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Efficiency and Maximum Temperature of Fins of Complex Geometry

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Abstract

Determining the efficiency of complex fins is important when performing calculations involving heat exchangers with individual finned tubes or continuous fin heat exchangers, i.e., plate-fin and tube heat exchangers (PFTHEs). Usually, these are circular or rectangular fins mounted on circular, elliptical, oval tubes or flattened tubes. With PFTHEs, the continuous fin is divided into virtual fins, which are either rectangular for an inline pipe layout or hexagonal for a staggered pipe arrangement. Maximum fin temperature is also important in light of a possible burnout of the material, especially when heat transfer from hot gas to fin is considered. This paper presents a procedure based on the finite element method (FEM) for determining the efficiency and maximum temperature of fins of any shape placed on tubes of any shape. It presents examples of calculating the efficiency of virtual fins in the most commonly used PFTHEs. It also depicts the efficiency of a fin as a function of heat transfer coefficient for the commonest two geometries and two more complex geometries in PFTHEs.

Keywords: complex-shaped fin, fin efficiency, numerical simulation, plate-fin, and tube heat exchangers, continuous fin

1 Introduction

Fin efficiency is defined as a ratio of heat transferred through the real fin to the heat flow rate transferred through the isothermal fin at the temperature of the fin base. The precise analytical formula for these calculations can be determined for a fin of simple geometry [1]. However, for complex geometrical fins approximate formulas need to be used, such as the sector method [2], Schmidt method [3] or various numerical methods: the finite element method (FEM) or finite volume method (FVM) [1].

The sector method is more accurate, but more complicated than the Schmidt method. In current software such as ANSYS CFX or ANSYS Fluent fin average temperature in the steady-state can be determined quite easily using FEM or FVM. Numerical methods can determine fin temperature distribution or fin efficiency for both simple and complex fin geometries [1]. Several fin geometries have been analyzed to date [4]. Experimental and computational analysis of ribbing structure in a PFTHE operating with non-uniform inflow of media was presented by Bury and Hanuszkiewicz-Drapała [5]. Hanuszkiewicz-Drapała et al. [6] also used a rectangular fin on an oval tube in a cross-flow heat exchanger in numerical modeling.

This paper compares analytical and numerical (CFD) methods for standard geometrical fins (such as straight or circular) and complex geometrical fins (elongated hexagonal and segmented). All cases of CFD simulations were supplemented by mesh independent study. The minimum or maximum temperature on the fin end dependent on the number of elements was calculated (Fig. 3, 4, 9, 16). Subsequently, mesh element size was selected with the assumption that the next few temperature values are at a constant level. CFD simulations were carried out for the following values for all fin shapes: 0.0003m, 0.0002m, 0.00015m, 0.0001m and 0.00005m. Since the surface area for each fin is similar for different geometries, the mesh elements are almost the same for the various mesh sizes. This article presents the following issues:

- analytical and numerical methods for simple fin geometries - straight and circular,
- numerical simulations for complex fin geometries - elongated hexagonal,
- numerical simulations for complex fin geometries - segmented.

2 Problem Statement

This paper consists of the calculation of fin efficiency under two-dimensional steady-state conditions based on the following assumptions:

- Omission of the radiation effect;
- Omission of heat transfer from the fin tip to the air (Fig. 1, 2);
- Omission of the heat transfer on the side if the fin is very thin (assumption: that w >> L_c is satisfied) (Fig. 2);
- Adiabatic boundary condition in the border of the virtual fin designated from the continuous fin because of the fin's symmetry;
- Constant fin base temperature;
- Constant temperature through the thickness of the fin;
- Constant heat transfer coefficient (α);
- Constant thermal conductivity (λ) for straight and circular fin;
- Variable thermal conductivity (λ) for elongated hexagonal and segmented fin as a function of temperature (λ = -7E-06T^2 - 0.0208T + 61.318).

The following heat transfer conduction equation (Eq. 1) was designed, taking into account all the above assumptions:

$$\frac{[?]T^2}{[?]x^2} + \frac{[?]T^2}{[?]y^2} = \frac{2\alpha}{\lambda\delta}\left(T - T_{cz}\right) \tag{1}$$

3 Comparing CFD software results with exact analytical solutions for determining fin efficiency and maximum temperature

Straight and circular fins have wide applications: electronic components, ventilation systems, cooling towers, hot water systems. Continuous plate fins are commonly used as a part of fin and tube heat exchangers, which have several applications: heating and cooling systems, power plants and car radiators, among others.

This paper presents various methods of determining the temperature field and the fin efficiency of commonly used types of fins, such as straight, circular, imaginary elongated hexagonal and segmented. All of the presented methods were also applied to more complex geometries. Fin efficiency was calculated for a uniform coefficient of heat transfer on the fin surface using the following formula:

$$\eta = \frac{\left(\overline{T}_{fin} - T_{cz}\right)}{\left(T_b - T_{cz}\right)} \tag{2}$$

where T_{fin} means the average temperature of the fin surface at which heat exchange with the environment occurs. Relative differences between various solutions were calculated as follows:

$$\eta = \frac{\left(\eta_{CFD} - \eta_{analythical}\right)}{\left(\eta_{CFD}\right)} \tag{3}$$

However, it differs for an elongated hexagonal and segmented fin:

$$\eta = \frac{\left(\eta_{CFD(0.00005)} - \eta_{CFD(0.0003-0.0001)}\right)}{\left(\eta_{CFD(0.00005)}\right)} \tag{4}$$

Analytical methods for calculating simple fin efficiency are easy to use [2], but CFD simulations give richer options.

3.1 Simple straight and circular fin on a round tube

Straight and circular fins of constant thickness on a round tube are depicted in Fig. 1 and Fig. 2.

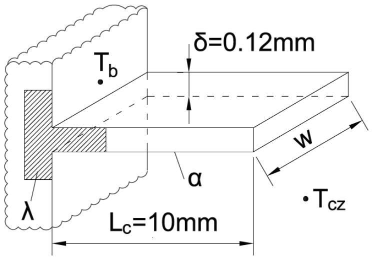

Figure 1: Fins of simple geometry - straight fin of constant thickness.

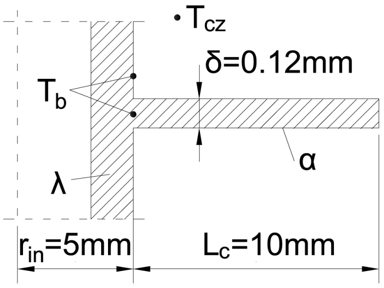

Figure 2: Fins of simple geometry - circular fin of constant thickness.

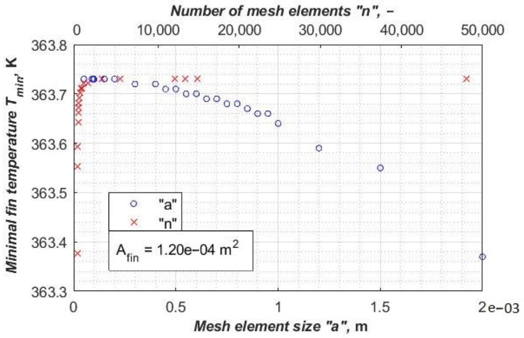

Figure 3: Minimum fin temperature (for heat transfer coefficient 25 W/m2K) as a function of the number of mesh elements ‘n’ and mesh element size ‘a’ - straight fin.

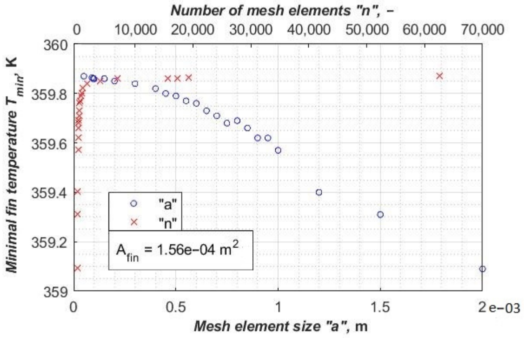

Figure 4: Minimum fin temperature (for heat transfer coefficient 25 W/m2K) as a function of the number of mesh elements ‘n’ and mesh element size ‘a’ - circular fin.

4

The following data were used for the calculation: $T_b = 373.15K$, $T_{cz} = 273.15K$, and $\lambda = 204$ W/(m.K). The minimum temperature and fin efficiency values obtained by the analytical method are very similar to those calculated by the numerical method.

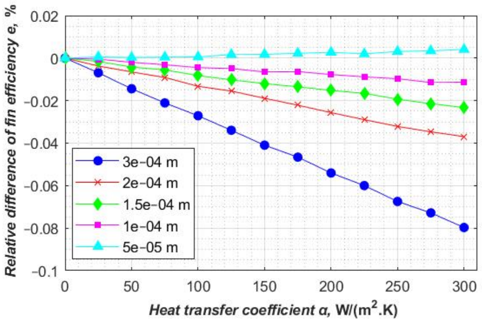

Figure 5: Relative differences of fin efficiency between analytical method and CFD simulation for several different mesh sizes as a function of heat transfer coefficient - straight fin.

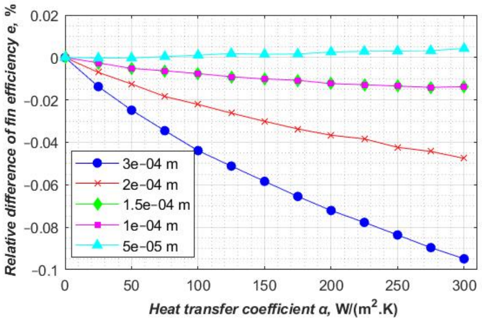

Figure 6: Relative differences of fin efficiency between analytical method and CFD simulation for several different mesh sizes as a function of heat transfer coefficient - circular fin.

Fig. 5 and Fig. 6 compare relative differences between analytical and CFD simulation results for five different mesh sizes. Relative differences are smaller than 0.1% for a mesh element size ranging from 0.0003 m to 0.00005 m (1000 – 70,000 mesh elements). The results are very similar, even for the largest mesh element size, which is 0.0003 m (1000 – 2000 mesh elements). It can be observed that the precision rises with the number of mesh elements (up to 70,000).

4.1 Complex elongated hexagonal fin on a flat tube

Continuous fins can have different geometries, from simple to complex (Fig. 7) [7]. There are no analytical methods to calculate such fins exactly. Approximate methods are precise, but often only for a particular range of heat transfer coefficient α. However, they are becoming increasingly accurate [8]. All of the above examples for straight and circular fins showed CFD simulation to be a precise, reliable method.

The simulation was validated by performing it independently on several types of meshes with different mesh element sizes (Fig. 9 and Fig. 16).

The method proposed in the article to calculate the fin temperature distribution and its efficiency may be applied to fins of any shape attached to pipes of any shape of cross-section. CFD modeling is used to determine fin efficiency and maximum fin end temperature on flattened pipes, which cannot be calculated by the analytical method. Fins and pipes are made from boiler steel type: P235GH-TC2. The thermal parameters for this steel are $T_b = 373.15$ K, $T_{cz} = 923.15$ K. In this case, we take heat from the environment and transfer it to the fin.

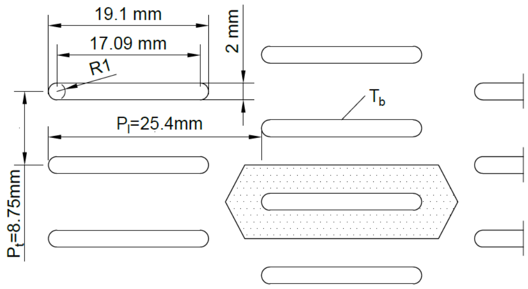

Figure 7: Plate finned tube heat exchanger made of flattened pipes with staggered pipe arrangement - elongated hexagonal imaginary fin for the inline pipe arrangement analyzed in [7].

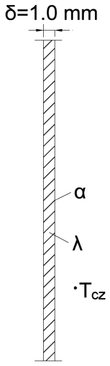

Figure 8: Plate finned tube heat exchanger made of flattened pipes with staggered pipe arrangement - cross-section of the fin.

This is the opposite situation to previously presented fins with simple geometries, where heat was transferred from the fin to the environment. It should be noted that thermal conductivity is lower than in previous cases (λ ˜ 50W/(m. K)) as is described in point "2. Problem Statement". That is why the temperature, which stabilizes during the mesh independent study, is falling. In previous cases, the temperature is rising.

The maximum temperature on the fin surface as a function of the number of mesh elements was calculated (Fig. 9). The maximum fin temperature was established as 379.23 K while the fin base temperature was 373.15 K.

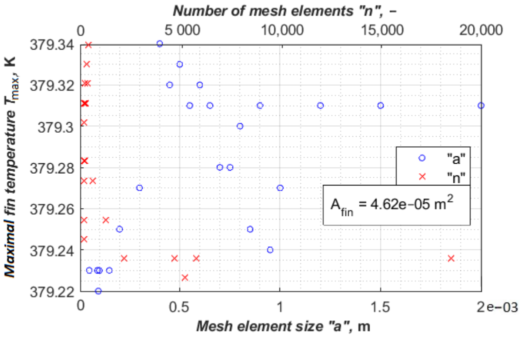

Figure 9: Maximum fin temperature (for heat transfer coefficient 25 W/(m^2.K)) as a function of the number of mesh elements 'n' and mesh element size 'a' for elongated hexagonal fin.

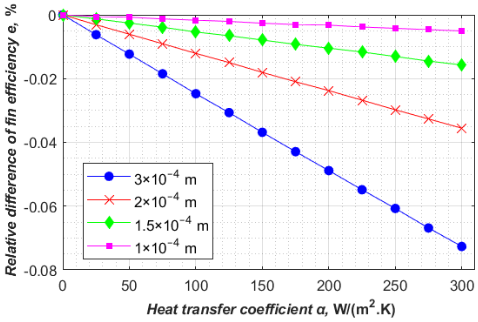

Figure 10: Relative differences of fin efficiency between several mesh element sizes and mesh element size of 0.0005 m.

Relative differences are less than 0.073% for mesh element size ranging from 0.0003 m to 0.00005 m (500 – 20,000 elements). The five results of independent simulations showed slightly different results. Comparability of results is satisfactory and is presented in Fig. 10.

Fin efficiency (η) for an elongated hexagonal fin for the numerical method is presented in Fig. 11. This figure shows fin efficiency as a function of the heat transfer coefficient (α) on the air-side.

The maximum fin temperature depends on the heat transfer coefficient, as shown in Fig. 12. This figure

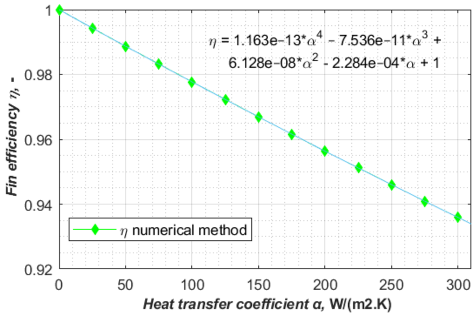

Figure 11: Fin efficiency (η) as a function of heat transfer coefficient (α) on the air-side for the elongated hexagonal fin.

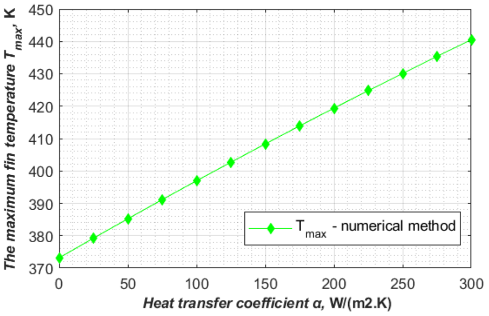

Figure 12: The maximum fin end temperature as a function of the heat transfer coefficient (α) on the air-side for the elongated hexagonal fin.

can indicate the maximum fin end temperature in the case of a possible fin burnout.

4.2 Complex segmented fin on a round tube

Fig. 13, Fig. 14 and Fig. 15 present numerical simulations for calculations of the efficiency of a segmented fin on round pipes. Analytical and approximate methods cannot be used for this kind of fin. A similar approach is shown in the case of the elongated hexagonal fin in point 4.1. Fins and pipes are made from boiler steel type: P235GH-TC2 as well as for the elongated hexagonal fin. The thermal parameters for this steel are $T_b = 773.15$ K, $T_{cz} = 923.15$ K. In this case, we take heat from the environment and transfer it to the fin.

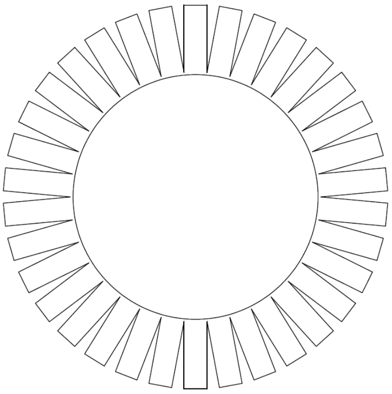

Figure 13: Complex segmented fins on a round pipe - view of the whole finned tube [10].

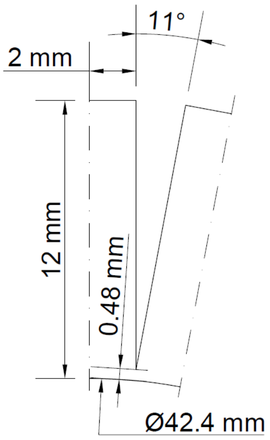

Figure 14: Complex segmented fins on round pipe - repeating fin element.

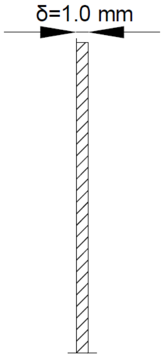

Figure 15: Complex segmented fins on round pipe - cross-section of the fin.

The fin maximum temperature stabilized around 785.56K (Fig. 16) for a mesh element size of 0.0003 m. Increasing the number of mesh elements beyond this point only slightly affects the quality of the result. Relative differences are less than 0.012 % for mesh element size ranging from 0.0003 m to 0.00005 m (number of mesh elements ranging from 630 to 23,501). The four results of independent simulations showed almost imperceptible differences (Fig. 17).

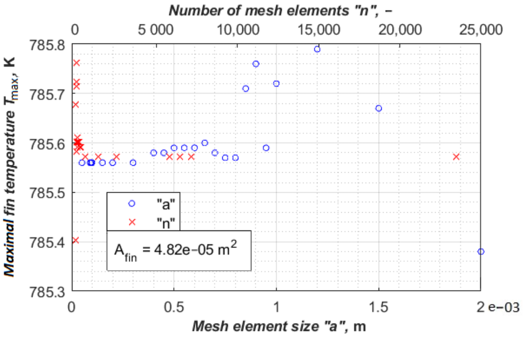

Figure 16: Maximum fin temperature (for heat transfer coefficient 25 W/(m^2.K)) as a function of the number of mesh elements ‘n’ and mesh element size ‘a’ for segmented fin.

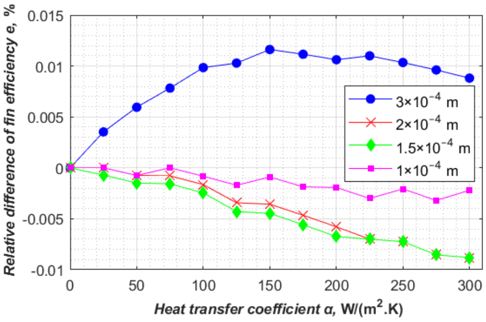

Figure 17: Relative differences of fin efficiency between several mesh element sizes and mesh element size of 0.0005 m.

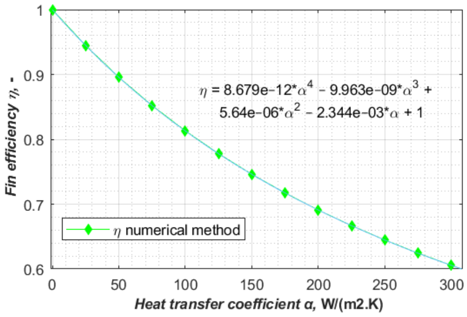

Figure 18: Fin efficiency (η) as a function of heat transfer coefficient (α) on the air-side for the segmented fin.

Fin efficiency (η) for a segmented fin for the numerical method is presented in Fig. 18. This figure shows fin efficiency as a function of heat transfer coefficient (α) on the air-side.

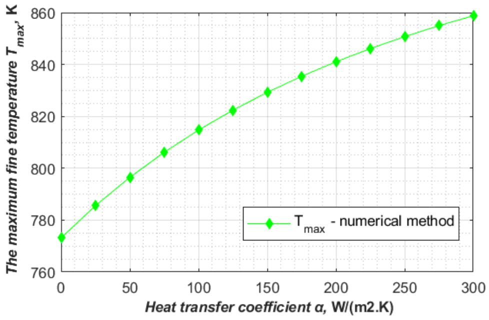

Figure 19: Maximum fin end temperature as a function of the heat transfer coefficient (α) on the air-side for the segmented fin.

The maximum fin temperature depends on the heat transfer coefficient as shown in Fig. 19. This figure can indicate the maximum fin end temperature in the case of a possible fin burnout.

5 Discussion

This paper presents the calculation of fin efficiency using exact analytical and numerical methods. It is known that the deviation of thermal performance between experimental and CFD results is less than 4% [9]. Padmanabhan et al. [10] also showed that the differences in temperature distribution between analytical results and CDF analysis are comparable. This paper extends the current state of knowledge about analytical, approximate, and numerical results as regards determining fin efficiency. The comparison results in this article show that the conducted investigation produced similar results: maximum relative differences between the analytical and numerical results are less than 0.1% (Fig. 5, 6). Fin efficiency in the case of complex geometry can be calculated only using numerical simulations. This research shows that numerical results are very precise for fin efficiency and maximum fin end temperature and can be used in further research and industrial calculations. Future research should factor in considerations about wet or dirty fins.

6 Conclusions

This paper presents a comparison of analytical and numerical (CFD) methods. The examples used demonstrate that CFD simulations are reliable, but fine element mesh and suitable validation are needed. In a comparison of the various calculation methods, heat transfer coefficient α varied between 0 and 300 W/m2K. The following conclusions can be drawn from the analyses and calculations:

- relative difference between analytical and CFD simulation results is smaller than 0.1 % for 0 [?] α [?] 300 W/(m2K) and less than 0.03 % for 0 [?] α [?] 100 W/(m2K) for mesh element size ranging from 0.0003 m (10-50 elements) to 0.00005 m (10,000 – 60,000 elements),
- in the mesh independent study, a close relationship was noted. The stable minimum temperature and constant fin efficiency occur for the maximum mesh element size of 0.0002 - 0.0003 m (700-1400 mesh elements). Values of fin efficiency changed slightly for mesh element size 0.003 m – 0.0005 m (1400 – 60,000 mesh elements),
- it is observed that relative difference does not always increase with heat transfer coefficient α,
- the examples above showed that numerical simulation can be an efficient tool to determine fin temperature distribution or maximum fin end temperature, to verify the possibility of burnout of fin material,
- the efficiency and speed of current computers, even home computers, enable highly advanced calculations to be performed in a relatively short time [11]. The heat conduction simulations presented in this article lasted several seconds for a mesh with 2000-5000 thousand elements. For meshes with 50,000-70000 elements, it took 20-30 s.

Nomenclature

A_{fin} – surface area of the fin, m^2

L_{ex} - extended length of the fin, m

L_c – length of the fin, m

w – width of the fin, m

r_{in} – outer radius of the plain tube, m

a – mesh size, m

n – number of mesh elements, -

T_b – temperature of fin base, K

T_{cz} – ambient temperature, K

P_l – longitudinal fin pitch, m

P_t – transversal fin pitch, m

δ – fin thickness, m

α - heat transfer coefficient, $W/(m^2.K)$

η – fin efficiency, -

References

1.Taler, D., and Taler, J. (2014) Steady-state and transient heat transfer through fins of complex geometry. *Archives of Thermodynamics*, **35** (2), 117–133.

2.Sekulić, R.K.S.;D.P. (2007) *Fundamentals of Heat Exchanger Design*, John Wiley & Sons.

3.Schmidt, T.E. (1949) Heat transfer calculations for extended surfaces. *Refrigeration Engineering*, 351–357.

4.Osorio, J.D., Rivera-Alvarez, A., and Ordonez, J.C. (2017) Shape optimization of thin flat plate fins with geometries defined by linear piecewise functions. *Applied Thermal Engineering*, **112**, 572–584.

5.Bury, T., and Hanuszkiewicz-Drapała, M. (2017) Experimental and computational analysis of ribbing structure modification effect in tube and fin cross-flow heat exchangers operating at non-uniform inflow of media. *Journal of Power Technologies*, **97**, 429–436.

6.Hanuszkiewicz-Drapała, M., and and, K.W.T.B. (2013) Numerical model of a cross-flow heat exchanger with non-uniform flow of media. *Journal of Power Technologies*, **93**, 295–302.

7.Thulukkanam, K. (2013) *Exchanger Design Handbook, 2nd ed.*, CRC Press.

8.Suárez, F., Keegan, S.D., Mariani, N.J., and Barreto, G.F. (2019) A novel one-dimensional model to predict fin efficiency of continuous fin-tube heat exchangers. *Applied Thermal Engineering*, **149**, 1192–1202.

9.Sathe, A., Sanap, S., Dingare, S., and Sane, N. (2021) Investigation of thermal performance of modified vertical rectangular fin array in free convection using experimental and numerical method. *Materials Today: Proceedings*, **38**, 2281–2290.

10.Padmanabhan, S., Thiagarajan, S., Kumar, A.D.R., Prabhakaran, D., and Raju, M. (2021) Investigation of temperature distribution of fin profiles using analytical and CFD analysis. *Materials Today: Proceedings*, **44**, 3550–3556.

11.Zhai, Z.J., Zhang, Z., Zhang, W., and Chen, Q.Y. (2007) Evaluation of Various Turbulence Models in Predicting Airflow and Turbulence in Enclosed Environments by CFD: Part 1—Summary of Prevalent Turbulence Models. *HVAC&R Research*, **13** (6), 853–870.