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Effects of Variable Fluid Properties and Mixed Convection on Biomagnetic Fluid Flow and Heat Transfer over a Stretching Sheet in the presence of Magnetic Dipole

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Abstract

This investigation covers the numerical analysis of a steady biomagnetic fluid flow (BFD) that passed through a two dimensional stretching sheet under the influence of magnetic dipole. The effect of fluid variable viscosity and thermal conductivity are also taken into consideration as assumed to vary as linear function of temperature. Our model mathematically formulated for BFD namely blood which consist of principles of magnetohydrodynamic (MHD) and ferrohydrodynamic (FHD), where blood treated as an electrically conducting fluid as well as polarization. Using similarity transformations, the governing system of partial differential equations are transferred into system of ordinary differential equations (ODE). The resulting coupled non linear ODE is numerically solved by employing bvp4c function technique available in MATLAB software. The effects of pertinent parameters namely ferromagnetic interaction parameter, magnetic field parameter, mixed convection parameter, viscosity variation parameter, Prandtl number, thermal conductivity parameter etc are plotted and discussed adequately for velocity and temperature profile as well as skin friction coefficient and rate of heat transfer. The results revels that velocity profile decreases as enhanced values of ferromagnetic number whereas temperature profile increased. Also found that skin friction coefficient reduces and rate of heat transfer increases by increasing values of thermal conductivity parameter and viscosity variation parameter. For numerical validation a comparison has been made for some specific values with previous investigators. We hope that the present analysis will present in bio-medical and bio-engineering sciences.

Keywords: Stretching sheet, Biomagnetic fluid, Magnetohydrodynamic, Ferrohydrodynamic, Magnetic dipole, Magnetization, Variable viscosity, Thermal conductivity

Introduction

Over the last few decades, research on biological fluid (which is also part of BFD) in presence of applied magnetic field has been adoption serious attention from research due to its numerous applications in medical and bio-engineering, for example: magnetic resonance imaging (MRI), in cancer tumor treatment (elctromagnetic hypothermia), magnetic particles used as drug delivery, development of magnetic devices for cell separation etc as early mentioned by [1-3]. It is an in disciplinary field of BFD which directly connected to finding and developing accomplishment of human body related diseases and disorders. Blood is one of the common peculiarities of BFD because blood behaves as a magnetic fluid due to presence of ions in plasma which slightly produce current.

For this reason, several mathematical model of BFD has been proposed by researchers which incorporate with principles of MHD and FHD. Haik et al. [1] was the first developed a mathematical model of BFD which consist principle of ferrohydrodynamic (FHD) and the dominant force in flow field is that of magnetization. Later on, Tzirtzilakis [2] explore this model with combining principle of ferrohydrodynamic (FHD) and magnetohydrodynamic (MHD). In that study he proposed that blood flow can be reduce up to 40% under the influence of strong magnetic field. A mathematical analysis of heated ferrofluid under the influence of magnetic dipole through a two dimensional linear stretching sheet perused by Tzirtzilakis et al. [3]. Studies on arterial blood flow with composited stenosis are mathematically presented by Rahman et al. [4]. To see flow feature of blood, Prallhad et al.[5] developed a arterial stenosis model.

The behavior of boundary layer over a two dimensional stretched surface was conducted by the mathematician Sakiadis [6, 7] in 1961. Further, Crane [8] elongated the idea of [6, 7] and the problem possesses to an exact solution with considering stretched velocity proportional to distance of origin. An incompressible two dimensional boundary layer flow over a stretching sheet under the influence of variable heat flux and variable wall temperature presented by Sharidan et al. [9] and numerically solved by using Keller-box method. The effect of variable fluid properties on continuous moving stretched surface examined by Carragher et al. [10] and Grubka et al. [11]. Brady et al. [12] presented an exact solution of the Navier-Stokes equations flow problem through a channel or tube with considering surface accelerated with velocity using similarity transformation.

Studies on MHD flow and heat transfer problem in a boundary layer has gained a tremendous attraction from researcher in last few decades owing to its wide range of applications especially in the area of chemical engineering, thermal insulation, power generation, metallurgy etc. The effect of fluid viscosity and thermal conductivity of an electrically conducting fluid through a continuous stretched sheet in presence of a magnetic field carried out numerically by Salem [13]. Anjali et al. [14] studied the MHD flow, heat and mass transfer over a two dimensional stretched surface in porous media under the influence of viscous dissipation. The effects of variable fluid properties like fluid viscosity, thermal conductivity, thermal radiation etc. on MHD flow over a steady/unsteady stretched surface including cylindrical surface/ oscillating surface under several boundary conditions has been numerically investigated by several researchers such as Mukhopadhay et al. [15], Malik et al. [16], Singh et al.[17], Ahmmed et al.[18], Ali et al.[19], Gul et al.[20], Mahmoud[21], Abel et al.[22], Makinde et al.[23], Abo-Eldahab et al.[24], Rana et al. [25], Alinejad et al.[26] and Gireeshal et al.[27] and found that the fluid velocity and temperature profile as well as skin friction coefficient and rate of heat transfer are significantly changed under the influence of above parameters.

The ultimate aim of the present analysis is to seek the effect of variable fluid properties on biomagnetic fluid flow over a two dimensional stretched sheet under the influence of magnetic dipole. The governing partial differential equations are converted into ordinary differential equations using suitable similarity transformations and numerically solved in MATLAB software by employing bvp4c function technique and numerical results are shown in graphical and tabular form. Numerical code also validates with some existing work of previous literature in order to check the accuracy of the solution.

Mathematical Formulation of the problem

Suppose an steady, incompressible, electrically conducting biomagnetic fluid (blood) flow and heat transfer past through a two dimensional stretching sheet with velocity $u = cx$ where c is a constant as shown in Fig. 1. Also suppose that the velocity components u and v are represents respectively in X - and Y -axes. T_w is the temperature of the sheet which kept fixed and the temperature of the ambient fluid is T_∞ which situated far away from the surface, where $T_w < T_\infty$. A magnetic dipole locate at below the sheet at distance d which generated by magnetic field of strength.

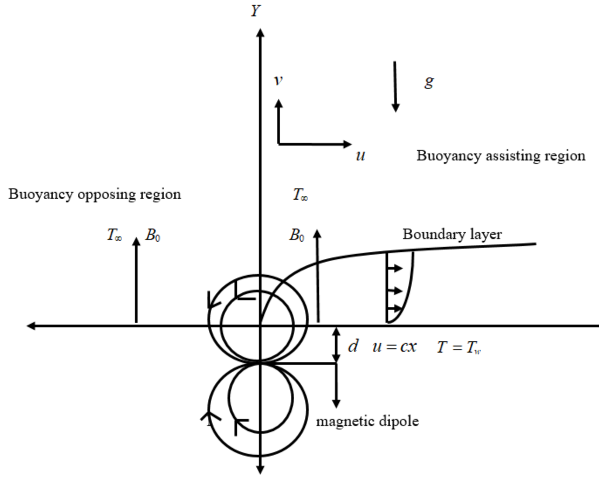

Fig. 1 Physical configuration and co-ordinate system

Under the above assumption we explore the idea of [28]and [35] and hence the governing boundary layer equations i.e. continuity, momentum and energy equations are in following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+ g\beta^*(T-T_\infty)-\frac{\sigma B_0^2}{\rho_\infty}u+\frac{\mu_0}{\rho_\infty}\mathrm{M}\frac{\partial \mathrm{H}}{\partial x} \tag{2}$$

$$\rho_\infty C_p\left(u\frac{\partial \mathrm{T}}{\partial x}+v\frac{\partial \mathrm{T}}{\partial y}\right)+\mu_0 T\frac{\partial \mathrm{M}}{\partial \mathrm{T}}\left(u\frac{\partial \mathrm{H}}{\partial x}+v\frac{\partial \mathrm{H}}{\partial y}\right)=\frac{\partial}{\partial y}\left(\kappa\frac{\partial \mathrm{T}}{\partial y}\right) \tag{3}$$

With applicable boundary conditions are:

$$y=0:\quad u=u_w=cx\,,\, v=0\,,\, T=T_w$$
$$y\to\infty:\; u\to 0\,,\, T\to T_\infty \tag{4}$$

Here, fluid density is ρ_∞ , κ is the thermal conductivity, B_0 is the uniform magnetic field, μ dynamic viscosity, g is the acceleration due to gravity, $\beta^*, \sigma, \mu_0, C_p$ represents the thermal expansion coefficient, electrical conductivity of the fluid, magnetic permeability and specific heat at constant pressure, respectively. Also, M indicates magnetization, H symbolizes as magnetic field of strength. The term $\mu_0 \mathrm{M}\frac{\partial \mathrm{H}}{\partial x}$ in equation (2) denote the component of ferromagnetic body force per unit volume and rely on the existence of magnetic gradient. The second term of equation (3) in left hand side i.e. $\mu_0 T\frac{\partial \mathrm{M}}{\partial \mathrm{T}}\left(u\frac{\partial \mathrm{H}}{\partial x}+v\frac{\partial \mathrm{H}}{\partial y}\right)$ presents the adiabatic heat due to magnetization.

On concern of the study [29-30], the components of magnetic field of strength H_x and H_y on horizontal and vertical direction are as follows:

$$H_x(x,y)=-\frac{\partial V}{\partial x}=\frac{\gamma}{2\pi}\frac{x^2-(y+d)^2}{[x^2+(y+d)^2]^2} \tag{5}$$

$$H_y(x,y)=-\frac{\partial V}{\partial y}=\frac{\gamma}{2\pi}\frac{2x(y+d)}{[x^2+(y+d)^2]^2} \tag{6}$$

Where,

$V=\frac{\alpha}{2\pi}\frac{x}{x^2+(y+d)^2}$ known as a scalar potential of the magnetic dipole.

Hence magnitude $\left\|\vec{\mathrm{H}}\right\|=\mathrm{H}$ of the magnetic field intensity are as follows:

$$\mathrm{H}(x,y)=[\mathrm{H}_x^2+\mathrm{H}_y^2]^{\frac{1}{2}}=\frac{\gamma}{2\pi}\left[\frac{1}{(y+d)^2}-\frac{x^2}{(y+d)^4}\right] \tag{7}$$

The relation between magnetization M and temperature T as defined by:

$$M = K(T - T_\infty) \text{ , where } K \text{ is a constant.} \quad (8)$$

Lai and Kulacki [31] define fluid viscosity vary as a inverse linear function of temperature

$$\left.\begin{aligned} \frac{1}{\mu} &= \frac{1}{\mu_\infty}\left[1 + \gamma\left(T - T_\infty\right)\right] \\ \text{Or, } \frac{1}{\mu} &= b\left(T - T_r\right) \end{aligned}\right\} \quad (9)$$

Here

$$b = \frac{\gamma}{\mu_\infty}, T_r = T_\infty - \frac{1}{\gamma}$$

Thermal conductivity of fluid as described by Salawu and Dada[32] in following way:

$$\kappa = k_\infty\left(1 + a\theta\right) \quad (10)$$

Here,

$$a = \frac{k_w - k_\infty}{k_\infty}$$

Where a presents thermal conductivity parameter, b, c, T_r are means constants. Where, numerical calculations for liquid are obtained when $b > 0$ and $b < 0$ for gases.

Solution Procedures

Following similarity transformations of [28] we convert the partial differential equation with associated boundary conditions are in ordinary differential equations:

$$\left.\begin{aligned} u &= cxf'(\eta) \\ v &= -\sqrt{c\vartheta_\infty}\, f(\eta) \\ \eta(y) &= \sqrt{\frac{c}{\vartheta_\infty}}\, y \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \\ \xi(x) &= \sqrt{\frac{c}{\vartheta_\infty}}\, x \end{aligned}\right\} \quad (11)$$

The continuity equation satisfied after defining the velocity component as follows $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ and the remaining momentum and energy equations are in following form:

$$f''' - \frac{\theta'}{\theta - \theta_r} f'' - \frac{\theta - \theta_r}{\theta_r} ff'' + \frac{\theta - \theta_r}{\theta_r} f'^2 + \frac{\theta - \theta_r}{\theta_r} Mf' + \frac{\theta - \theta_r}{\theta_r}\left(\frac{2\beta}{(\eta + \alpha)^4} - \lambda_1\right)\theta = 0 \quad (12)$$

$$(1 + a\theta)\theta'' + a\theta'^2 + \Pr f\theta' - 2\beta\lambda(\varepsilon + \theta)\frac{f}{(\eta + \alpha)^3} = 0 \quad (13)$$

and boundary conditions (4) become

$$\left.\begin{aligned} &\eta = 0 : f = 0\,,\ f' = 1\,,\ \theta = 1 \\ &\eta \to \infty : f' \to 0\,,\ \theta \to 0 \end{aligned}\right\} \quad (14)$$

Where, $\Pr = \frac{\mu C_p}{k_\infty}$ is the Prandtl number, $M = \frac{\sigma B_0^2}{c\rho_\infty}$ is the magnetic field parameter, $\lambda = \frac{c\mu^2}{\rho_\infty k_\infty (T_w - T_\infty)}$ is the viscous dissipation parameter, $\beta = \frac{\gamma}{2\pi}\frac{\mu_0 K(T_w - T_\infty)\rho}{\mu_\infty^2}$ is the ferromagnetic interaction parameter, $\varepsilon = \frac{T_\infty}{T_w - T_\infty}$ is the dimensionless Curie temperature, $\alpha = \sqrt{\frac{c}{\vartheta_\infty}}d$ is the dimensionless distance, $R_e = \frac{xU_w}{\vartheta_\infty}$ is the local Reynolds number, $G_r = \frac{g\beta^*(T_w - T_\infty)x^3}{\vartheta_\infty^2}$ is the Grashof number, $\lambda_1 = \frac{G_r}{R_e^2}$ is the mixed convection parameter, $\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{-1}{\gamma(T_w - T_\infty)}$ is the viscosity variation parameter.

Note that for liquid $\theta_r < 0$ and for gases $\theta_r > 0$. Also need to consideration that when $\lambda_1 > 0$ represent the assist flow and $\lambda_1 < 0$ opposes the flow; while $\lambda_1 = 0$ $(T_w = T_\infty)$ represents the case when the buoyancy forces are absent.

The most important part of the present study is to skin friction coefficient C_f and rate of heat transfer Nu, which are defined as following way:

$$C_f = \frac{\tau_w}{\rho_\infty u_w^2} \text{ and } Nu = \frac{xq_w}{k(T_w - T_\infty)} \tag{15}$$

Where τ_w is the surface shear stress and q_w being surface heat defined as following way:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{16}$$

By using (16) and (17), equation (15) reduces to

$$C_f = -\frac{\theta_r}{\theta - \theta_r} R_e^{\frac{-1}{2}} f''(0) \tag{17}$$

$$Nu = -R_e^{\frac{1}{2}} \theta'(0) \tag{18}$$

Numerical Method for solution

To find an exact solution of such kind of problems in fluid dynamics several numerical techniques have been proposed. Among these bvp4c function technique is most useful technique to solve higher non-linear differential equation. For this we need to convert the equations (12) and (13) with suitable boundary conditions (14) are in first order differential equation as assumed by new variables such as : $f = y_1, f' = y_2, f'' = y_3, \theta = y_4, \theta' = y_5$. All these process are simplified in MATLAB software. So after introducing new variables in equations (12), (13) and (14) we get the following form.

$$f' = y_2$$

$$f'' = y_2' = y_3$$

$$f'''=y_3'=\frac{y_5y_3}{y_4-\theta_r}+\frac{y_4-\theta_r}{\theta_r}y_1y_3-\frac{y_4-\theta_r}{\theta_r}y_2^2$$
$$-\frac{y_4-\theta_r}{\theta_r}\left(\frac{2\beta}{(\eta+\alpha)^4}-\lambda_1\right)y_4-\frac{y_4-\theta_r}{\theta_r}My_2$$
$$\theta'=y_5$$
$$\theta''=y_5'=-\frac{ay_5^2}{(1+ay_4)}-\frac{\Pr y_1y_5}{(1+ay_4)}+\frac{2\beta\lambda(y_4+\varepsilon)y_1}{(1+ay_4)(\eta+\alpha)^3} \quad (19)$$

Boundary conditions are:

$$y_1(0)=S, y_2(0)=1, y_4(0)=1, \quad (20)$$
$$y_2(\infty)=0, y_4(\infty)=0$$

Set of equation (19) as well as boundary condition (20) are integrated numerically as an initial value problem to a given terminal point.

Parameter Estimated

In this paper the steady biomagnetic fluid flow namely blood over a two-dimensional stretching sheet under the influence of a magnetic dipole has been investigated numerically while the effect of fluid viscosity and thermal conductivity also taken into consideration. Before obtain numerical solution we need to put some realistic value relevance to this paper. As we early mentioned that we consider the fluid is blood. So the whole numerical calculations carried only for blood and we survey the previous published literature related to blood and considered to those values in this study, where from [34,35]

$\mu=3.2\times10^{-3}\,kg\,m^{-3}s^{-3}$,

$C_p=14.65\,j\,kg^{-1}k^{-1}$,

$\kappa=2.2\times10^{-3}\,j(msk)^{-1}$

As described in studies [40-42] that human body temperature is $T_w=37^0c$ and the body Curie temperature is $T_\infty=41^0c$ where dimensionless temperature $\varepsilon=78.5$.

Using these values we have, $\Pr=\frac{\mu C_p}{k}=21$

That is for human blood flow the Prandtl number is 21.

We utilize the following parameters to perform in the following figure 2– 21:

1. Ferromagnetic interaction parameter $\beta=0, 5, 10$ as in [3, 37, 38, 39].
2. Thermal conductivity parameter $a=0.0, 0.12, 0.2$ as in [40].
3. Viscosity variation parameter $\theta_r=-0.2, -0.4, -0.6$ as in [40].
4. Prandtl number $Pr=21, 23, 25$ as in [30, 35]
5. Values of dimensionless distance $\alpha=1$ as in [3].
6. Magnetic field parameter $M=1, 2, 3$ as in [36]
7. Mixed convection parameter $\lambda_1=-0.5, 0.0, 0.5$ as in [41]
8. Viscous dissipation parameter $\lambda=1.6\times10^{-14}$

Results and Discussion

To check the numerical accuracy we compared our present results of the rate of heat transfer $-\theta'(0)$ and skin friction coefficient $-f''(0)$ with the existing work of IOAN POP et al. [33] for the values of $\Pr=0.7$ and $\Pr=10$ respectively while viscosity parameter θ_r ranging from -10 to +10. The comparison shows an excellent agreement as present in Table 1and Table 2.

Table 1: Comparison results of $-f''(0)$ and $-\theta'(0)$ when $\Pr = 0.7$

θ_r	**Present results**		IOAN POP et al. [33]	
	$-f''(0)$	$-\theta'(0)$	$f''(0)$	$\theta'(0)$
-10	-0.470211	-0.349787	-0.470990	-0.3503751
-8	-0.476535	-0.348736	-0.4773578	-0.349318
-6	-0.486866	-0.347014	-0.4877456	-0.3475870
-4	-0.506799	-0.343672	-0.5077877	-0.3442274
-2	-0.561597	-0.334383	-0.5628924	-0.3348913
-1	-0.654716	-0.312208	-0.6565296	-0.3189275
-0.1	-1.502157	-0.219010	-1.5061732	-0.2991391
-0.01	-4.480144	-0.154461	-4.4856641	-0.1544918
-0.001	-14.056737	-0.134081	-14.0654213	-0.1340890
2	-0.278513	-0.377913	-0.2783288	-0.3806688
4	-0.369609	-0.363435	-0.3698711	-0.3667289
6	-0.39591	-0.359051	-0.3963122	-0.3625422
8	-0.408455	-0.356932	-0.4089153	-0.36055226
10	-0.515796	-0.355682	-0.5162948	-0.359334

Table 2: Comparisons results of $-f''(0)$ and $-\theta'(0)$ when $\Pr = 10$

θ_r	**Present results**		IOAN POP et al. [33]	
	$-f''(0)$	$-\theta'(0)$	$f''(0)$	$\theta'(0)$
-10	-0.50644	-1.671878	-0.5067231	-1.6815592
-8	-0.515606	-1.670482	-0.5157982	-1.6731001
-6	-0.530751	-1.668175	-0.5310019	-1.6706682
-4	-0.56038	-1.663629	-0.5607505	-1.6658760
-2	-0.644386	-1.650514	-0.6450530	-1.6559052
-1	-0.654716	-1.626383	-0.6565296	-1.6330620

-0.1	-1.873608	-1.492917	-1.8733513	-1.5761532
2	-0.258092	-1.708215	-0.2570983	-1.7128521
4	-0.368489	-1.692373	-0.3680423	-1.6961652
6	-0.402973	-1.687323	-0.4026855	-1.6908461
8	-0.419812	-1.684841	-0.4196006	-1.6882310
10	-0.429791	-1.683364	-0.4296234	-1.6866740

Fig.2and Fig.3 presents the variation of fluid velocity and temperature profile for various values of viscosity parameter θ_r. Where from Fig.2it clearly notice that as the value of θ_r increased velocity profile decreased and asymptotically tends to zero while reverse trend was found in $\theta(\eta)$ (see Fig.3) due to the fact that rising of thermal boundary layer thickness.

The influence of thermal conductivity on velocity and temperature profile is observed from Fig.4 and Fig.5. These two figures reveal that by enhancing values of conductivity parameter (a), velocity distribution is decreasing whereas temperature distribution increased. The reason behind is that when the values of thermal conductivity parameter gradually increased heat transferred from sheet to fluid (blood) significant than velocity profile which cause to enhancement of temperature distribution.

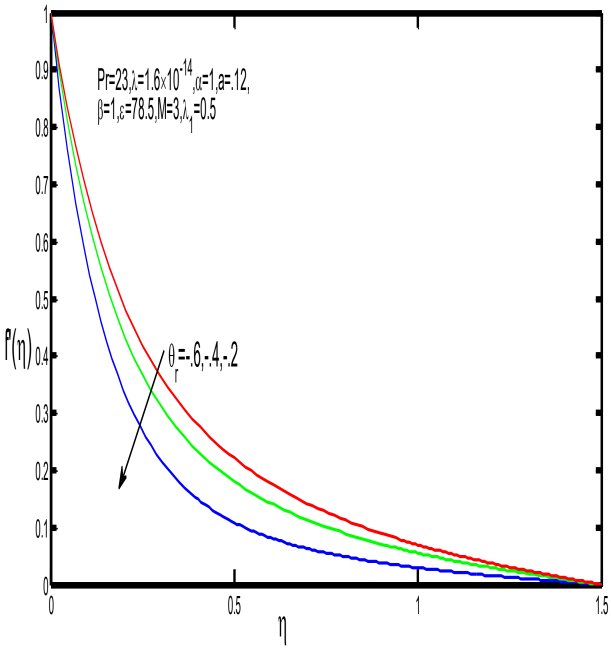

Fig.2 Variation of $f'(\eta)$ with θ_r

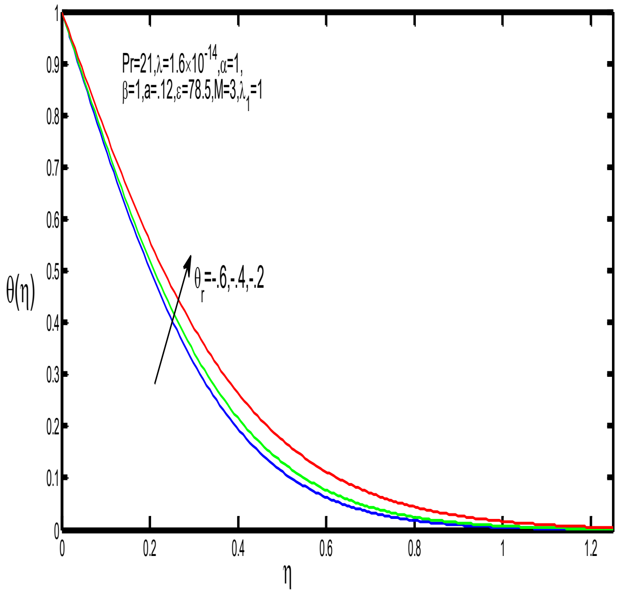

Fig.3 Variation of $\theta(\eta)$ with θ_r

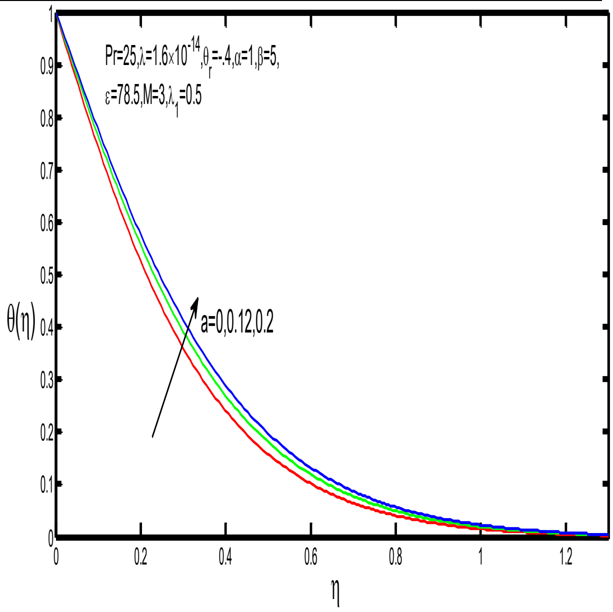

Fig.5 Variation of $\theta(\eta)$ with a

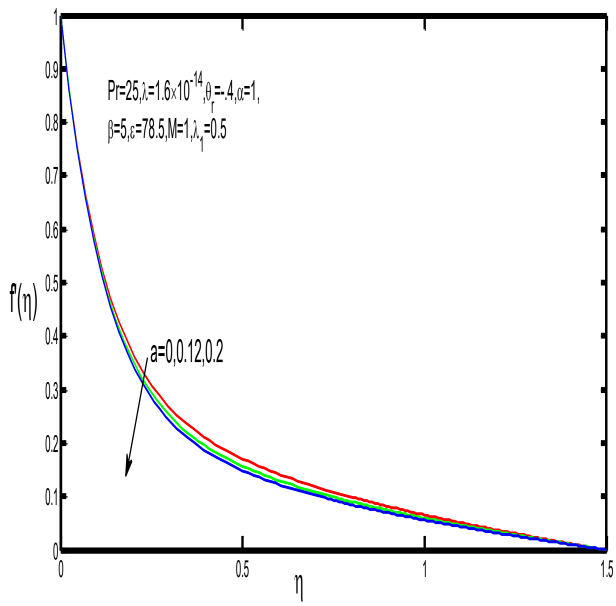

Fig.4 Variation of $f'(\eta)$ with a

Fig.6 and Fig.7 depict the effect of ferromagnetic interaction parameter (β) on fluid velocity and temperature profile. Where Fig.6 shows that velocity profile decreases with increment values of ferromagnetic number but temperature profile increases in this case. Because the ferromagnetic number is directly related to Kelvin force, which is also known as drug force or resist force.

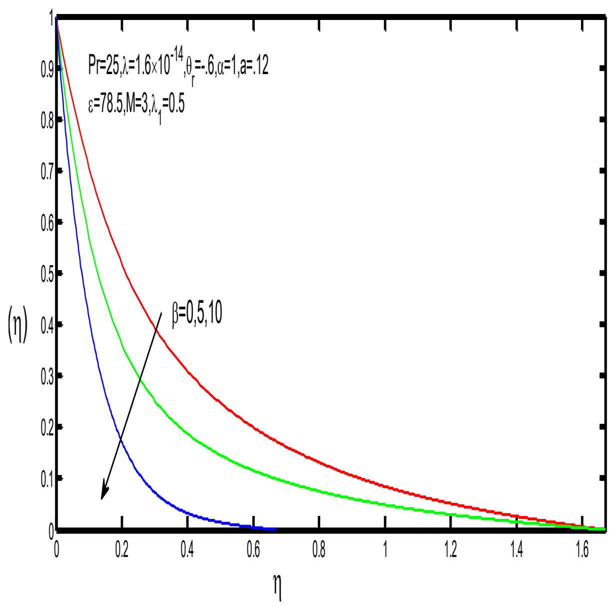

Fig.6 Variation of $f'(\eta)$ with β

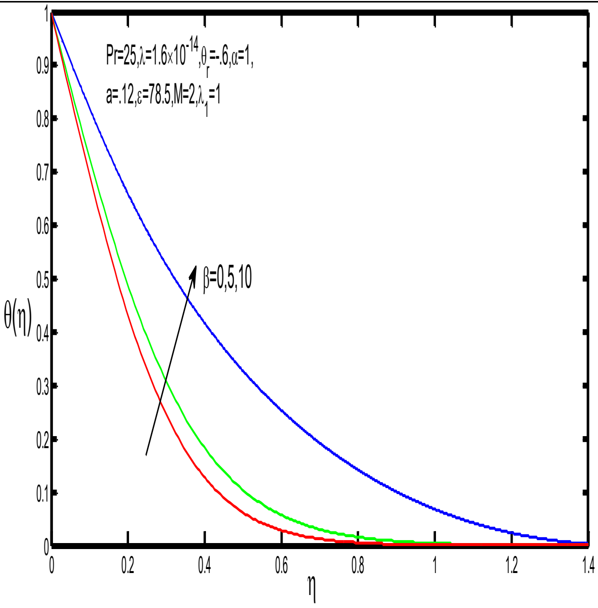

Fig.7 Variation of $\theta(\eta)$ with β

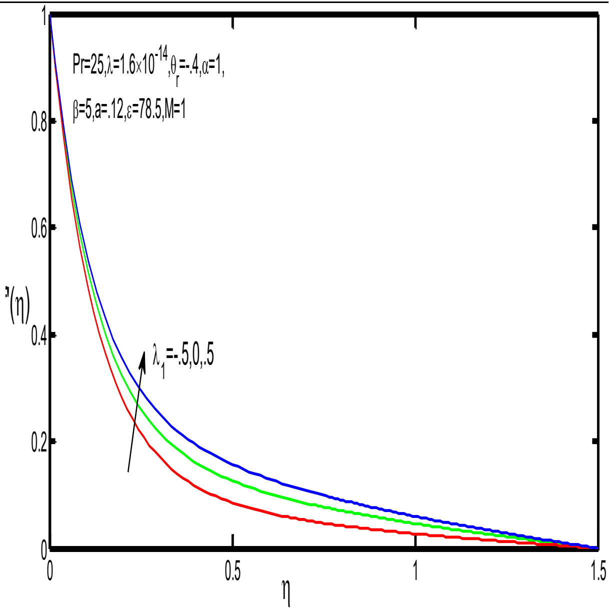

Fig.8 Variation of $f'(\eta)$ with λ_1

The impact of mixed convection parameter (λ_1) on fluid velocity and temperature distributions can be found in Fig.8 and Fig. 9. It is observed from Fig.8 that with rising values of λ_1 fluid velocity increased while temperature profile behave reverse as velocity profile. Due to the fact that temperature difference $(T_w - T_\infty)$ increased when values of λ_1 enhanced gradually and it causes to rising the fluid velocity along horizontal direction.

Fig.10 and Fig.11 depict the velocity and temperature profiles under the influence of Prandtl number (Pr). As we know the ratio of momentum diffusivity to thermal diffusivity is known as Prandtl number. So, higher values of Prandtl number decline the thermal boundary layer which is expected and found in Fig.11.

The effect of magnetic field parameter (M) for various values is present in Fig.12 and Fig.13. By increasing values of M, fluid velocity decreases because of Lorentz force which acts oppose to fluid velocity and consequently temperature profile enhanced.

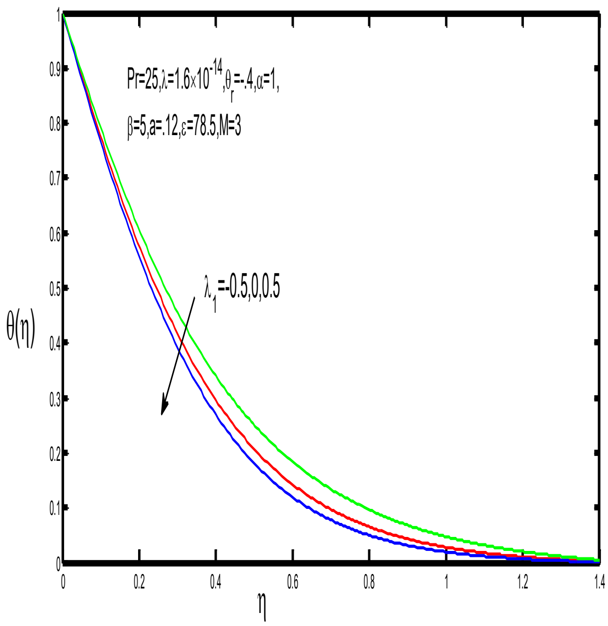

Fig.9 Variation of $\theta(\eta)$ with λ_1

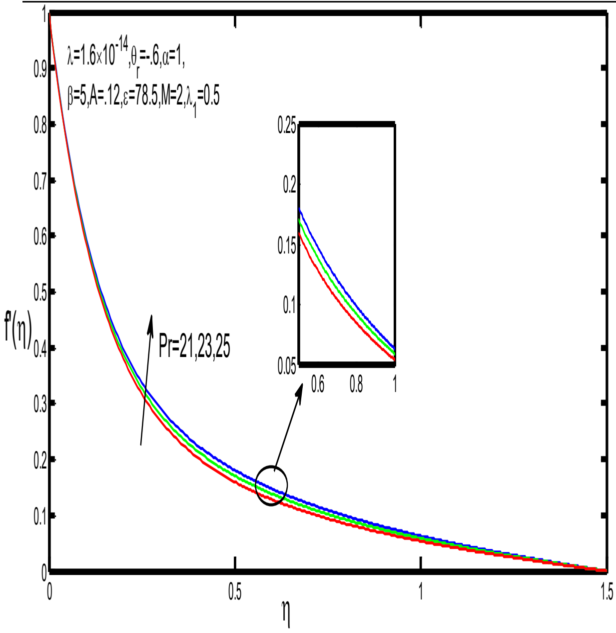

Fig.10 Variation of $f'(\eta)$ with Pr

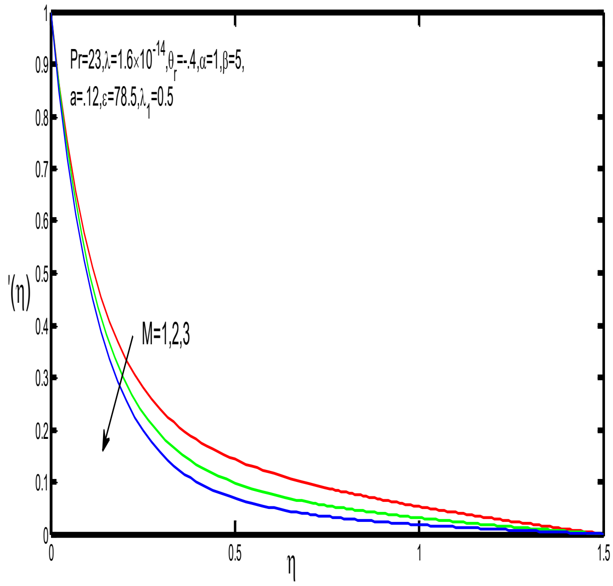

Fig.12 Variation of $f'(\eta)$ with M

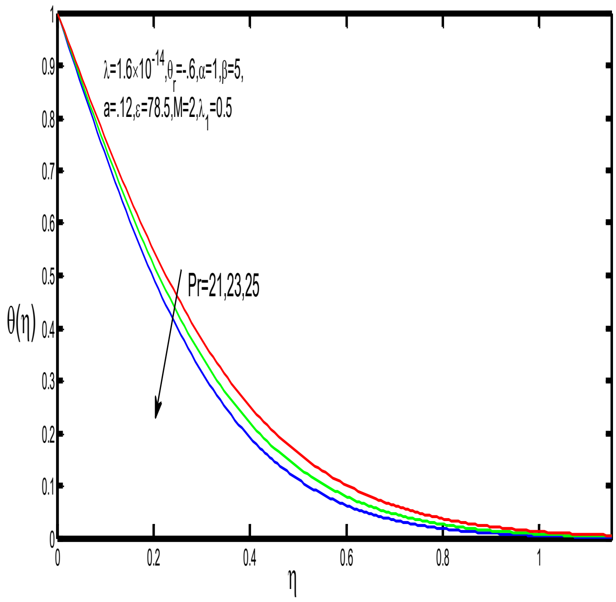

Fig.11 Variation of $\theta(\eta)$ with Pr

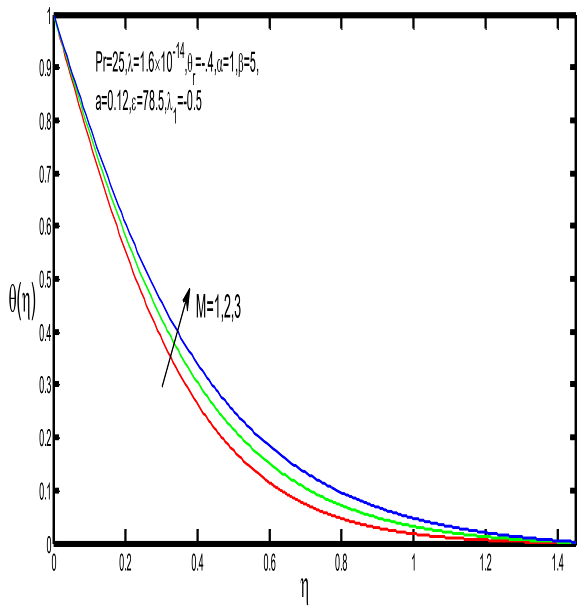

Fig.13 Variation of $\theta(\eta)$ with M

Figures 14-21 depict the variation of skin friction coefficient $-f''(0)$ and Local Nusselt number $-\theta'(0)$ with regard to the mixed convection parameter

(λ_1) for several values of ferromagnetic number (β), viscosity variation parameter (θ_r), thermal conductivity parameter (a) and magnetic field parameter (M) respectively. It is noticed from these figures that with increasing values of ferromagnetic number (β), viscosity parameter (θ_r), thermal conductivity parameter (a) and magnetic field parameter (M), $-f''(0)$ decreased and reversed trend was found in $-\theta'(0)$.

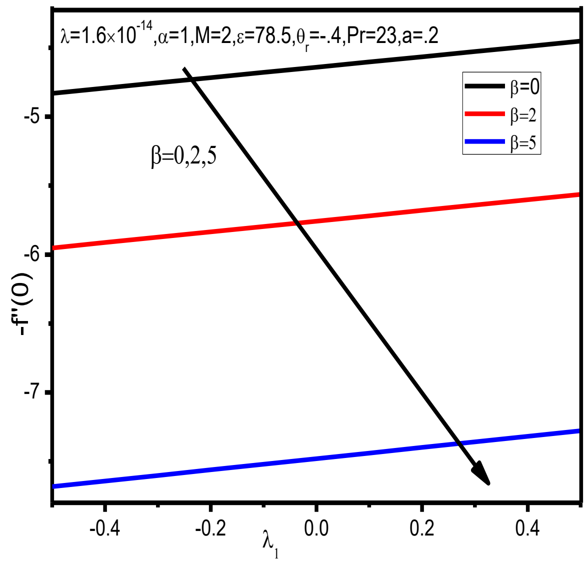

Fig.14 Variation of $-f''(0)$ for various values of β with regard to λ_1

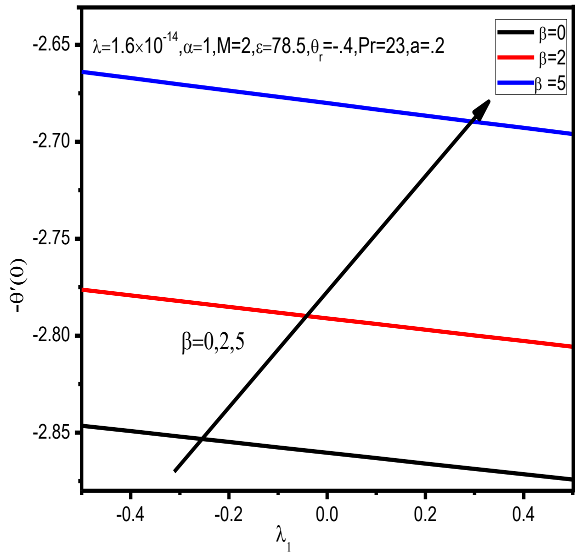

Fig.15 Variation of $-\theta'(0)$ for various values of β with regard to λ_1

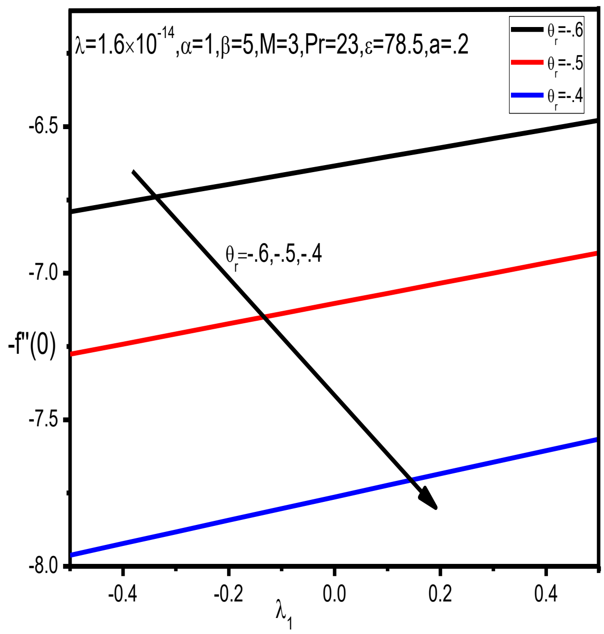

Fig.16 Variation of $-f''(0)$ for various values of θ_r with regard to λ_1

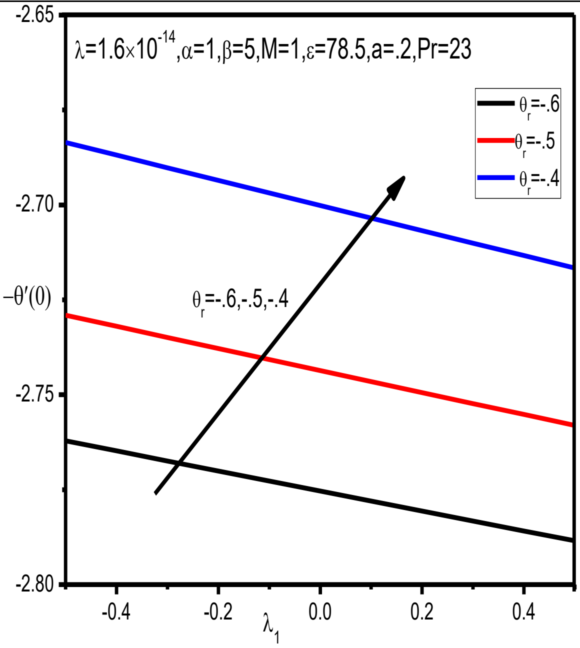

Fig.17 Variation of $-\theta'(0)$ for various values of θ_r with regard to λ_1

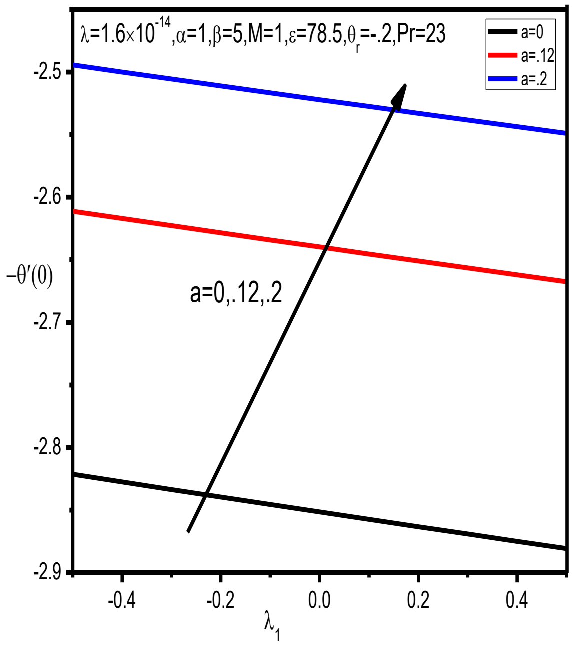

Fig.19 Variation of $-\theta'(0)$ for various values of a with regard to λ_1

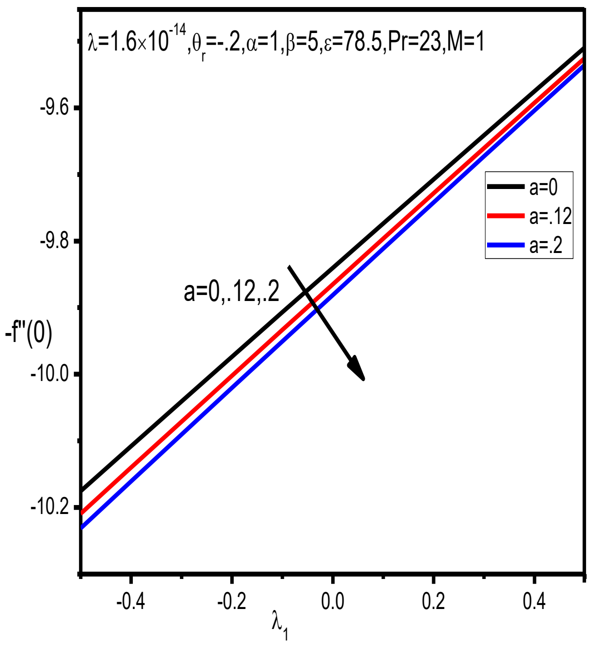

Fig.18 Variation of $-f''(0)$ for various values of a with regard to λ_1

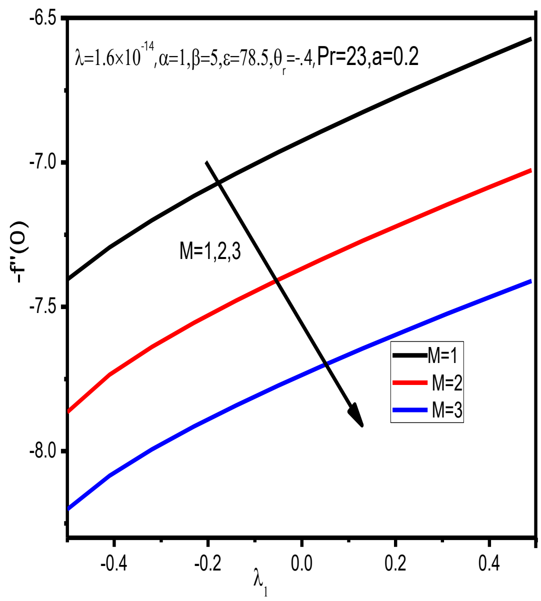

Fig.20 Variation of $-f''(0)$ for various values of M with regard to λ_1

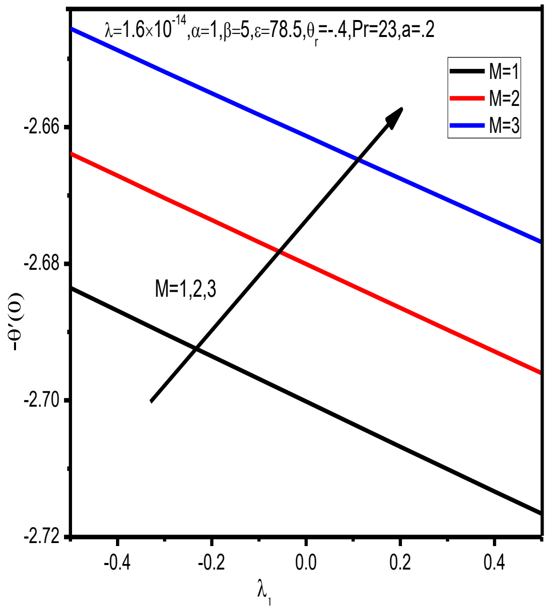

Fig.21 Variation of $-\theta'(0)$ for various values of M with regard to λ_1

Conclusions

In this work, two dimensional steady, viscous, incompressible biomagnetic fluid (blood) over a stretching sheet was numerically investigated under the influence of magnetic dipole as well as fluid viscosity and thermal conductivity. Both two principle MHD and FHD are also considered in this study. Governing partial differential equations are transformed into ordinary differential equations along with boundary conditions by using similarity transformations and numerically calculations were carried out for blood by employing bvp4c function technique available in MATLAB software. For better understand of such kind of flow problem we also calculated the characteristics of skin friction coefficient and rate of heat transfer.

From the above analysis we conclude that:

(1) The fluid velocity increases with increasing values of mixed convection parameter, Prandtl number; whereas temperature profile decreases in all this cases.
(2) The fluid velocity decreases with thermal conductivity parameter, viscosity parameter, Ferromagnetic interaction parameter, magnetic field parameter; whereas temperature increases in all cases.
(3) By increasing values of thermal conductivity parameter, viscosity parameter, Ferromagnetic interaction parameter, Magnetic field parameter, skin friction coefficient decreases; while rate of heat transfer enhanced in all cases.

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