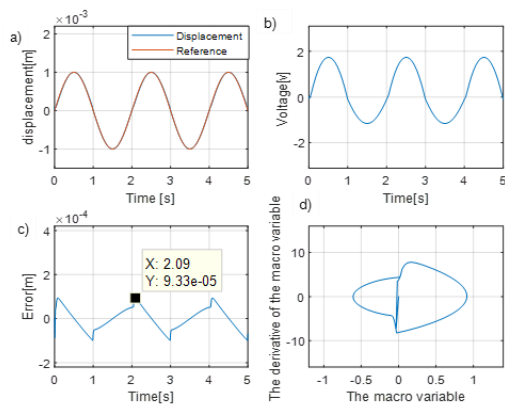


Figure 6: Periodic sinusoidal control results at 1mm, 0.5 Hz for  $p = q = 1$ : tracking response; tracking effort; tracking error the plane of phase ( $r, \dot{r}$ )

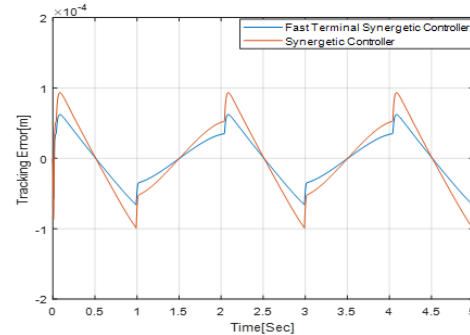


Figure 7: Tracking error evolution for the fast terminal synergetic controller and the classic synergetic controller

## Conclusions

The work discussed in this article consists of a robust fast terminal synergetic controller based on classical synergetic control and a nonlinear macro-variable equation of choice. The algorithm developed was used on the P.E.A. The simulation results have highlighted the system's state to converge in a finite time. A comparison of the two fast terminal synergetic controllers and traditional synergetic techniques revealed that the fast terminal synergetic controller achieves the best understanding between tracking error in displacement and control effort provided.

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