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Analysis of a hydro-osmotic power plant using a simple mathematical model

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Abstract

Osmotic energy created by salinity gradient is a novel energy source. The chemical potential difference of two liquids with dissimilar salinities can be used to generate a flow of water across a semi-permeable membrane. The Pressure-Retarded Osmosis (PRO) concept controls the flow of the water across the membrane and contributes to the production of electrical energy with a hydraulic turbine. This study describes a simple mathematical model for evaluating the effectiveness of application of the PRO process in a hydro-osmotic power plant. The influences of pressures and salt concentrations of the inlet streams as well as the concentration polarization across the membrane on the membrane power density and plant efficiency were investigated. The effects of liquid temperature and membrane characteristics (water and salt permeability) on power plant performance were also studied.

Keywords: Renewable energy; pressure retarded osmosis; osmotic power plant; mathematical modelling

1. Introduction

Useful power can be obtained from many renewable energy sources such as directly from solar, geothermal or gravitation energy collected in water reservoirs, from the wind, ocean waves, currents and tides as well as temperature gradient in the sea. Generation of electrical energy from most of these sources is possible due to differences in altitude, temperature or pressure. A less popular type of renewable energy source derives from presence of different liquid solutions in the environment. Since the thermodynamic potential (Gibbs free energy) of a solution depends on its composition, differences in species concentration between the same type of solutions can be used to generate electric energy by applying a controlled process of separation of their components. The process is carried out by using a semi-permeable membrane through which one of the components diffuses, thus contributing to the increased pressure or amount of this component on the other side of the membrane. The process is known as osmosis. The increased pressure or amount of component on one side of the membrane can be subsequently used, e.g., in a hydraulic turbine with the electric generator to obtain useful electric energy [1].

The most common liquid solutions available worldwide are water as a solvent with salts as the dissolved substances. They are present in many water reservoirs such as seas, lakes and rivers. The mixing of low salt concentration river water with salty sea water occurs naturally in estuaries. The process is spontaneous and leads to generation of entropy and loss of the available energy, which can be converted to a useful form in a controlled process.

Sea water can be treated as a mixture of water and mineral salts dissolved in it. Generally, the word “salt” is used to denote sodium chloride, NaCl, but sea water contains other salts such as: magnesium chloride, sulfates and carbonates. The average salinity of sea water is 35 g/l including: 27.2 g/l NaCl, 3.8 g/l $MgCl_2$, 1.7 g/l $MgSO_4$, 1.26 g/l $CaSO_4$, 0.86 g/l K_2SO_4. Sea and lake salinities (NaCl salt concentration) cover a broad range of values: Caspian Sea: 13 g/l, surface waters of Antarctica: 34 g/l, Pacific Ocean: 34.5 g/l (but only 33 g/l for the North Pacific), Atlantic Ocean: 35 g/l, Mediterranean Sea: from 36 to 38 g/l, Red Sea: 41 g/l, Great Salt Lake (USA, Utah): 150 g/l, Dead Sea: 270 g/l, Indian Ocean: from 34.5 g/l, Baltic Sea: 10 g/l [2, 3, 4]. These data vary locally, especially in the oceans, because of currents and changes in temperature.

In the seas of warm countries, the amount of water that evaporates is much greater than the amount of water returned to the sea via rivers and streams. These seas are saltier, but they also reject part of this salt by giving it to other, less salty seas. Inland seas that are not connected to other seas and oceans, are unable to reduce their amount of salt so they can contain up to 5 or 6 times more salt per liter of water than other seas.

Useful energy can be obtained not only by mixing river and

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Table 1: Theoretical energy potential of selected rivers in the world [5]. Water temperature was assumed as 20°C with a Van't Hoff coefficient of 1.85—see eq. (7)

River name	Flow rate, m^3/s	Power, GW	Energy, TWh/year
Amazon	175000	472.5	4139.1
Nile	2622	7.1	62.0
Ganges	13000	35.1	307.5
Mississippi	19000	51.3	449.4
Congo	42000	113.4	993.4
Yangtze	35000	94.5	827.8
Yenisei	19800	53.5	468.3
Vistula	1080	2.9	25.5
World	1182394	3192.5	27966.0

Table 2: The practical energy potential of selected rivers in the world [5]. Efficiency of conversion of primary to final energy was assumed as 40% and 15% of the volumetric flow rate of water assumed due to ecological aspects

River name	Power, GW	Energy, TWh/year
Amazon	28.4	248.3
Nile	0.4	3.7
Ganges	2.1	18.4
Mississippi	3.1	27.0
Congo	6.8	59.6
Yangtze	5.7	49.7
Yenisei	3.2	28.1
Vistula	0.2	1.5
World	191.5	1678.0

sea water, but also wherever there are waters in which a difference in salinity occurs. For example, sea water can be treated as a low salt concentration fluid which is to be mixed with a stream of water coming from a lake of high salt content. Other two-fluid systems can also be considered, e.g., sea water and concentrated community waste sewage water, or sea water and concentrated salty water obtained from reverse osmosis. Further examples are: industrial effluent or high salinity geothermal water and treated household sewage water or ground water with low dissolved species concentration [6].

The amount of energy released during the mixing process depends on the water volumetric flow rate and difference in the dissolved species concentration of the two water streams. Knowing the flow rate in rivers running into the sea and assuming mean salinity of sea water of 35 g/l, the theoretical energy potential of rivers was evaluated and presented in Table 1.

The real energy that can be converted into useful energy also depends on the method of energy conversion, losses in the system used and the amount of river water available due to ecological aspects—see Table 2. The values of the available energy will in reality also differ due to differences in water salinity and temperature at the particular location in the world. For example, Baltic Sea salinity and temperature are significantly lower than in the Mediterranean. Therefore a hydro-osmotic power plant will be theoretically more effective at the mouths of rivers flowing into the Mediterranean.

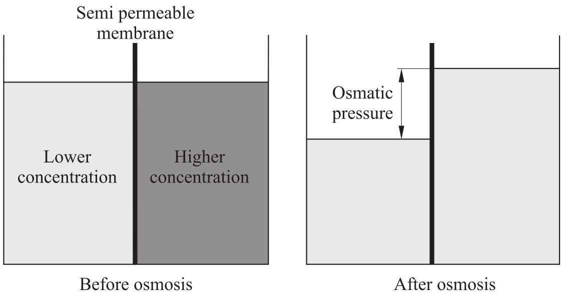


Figure 1: The phenomenon of free (forward) osmosis

2. Osmosis and the flow of water through membranes

If a membrane that is semi-permeable to water is inserted between the first low salinity solution and the second high salinity solution, then the flow of pure water from the first solution to the second one will be initiated. During the water flow across the membrane the level of the more concentrated solution tends to rise and the solution becomes increasingly diluted—see Fig. 1. The phenomenon is known as osmosis [7] and is the natural process of water and dissolved species exchange between plants and soil driving the photosynthesis process in plants. Due to the rise in water level the pressure on the high concentration side of the membrane increases. The process will continue until thermodynamic equilibrium is attained, which manifests itself by both sides of the semi-permeable membrane having the same value of the chemical potential of water.

The pressure difference between solutions containing the same i^{th} species when the equilibrium state is attained (osmotic pressure) can be derived in the following way. The chemical potential for the species in the solution is expressed as:

$$\mu_i(T,p,x_i) = \mu_i^{\oplus}(T,p) + R_i T \ln a_i \tag{1}$$

where: T, p, a_i and R_i denote temperature, pressure, activity of i^{th} species in the solution and its particular gas constant, respectively. The symbol $\mu_i^{\oplus}$ denotes the chemical potential for the pure i^{th} component and x_i stands for the mole fraction of i^{th} species. Since the chemical potential corresponds to the specific free enthalpy (Helmholtz energy) of the species, the presence of the species in the solution decreases its specific enthalpy.

If the thermodynamic equilibrium is attained between solutions with different pressures and concentrations present on both sides of the membrane then the chemical potentials of the i^{th} species follow the relation:

$$\mu_i^{\oplus}(T,p_1) + R_i T \ln a_{i,1} = \mu_i^{\oplus}(T,p_2) + R_i T \ln a_{i,2} \tag{2}$$

From the known relation for variation of the chemical potential with temperature and pressure for the pure species at constant temperature and incompressible fluid (liquid solution):

$$\mu_i^{\oplus}(T,p_2) = \mu_i^{\oplus}(T,p_1) + \int_{p_1}^{p_2} v_i dp = \mu_i^{\oplus}(T,p_1) + v_i \Pi \tag{3}$$

where: $\Pi = p_2 - p_1$ is the osmotic pressure and v_i is the partial volume of i^{th} species in the solution. When the relation above is substituted into eq. (2) the following formula is obtained:

$$\begin{aligned}\Pi = \tfrac{R_i T}{v_i} = \tfrac{\bar{R}T}{M_i v_i}\left(\ln a_{i,1} - \ln a_{i,2}\right) = \\ = \tfrac{\bar{R}T}{M_i v_i}\left[\ln\left(\gamma_{i,1} x_{i,1}\right) - \ln\left(\gamma_{i,2} x_{i,2}\right)\right] = \\ = \tfrac{\bar{R}T}{M_i v_i}\left[\ln\left(x_{i,1}\right) - \ln\left(x_{i,2}\right) + \ln\left(\gamma_{i,1}\right) - \ln\left(\gamma_{i,2}\right)\right]\end{aligned} \tag{4}$$

where: M_i is the molecular mass and γ_i is the activity coefficient of i^{th} species, while $\bar{R}$ is the universal gas constant. The activity coefficient depends on temperature, pressure and solution composition.

In the case of dilute and ideal solutions when species $i = 1$ is the solvent $\gamma_1 = 1$ and $\ln x_1 = \ln\left(1 - \sum_{j=2} x_j\right) \approx -\sum_{j=2} x_j$ hence:

$$\Pi = \frac{\bar{R}T}{M_1 v_1} \sum_{j=2} \left(x_{j,2} - x_{j,1}\right) = \frac{\bar{R}T}{V_{m,1}} \sum_{j=2} \left(x_{j,2} - x_{j,1}\right) \tag{5}$$

where: $V_{m,1}$ is the molar volume of the solvent. Moreover for the dilute solutions the mole fraction can be expressed as $x_j = n_j / \sum_k n_k \approx n_j / n_i$ and $V_{m,1} n_1 \cong V$ so finally:

$$\Pi = \bar{R}T \sum_{j=2} \Delta c_j \tag{6}$$

where: $c_j = n_j / V$ is the molar concentration of the j-th dissolved species.

For the dissolved species which undergo dissociation, the summation in eq. (6) extends over all ions formed. For a binary solution of salt in water 1 mole of sodium chloride (NaCl) dissociates into 2 moles of ions (Na^+ and Cl^-) and the summation in eq. (6) extends to these two ions.

For the real solution of concentration c_2 of NaCl in water the osmotic pressure is often expressed in the form:

$$\Pi = \varphi \bar{R}T \Delta c_2 \tag{7}$$

where Δc_2 is the concentration difference across the membrane while φ is the Van't Hoff coefficient dependent on the discrepancy between the ideal and real solutions and the number of ions into which the dissolved species dissociates [8]. Therefore for the ideal solution of NaCl in water $\varphi = 2$. For mean salinity of sea water of 35 g/l and negligible salinity of the fresh water used, the osmotic pressure is about 2.7 MPa.

2.1. Flow of water, concentration polarization and salt leakage

Flow of water across the semi-permeable membrane follows from the difference between the chemical potentials of water on both sides of the membrane. It is described by the formula:

$$j_w = A\left(\Pi - \Delta p\right) \tag{8}$$

where: j_w is the volumetric flux of water while A is the membrane permeability. The permeability of the membranes depends on the type and structure of the membrane and varies in the range $0.05.10^{-11}$–5.10^{-11} $m^3/(m^2 sPa)$ [2]. If the pressure difference Δp between the solutions is smaller than the osmotic pressure, the flow of water occurs from the low salt to the high salt concentration solution (forward osmosis). If the opposite is true, then the flow of water occurs in the reverse direction (reverse osmosis).

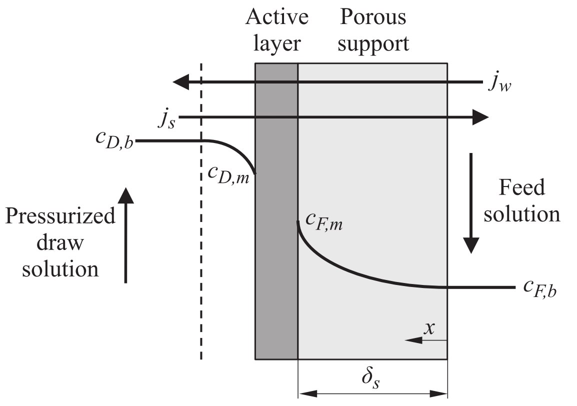


Figure 2: Concentration polarization across the membrane (the symbols in this figure are explained in section 5)

The membrane is usually not perfectly selective and therefore the transfer of both the solvent (water) and solute (salt) occurs through the membrane due to solubilization and diffusion. The permeate water flows from the fresh water into the draw solution under the action of osmotic pressure, while salt permeates from the salty water across the membrane into the fresh water due to the salt concentration gradient [9]. The membrane is usually composed of two layers: a thin active layer and a thicker support layer. The active layer faces the feed solution (of low salt concentration). The support layer facing the draw solution (of high salt concentration) does not take part in the selective transfer of solution components. However, it contributes to appearance of the salt concentration difference, known as the Internal Concentration Polarization (ICP), due to resistance to the salt flow.

The water flow across the membrane is initially higher and then starts to decrease until the steady state is attained. The reason for this is the drop in osmotic pressure. This results from the flow of salt and the formation of boundary layers of salt close to the membrane's external surfaces. In the more concentrated solution (draw solution) the salt concentration decreases close to the membrane surface while in the less concentrated solution (feed solution) it decreases—see Fig. 2. This process is known as External Concentration Polarization (ECP). The effect of the ECP can be decreased if in the boundary layer the flow along the membrane is turbulized.

These concentration polarization phenomena lead to a decrease in the resulting salt concentrations and in turn to a drop in osmotic pressure and in the flow of water across the membrane.

There are also other reasons for clogging of the mass transfer across the membrane, such as precipitation of inorganic compounds on the membrane, the formation of deposits of particles or organic materials on the membrane surface and adsorption in the membrane. This additional blockage lends additional resistance to the mass transfer across the membrane and contributes to a significant decrease in overall membrane performance. Therefore periodical cleaning is required to maintain membrane performance.

The power generated per unit surface of the membrane when concentration polarization phenomena are absent can be written as:

$$\dot{w} = j_w \Delta p = A\left(\Pi - \Delta p\right)\Delta p \qquad (9)$$

Therefore, the power generated varies with the pressure difference across the membrane. Maximum power can be attained when $\Delta p = \Pi/2$ and thus

$$\dot{w}_{\max} = A\Pi^2/4 \qquad (10)$$

This value is known as the power density of the membrane and for the mean salt concentration in the world's oceans it usually varies in the range 3 to 10 W/m^2 although much higher values for new membranes attaining 30 W/m^2 were recently reported.

3. Use of osmosis to generation of useful energy

There are three basic ways of extracting power from the difference in salt concentrations in solutions. The first one is Pressure-Retarded Osmosis (PRO) based on applying the constant pressure difference between two solutions, which is less than the osmotic pressure [2]. This results in the flow of pure water across a semi-permeable membrane from the dilute and low pressure solution to the high salt concentration pressurized solution and retaining the solute (dissolved salt). Through applying higher pressure to the concentrated solution, the water transport is partly retarded. The transport of water from the low-pressure diluted solution to the high-pressure concentrated solution results in the pressurization of the volume of transported water. This pressurized volume of transported water can be used to run a turbine and finally generate electrical power.

The second method is Reversed Electro-Dialysis (RED) where two types of ion-selective (cation and anion) exchange membranes are used [10]. A number of these membranes are arranged in a variable pattern between a cathode and an anode. The cells between the membranes are alternately filled with concentrated salt solution and diluted salt solution. The difference in chemical potential due to the difference in salinity causes the transport of ions through the membrane from the concentrated solution to the diluted solution. In a sodium chloride solution, sodium ions permeate through the cation exchange membrane toward the cathode and chloride ions permeate through the anion exchange membrane toward the anode. Electro-neutrality of the solution in the anode cell is maintained by oxidation on the surface of the anode. Electro-neutrality of the solution in the cathode cell is maintained by reduction on the cathode surface. The electric potential difference builds up between the two neighboring electrodes. The difference in electrical potential between the outer cells of the membrane stack is the sum of the potential differences over each membrane. This potential difference means that an electron can be transferred from the anode to the cathode via an external electrical circuit when the external load or energy is connected to the circuit.

The third method is the Capacitive Method (CP), which is based on electric double-layer capacitor technology [11]. The capacitor is first immersed in the salt solution and charged. It is then placed in the freshwater and discharged. Due to the formation of an electric double layer, the voltage over the electrodes remains low during the charging step and after bringing the electrodes into the freshwater the voltage over the electrodes increases; thus each completed cycle effectively produces electric.

4. Operation of osmotic power plants using PRO

A schematic diagram of a PRO based hydro-osmotic power plant is shown in Fig. 5. The basic components of the plant are: filters, pumps, osmotic chambers with semi-permeable membranes (membrane unit), pressure exchanger, hydraulic turbine, electric generator and connecting pipes. The fresh (feed solution) water is let into the plant and filtered before entering the low pressure osmotic chamber, which is supplemented with a semi-permeable membrane. Part of the feed water permeates through the membrane and the rest is fed back into the river.

Saltwater (draw solution) is pumped from the sea with volumetric flow rate $\dot{V}$ and is filtered. Experience from Norwegian water treatment plants shows that mechanical filtration down to 50 mm in combination with a standard cleaning and maintenance cycle is enough to sustain membrane performance for 7–10 years [12]. Similar lifetime data are assumed for osmotic power plants. The volumetric feed of sea water is about twice that of fresh water. After being pressurized by Δp in the boost pump, the draw solution flows through pipes carrying it to the high pressure osmotic chamber. Two water chambers in the membrane unit are in contact via the semi-permeable membrane. The membrane should enable high water flux and high salt retention [13]. For significant volumes of fresh water to cross the semi-permeable membrane, a large membrane area is required. Therefore, the membranes are packed or coiled in modules to save space, with a packing density of 1000 m^2 membrane surface area per 1 m^3 modules [14]. Different types of membrane modules were considered such as: flat-sheet, hollow-fibre, spiral wound or tubular modules [15]. As membrane modules have been used in desalination plants for decades, well-developed membrane technology can be adopted for osmotic power plants. In the membrane module a significant amount (80–90%) of the fresh water is transferred by osmosis across the membrane into the pressurized sea water. The fresh water flowing through the membrane is thus pressurized while the sea water is diluted by the fresh water permeating through the membrane. The volumetric flow rate of the draw water is increased by $\dot{V}_m = \triangle\dot{V} = j_w A$ due to diffusion of fresh water through the membrane. Sea water leaving the osmotic chamber is therefore pressurized to $p_0 + \Delta p$ and has the volumetric flow rate $\dot{V} + \Delta\dot{V}$. Thus the osmotic process increases the volumetric flow of high pressure water and is therefore the key energy transfer mode in the plant.

The diluted sea water (brackish water) is subsequently directed to the hydraulic turbine where it depressurizes to the initial sea water pressure p_0. In the ideal case when there is no need of pumping water from the river, the efficiency of the pump and turbine is 100% and no friction due to liquid viscosity is present (manifesting itself by lack of pressure drop in the pipes), the power delivered to the system by the boost pump is equal to $\Delta p\dot{V}$ while the power taken away from the system is $\Delta p\left(\dot{V} + \Delta\dot{V}\right)$. Therefore the net power of the osmotic power plant is $\Delta p\Delta\dot{V}$ or if expressed per 1 m^2 of the membrane $\Delta p j_w$.

As the turbine uses brackish water but there is only a fresh water supply, the salt concentration on the brackish water side decreases during its flow along the membrane. To achieve a constant performance, the salt concentration of the brackish water must be kept constant (and ideally close to the concentration of the salt water reservoir, e.g., sea water salinity.

The use of a pressure exchanger was proposed in order to keep the salt concentration constant and to increase the effectiveness of energy conversion in the system. The device is mounted before the osmotic chambers, i.e., the membrane module. The fresh sea water—before entering the membrane module—flows into the pressure exchanger where its pressure is increased. The brackish water from the membrane module is split into two flows. About 1/3 of the water goes to the turbine to generate power and about 2/3 returns to the pressure exchanger to pressurize the sea water feed. The brackish water from the pressure exchanger with decreased pressure is joined with the depressurized water from the turbine or is discharged directly to the sea. Due to the presence of the pressure exchanger the draw pump, which is fed with the fresh sea water, requires much less power—only to push the fresh water to the pressure exchanger where the specified increase of pressure occurs. The smaller power of the pump leads to smaller energy loss. Use of the pressure exchanger increases the efficiency of the hydro-osmotic power plant by several percent. This pressure exchanger is commercially available and has an efficiency of up to 0.97 [13]. It exchanges a volume of brackish water with, ideally, an equal volume of salt water, by using the mechanical energy of the brackish water volume to pump the salt water volume against the operating pressure in the brackish water chamber. Pressure exchangers have been used in desalination plants for decades. To date they offer the most efficient way to exchange water volumes of different pressures. However, an additional pump is required to compensate for the pressure loss in the pressure exchanger. Furthermore, on the fresh water side, there is a small amount of flushing required to avoid a build-up of salinity and this is provided by a fresh water outlet.

In reality all parts of the hydro-osmotic power plant such as pumps, connecting pipes, pressure exchanger, hydraulic turbine and electric generator have energy efficiency of less than 100% due to energy dissipation processes occurring in them. Moreover, sea water and fresh water have contaminations in the form of sand, suspensions of small mineral and organic particles, microorganisms and therefore they should be filtered before entering modules with semipermeable membranes otherwise intensive fouling will occur. This requires additional expenditure of energy for pumps equipped with filters. All these factors together with the lack of full selectivity of the membranes, manifesting itself by a small stream of salt from the sea water to the fresh water that is present, lead to decreases in the degree of energy conversion in the hydro-osmotic power plant.

5. Existing prototype hydro-osmotic power plant using PRO

The first experimental hydro-osmotic power plant operating on PRO was opened in Tofte (Norway) in November 2009 and run by Statkraft until December 2013. The plant was built to test and evaluate the effectiveness of the membrane and the whole energy generation system in real-world conditions. The project envisaged 2000 m^2 of membranes with power density 5 W/m^2 giving total power of 10 kW [12].

A second prototype hydro-osmotic power plant was opened in Fukuoka (Japan) also in 2009. The project focused on using the plant to supply electric energy to a neighboring desalination plant. The power plant achieved power density of 7.7 W/m^2 in 2011 and 13.3 W/m^2 in 2013 [6]. The salt water was sourced from the desalination plant while fresh water was taken from the sewage water from a nearby sewage treatment plant. Before entering the membrane containing modules the sewage was initially subjected to ultrafiltration and then low-pressure reverse osmosis. A pressure exchanger was used in the hydro-osmotic power plant while the streams of desalination plant brine and sewage were 460 and 420 t/day, respectively. A new hydro-osmotic power plant with total power of 100 kW is planned in Japan in the short term.

Other plans for building hydro-osmotic power plants were announced in Canada and Iran. In Iran a complete study and design was presented for a 25 MW hydro-osmotic power plant on Bahmanshir River, which flows into the Persian Gulf (Arabian Gulf) [16].

6. Mathematical model of the hydro-osmotic power plant

In this paper a simplified mathematical model of operation of the hydro-osmotic power plant was proposed. According

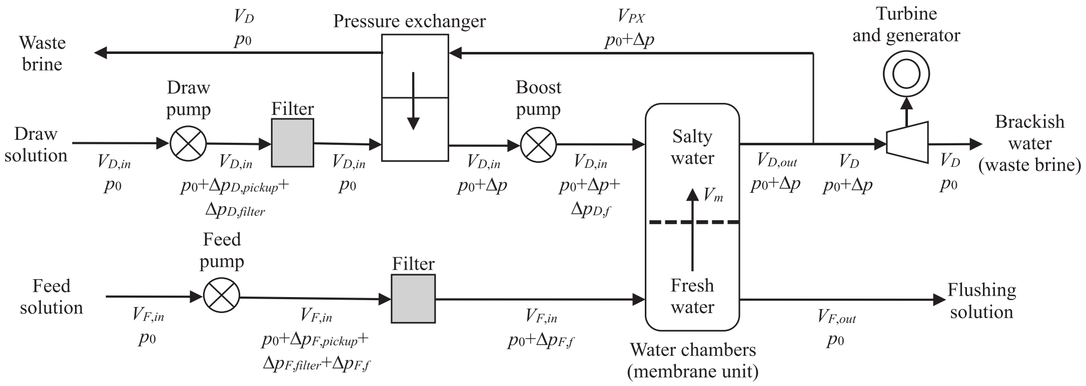


Figure 3: A schematic diagram of the hydro-osmotic power plant using PRO. The symbol Δp denotes pressure difference across the membrane; for other symbols please see the explanations in section5

to eq. (9) the power generated by the hydro-osmotic power plant depends on the volumetric flow rate of water through the membrane $\dot{V}_m = A_m j_w$ and the pressure difference Δp.

This volumetric flow of water is dependent on the osmotic pressure Π—see eq. (7). The osmotic pressure is proportional to the difference in the salt concentration across the membrane. Due to the concentration polarization (external and internal), see section 1, the difference in the salt concentration across the semi-permeable membrane is reduced. This leads to a decrease in osmotic pressure between the draw and feed solution to Π^* according to the following relation [17, 9]:

$$\Pi^* = \bar{R}T\Delta c_m \tag{11}$$

$$\begin{aligned}\Delta c_m &= c_{D,m} - c_{F,m} = \\ &= \frac{c_{D,b}\exp(-j_w k_D) - c_{F,b}\exp\left[j_w(k_F+S)\right]}{1+\frac{B}{j_w}\left\{\exp\left[j_w(k_F+S)\right]-\exp(-j_w k_D)\right\}}\end{aligned} \tag{12}$$

where: $c_{D,m}$ and $c_{F,m}$ as well as $c_{D,b}$ and $c_{F,b}$ are the volumetric salt concentrations in the draw as well as in the feed solution at the membrane surface (subscript m) and in the bulk fluid (subscript b), respectively, and k_D and k_F are the convective mass transfer coefficient on the draw and feed sides, respectively. The symbol B is the permeability of the membrane to the reverse salt flow, given by the following formula:

$$j_s = B\left(c_{D,m} - c_{F,m}\right) \tag{13}$$

The symbol S stands for [18]:

$$S = \frac{\delta_s}{D_{es}} \tag{14}$$

where δ_s is the thickness of the membrane support layer (see Fig. 2) and D_{es} denotes the effective diffusion coefficient of the salt in this layer. The latter can be expressed as:

$$D_{es} = \frac{D\varepsilon}{\tau} \tag{15}$$

where: D is the diffusion coefficient of salt in water, ε—the support layer porosity and τ is the tortuosity coefficient for the salt flow in the porous medium (membrane support layer). The diffusion coefficient D varies with temperature and salt concentration according to the following relation [19]:

$$D = 6.725\cdot 10^{-6}\exp\left(1.546\cdot 10^{-4}c - \frac{2513}{T}\right) \tag{16}$$

The feed and draw mass transfer coefficients appearing in eq. (12) were found from the correlation [20]:

$$k = 5.58\left(\frac{uD^2}{d_h L}\right)^{0.33} \tag{17}$$

The volumetric flow of the draw and feed water at the inlet and exit to the membrane module unit, see Fig. 3, are different due to exchange of water through the membrane and can be calculated from the formulae:

$$\dot{V}_{D,out} = \dot{V}_{D,in} + \dot{V}_m \quad \text{and} \quad \dot{V}_{F,out} = \dot{V}_{F,in} - \dot{V}_m \tag{18}$$

The flow of fresh water through the membrane also changes the salt concentration in the bulk stream of the draw and feed water at the exit of the membrane unit compared to the unit inlet:

$$\begin{gathered} c = \frac{c_{D,b,in}\dot{V}_{D,in} - j_s A_m}{\dot{V}_{D,out}} \\ \text{and} \\ c_{F,b,out} = \frac{c_{F,b,in}\dot{V}_{F,in} + j_s A_m}{\dot{V}_{F,out}} \end{gathered} \tag{19}$$

The main pressure losses occur in the membrane unit, compared to the pressure loss in the feeding pipes. The reason for this are the small liquid passages between the membranes in the unit. The frictional pressure drop on the draw and feed water sides was evaluated from the expressions [17]:

$$\begin{gathered} \Delta p_{D,f} = f_D \frac{L}{d_{h,D}} \frac{\rho u_D^2}{2} \\ \text{and} \\ \Delta p_{F,f} = f_F \frac{L}{d_{h,F}} \frac{\rho u_F^2}{2} \end{gathered} \tag{20}$$

where L is the membrane length, d_h is the passage hydraulic diameter while u is the water velocity respectively for the draw (D) and (F) water streams. Laminar flow was assumed

Table 3: Modelling data (flat sheet membrane)

Parameter	Value
Membrane parameters	
Water permeability A, m/Pa/s	$1.87 \cdot 10^{-12}$
Salt permeability B, m/s	$1.11 \cdot 10^{-12}$
Structure parameter S, m	$6.78 \cdot 10^{-4}$
Salt diffusion coefficient ($T = 25°$C) D, m^2/s	$1.5 \cdot 10^{-9}$
Membrane module length L, m	1.5
Membrane surface area A_m, m^2	2000.0
Membrane module hydraulic diameter d_h, m	0.5
Inlet flow parameters	
Concentration of feed bulk $c_{F,b,in}$, g/l	0
Concentration of draw bulk $c_{D,b,in}$, g/l	35–60
Inlet feed velocity u_F, m/s	0.25
Inlet draw velocity u_d, m/s	0.25
Temperature T, °C	0–30
Inlet hydraulic pressure difference $\triangle p_{in}$, MPa	0–0.97
Inlet feed volumetric flow rate $\dot{V}_{F,in}$, m^3/s	0.013
Inlet draw volumetric flow rate $\dot{V}_{D,in}$, m^3/s	0.02
Equipment parameters	
Pump and motor efficiency η_{pump}, η_{motor}	0.77
Turbine and generator efficiency η_t, η_g	0.85
Pressure exchanger efficiency η_{PX}	0.95
Pickup and filter pressure loss Δp_{pickup}, Δp_{filter}, Pa	0.0
Volumetric flow rate of the brackish water to the turbine	$\dot{V}_D = \dot{V}_m$

in these passages and the friction coefficient was calculated from the formula: $f = 64/Re$, while the Reynolds number was defined as: $Re = ud_h/\nu_w$ with the symbol ν_w denoting the kinetic viscosity of water.

The pressure difference between the draw and feed solution at the outlet from the membrane unit Δp_{out}, corresponding to the pressure drop in the hydraulic turbine, was determined from the relationship:

$$\Delta p_{out} = \Delta p_{in} - \left(\Delta p_{D,f} - \Delta p_{F,f}\right) \tag{21}$$

where Δp_{in} is the pressure difference between the draw and feed solution at the inlet to the membrane unit, see Fig. 3.

The electric power generated by the hydro-osmotic power plant can be found from the expression:

$$\dot{W}_{net} = \dot{V}_D \Delta p_{out} \eta_t \eta_g - \sum_{i=1}^{3} \dot{W}_{p,i} \tag{22}$$

where: η_t and η_g are the efficiency of the turbine and electric generator while the last term in eq. (22) corresponds to the power required for running draw, feed and boost pumps and $\dot{V}_D$ is the volumetric flow rate of the brackish water to the turbine—see Fig. 3.

The volumetric flow rate into the turbine is smaller than the volumetric flow rate of the brackish water leaving the membrane unit due to the water stream feeding the pressure exchanger:

$$\dot{V}_D = \dot{V}_{D,out} - \dot{V}_{PX} \tag{23}$$

where $\dot{V}_{PX}$ denotes the flow rate of the brackish water into the pressure exchanger.

The power needed to run the draw and feed pumps can be calculated from the following formulae:

$$\dot{W}_{p,D} = \frac{\left(\Delta p_{D,pickup} + \Delta p_{D,filter}\right) \dot{V}_{D,in}}{\eta_{pump} \eta_{motor}} \tag{24}$$

$$\dot{W}_{p,F} = \frac{\left(\Delta p_{F,pickup} + \Delta p_{F,filter} + \Delta p_{D,f}\right) \dot{V}_{F,in}}{\eta_{pump} \eta_{motor}} \tag{25}$$

and for the boost pump from the expression:

$$\dot{W}_{p,B} = \frac{\left[\Delta p_{D,f} + \Delta p_{out} \left(1 - \eta_{PX}\right)\right] \dot{V}_{D,in}}{\eta_{pump} \eta_{motor}} \tag{26}$$

where Δp_{pickup}, Δp_{filter} denote the required pressure for picking up the draw or feed water from their reservoirs and pushing it through a filter, respectively. The symbols η_{pump} and η_{motor} are the energy conversion efficiencies of the pump and the motor that runs it, respectively, while η_{PX} is the efficiency of the pressure exchanger. The last efficiency can be determined from the relationship [13]:

$$\eta_{PX} = \frac{E_{salty\ water,out}}{E_{brackish\ water,in}} \tag{27}$$

where E stands for the energy of the salt water or brackish water streams being a product of their flow rates and pressures. Efficiency is a function of the pressure differentials and the volumetric losses (leakage) through the device.

The efficiency of the hydro-osmotic power plant can be found from the following expression:

$$\eta_{plant} = \frac{\dot{W}_{net}}{A_m w_{\max}} \tag{28}$$

7. Results of the numerical analysis of operation of the hydro-osmotic power plant

The model described in the previous section is summarized by a flow chart—see Fig. 4 and was simulated in MATLAB-SIMULINK software. The assumed membrane, inlet and equipment parameters are presented in Table 3.

The following cases were considered in the analysis of the hydro-osmotic power plant:

1. Operation of the power plant without any losses or concentration polarization.
2. Operation of the power plant without losses except for concentration polarization.
3. Operation of the power plant with pressure and concentration polarization losses.
4. Effect of water temperature on operation of the power plant.
5. Effect of the membrane water and salt permeability on operation of the power plant.

The selected results of calculations are presented in Fig. 5–9. Initially all parameters important for operation of the hydro-osmotic power plant, i.e., osmotic pressure, water and salt fluxes through the membrane, power density and overall efficiency were calculated for the different feed and draw salt concentrations. Power density of 7.47 W/m^2 was obtained at osmotic pressure of 5 MPa, inlet pressure of 0.97 MPa and inlet NaCl concentration differences between

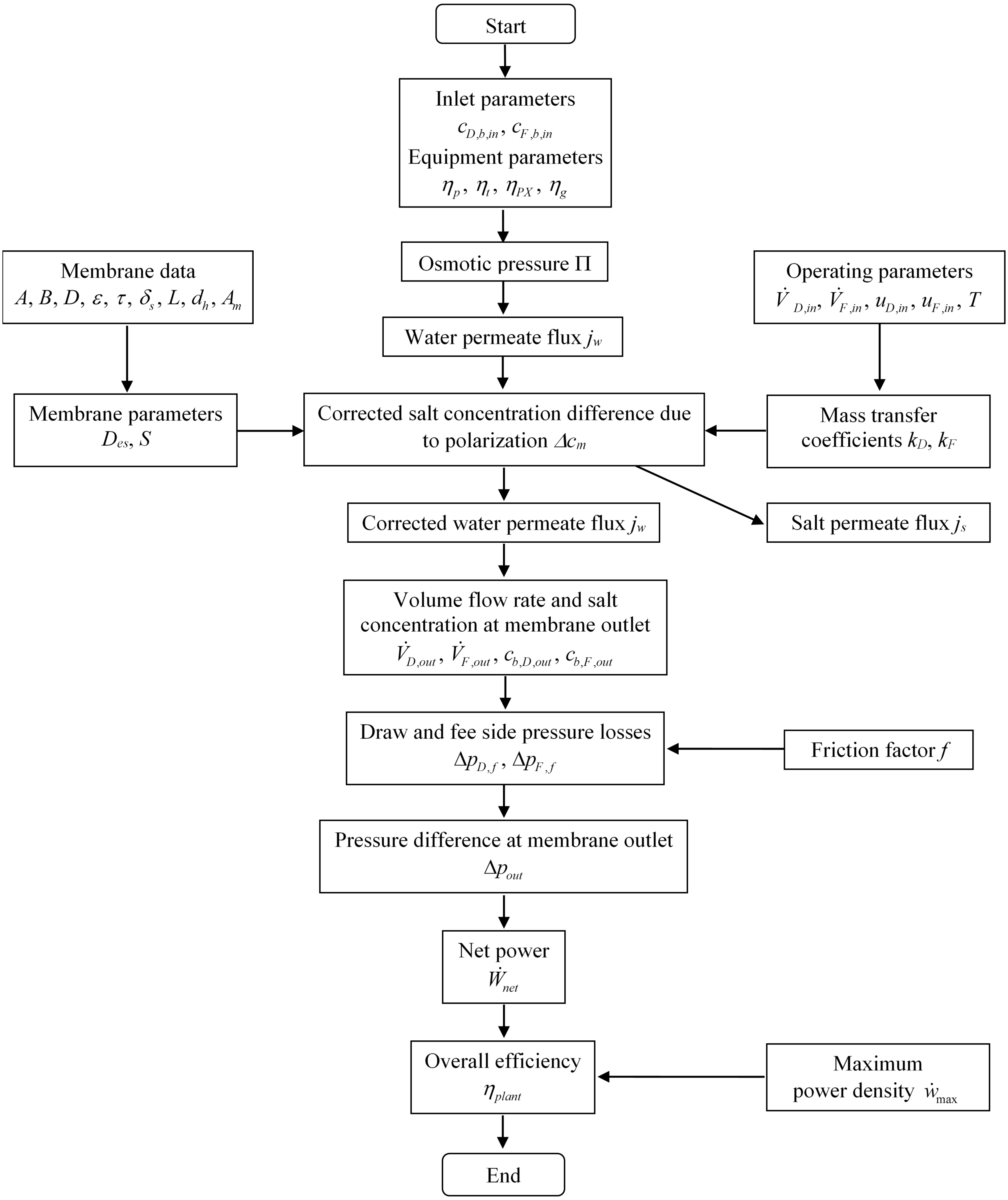
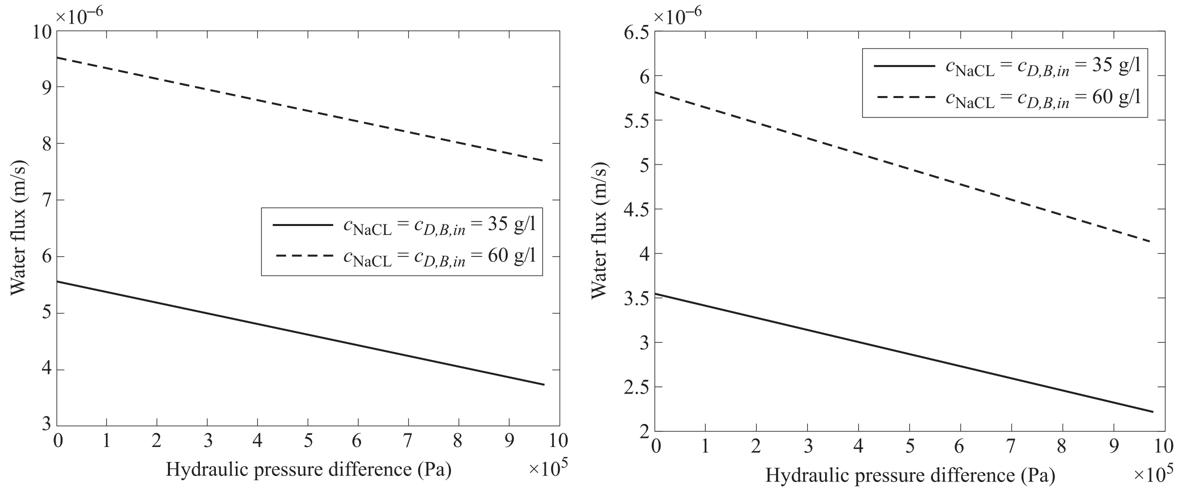


Figure 4: A flow chart of calculations of the hydro-osmotic power plant using PRO



(A)

(B)

Figure 5: Permeate water flux through the membrane versus the inlet pressure difference and NaCl concentration for: A) case a without any losses or concentration polarization and B) case c with pressure and concentration polarization losses

the draw and feed streams of 60 g/l. On the other hand power density of 1.54 W/m^2 was obtained at osmotic pressure of 2.9 MPa, inlet pressure of 0.31 MPa and NaCl concentration differences of 30 g/l. This means that these parameters have a major effect on operation of the hydro-osmotic power plant. The values of water flux flowing through the membrane decrease when the inlet pressure difference increases – Fig. 5. In contrast, increases in power density and efficiency of the power plant are observed with increasing inlet pressure difference—Fig. 6 and 7. The effect of concentration is important. When the salt concentration in the feed stream increases, the permeate water flux and power density decrease because of the reduction in osmotic pressure. When the draw solution has a higher salt concentration, the permeate water fluxes and power densities are much higher. This is due to higher osmotic pressure.

The effect of concentration polarization was discussed. This effect caused a reduction in performance of the hydro-osmotic power plant . Significant drops in the permeate water flux, power density and plant efficiency were observed—see Fig. 5 B, 6 B and 7 B. The drops occurred because the concentration polarization reduces the driving force across the membrane. The effect varies depending on the salt concentrations of the feed and draw solutions.

The effect of water temperature on performance of the hydro-osmotic power plant was also investigated. The osmotic pressure is directly proportional to temperature, but the salt diffusion coefficient through the support layer of the membrane and the streams boundary layers decreases—see eq. (16). When the osmotic pressure increases with temperature, the water permeate flux and power density also increase. The salt diffusion coefficient affects the mass transfer coefficient and is related to the concentration polarization effect and, therefore, overall power plant efficiency decreases—Fig. 8.

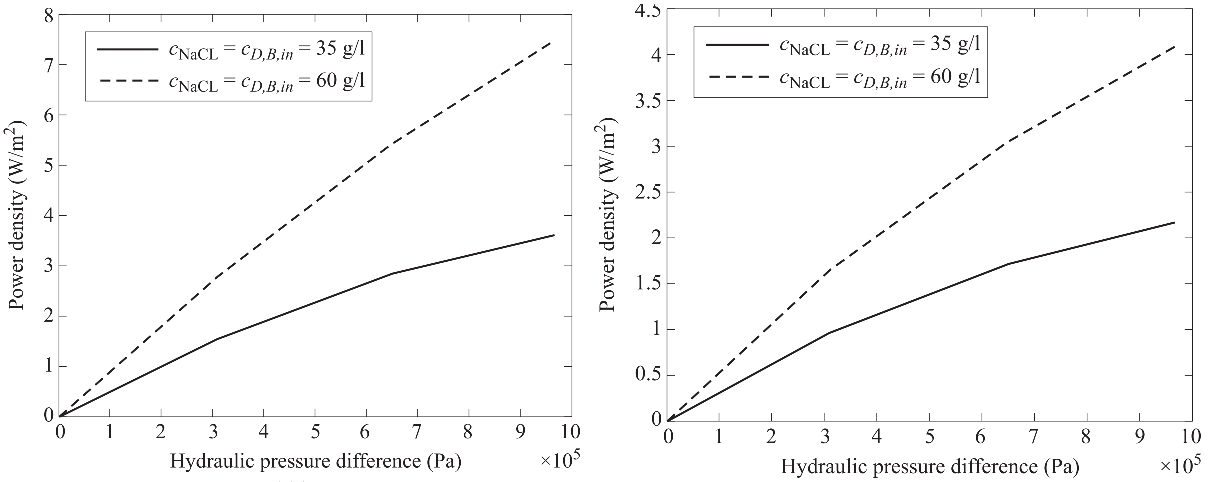
Finally, the model was used to study the effect of water permeability on power plant operation—see Fig. 9. Power density increases with higher water permeability of the membrane due to the increased permeate water flux through the membrane.

8. Conclusions

A model of the hydro-osmotic power plant using differences in salt concentrations between two streams of water and operating in open cycle was proposed in the paper. The model adopts many assumptions: steady state operation of the hydro-osmotic power plant, isothermal process, simplified relations for calculation of the pressure drop in the membrane modules and the mass transfer coefficients not accounting for the water permeate flux affecting the velocity and concentration boundary layers on both sides of the membrane. Moreover, the flow through the membrane is treated as 1D and no variation of the osmotic pressure along the membrane is assumed. These assumptions can be released in the more advanced mathematical model of the hydro-osmotic power plant.

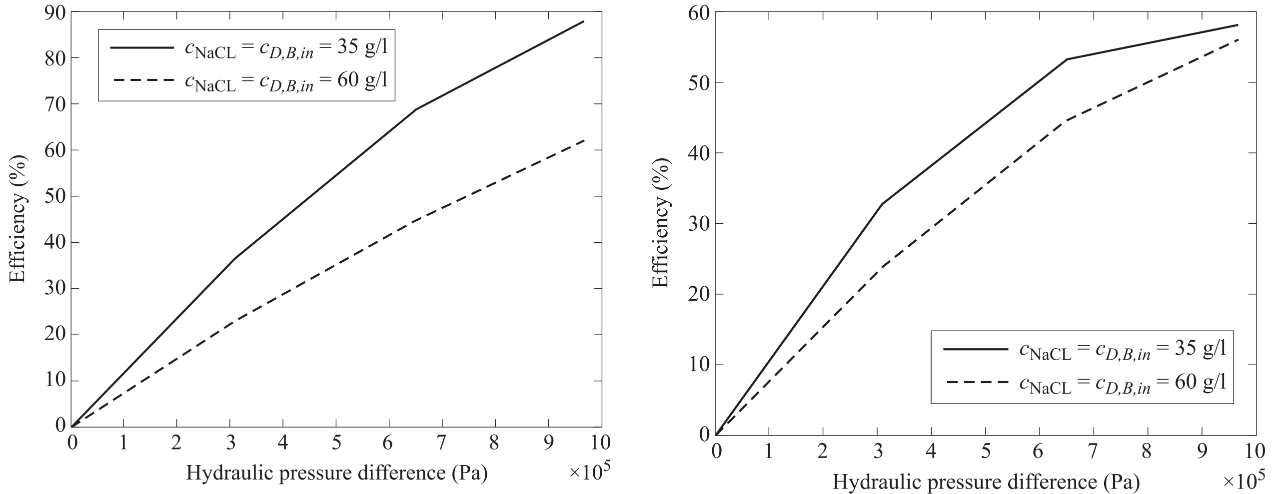
It was found that the main factors affecting the efficiency of energy conversion in this power plant are associated with the quality of membranes. Higher water permeability of the membrane and its selectivity (no reverse flow of the salt) significantly contribute to achieve this goal. The temperature of the draw and feed streams in the range 0–30°C exerts a relatively minor influence on plant efficiency, leading to a fall of a few percent.

The technology of generating electric energy using osmosis is at an early stage of development therefore the investment costs for implementation are high. Nevertheless, it is



(A) **(B)**

Figure 6: Power density of the membrane versus the inlet pressure difference and NaCl concentration for A) case a without any losses or concentration polarization and B) case c with pressure and concentration polarization losses



(A) **(B)**

Figure 7: Overall efficiency of the hydro-osmotic power plant versus the inlet pressure difference and NaCl concentration for A) case a without any losses or concentration polarization and B) case c with pressure and concentration polarization losses

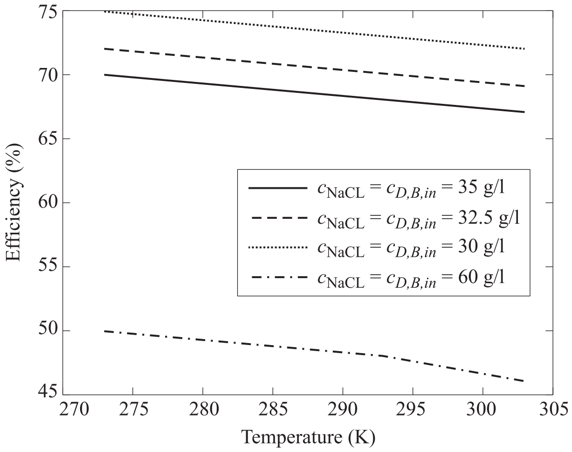


Figure 8: Variation of overall efficiency of the hydro-osmotic power plant with temperature and NaCl concentration

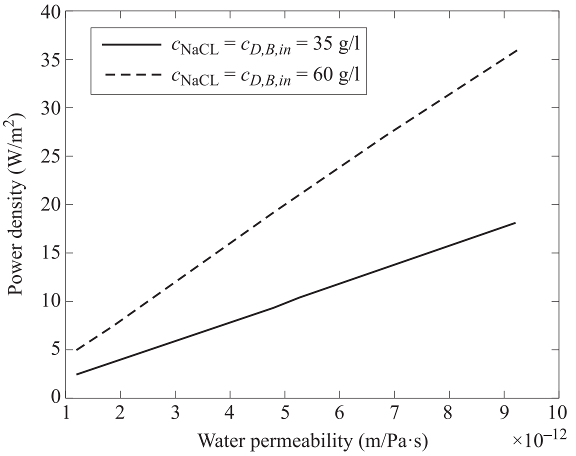


Figure 9: Variation of power density with membrane water permeability and NaCl concentration

promising due to the lack of emissions and stability in the energy supply. Studies on improving the efficiency of energy conversion in the hydro-osmotic power plant are currently being carried out, concentrating on developing new highly selective membranes with better water permeability, mathematical modelling and optimizing of the plant system. The presented open system uses salt water, both draw and feed streams, taken from the environment. However it is possible to imagine an osmotic power plant operating as a closed system. Such plants do not have to be located at river estuaries or can use fluids other than salt and fresh water and can attain high efficiencies [19].

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