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A mathematical analysis of two dimensional steady state heat conduction in the coil of an induction heater using finite element method

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Abstract

In developing heaters typically an induction heater within specific temperature limits can be a key issue impacting the efficiency of the overall policy, as the typical loading of an induction heater is costly. Mathematical modelling is highly useful in terms of estimating the rise in temperature and in shedding light on the wider processes. The projected model might in addition reduce computing prices. The paper develops a 2-Dimensional (2-D) steady state thermal model in polar co-ordinates by means of finite element formulation and arch shaped components. A temperature time methodology is utilized to calculate the distribution of loss in various elements of the induction heater and used as input for finite element analysis. Additional precise temperature distributions are obtained. The projected model is applied to predict the temperature rise within the coil of the induction heater 3200 W totally encircled fan-cooled induction heater. The temperature distribution was determined considering convection from the outer air gap surface and circular finish surface for each entirely encircled and semi encircled structures.

Keywords: Heater; Coil; Design Performance; FEM

1. Introduction

In electrical engineering design problems, heating is the most vital burden that limits design. Heating reduces the life span of a product considerably. Different types of losses in induction heater equipment are the main sources of heat. Beyond predicting heat generation, a proper cooling method has to be established to limit the heating effect. For this, the exact point source of the heat is to be determined. To be specific, in the case of an induction coil, in its normal operating condition it is not easy to determine the exact region where the heat is being generated, as it is to be found below the surface and its high power causes problems for any measuring device. But if it is malfunctioning any an abrupt rise in temperature must be identified or better predicted before the temperature builds to a point when jeopardizes the safety of the insulation. There the problem arises when we need to determine the exact point where the heat is originated, as the heat generation may be unequal for a induction heating coil when it delivers heat. Now, to solve this problem, differential equations enjoy some advantages. However, as computers and microprocessors have developed, various types of numerical methods are commonly used. The finite element method, a method that has many advantages over any other type of solution, is extensively used to solve heat conduction problems in 2-Dimensional (2-D) and in 3-Dimensional (3-D). After considering some geometrical restrictions, we obtain a solution that is almost equal to the original values and hence this method is advantageous and used in herein.

The paper deals with a basic problem related to heat flow through the induction coil in common steady state working conditions. The body of induction coil consisting of copper, lead, insulations, coolant etc may be considered as a composite. Because of volumetric, line or point source, heat may be produced in the induction coil. Maximum heat is produced or generated due to eddy current loss in the induction coil. In the usual operating condition of the induction heater, heat is produced continuously in the induction coil [1–3]. If the induction heater is in a faulty condition, heat can discrete in the induction coil. The 2-Dimensional (2-D) steady state finite-element procedure for the thermal analysis of an induction coil provides a chance to conduct in-depth studies

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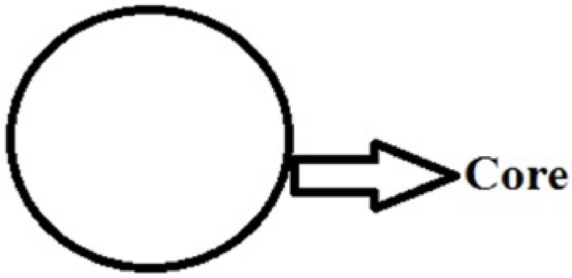

Figure 1: Cross-sectional area of the coil

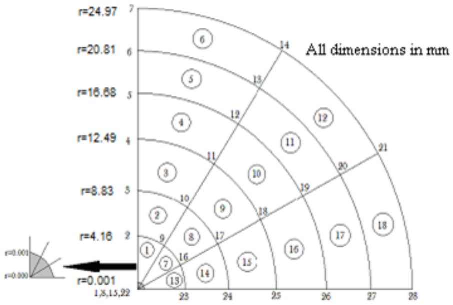

Figure 2: Slice of core conductor delimited by two 90° inclined planes divided into arc formed components

of coil heating issues using a new, expressly derived arch part, along with associates economic data and Gauss routine. A novel two-dimensional finite element procedure with cylindrical polar co-ordinates and expressly derived answer matrices was applied to the answer of the heat conduction equation throughout steady state condition. Though the results are approximate, the strategy is quick, cheap and leads itself to immediate visual imaging of the temperature pattern in an exceedingly 2-Dimensional (2-D) slice of core conductor (single)bounded by two 90° inclined planes divided into arc shaped elements of the coil.

For this end the heat of the conductor is limited to a definite limit [4–7]. For full solution of the heat [8, 9] distribution a 2-D cross-sectional area of the single-core induction coil is calculated to solve the problem [10–12]. The thermal conduction of copper within the core is taken in order to simplify the calculation. In this numerical analysis, the 2-Dimensional (2-D) planes divided into arch shaped elements for symmetry and these are divided into finite elements as shown in Fig. 2. Throughout the solution region arch shaped elements are used [13–15]. This is shown in Fig. 2, taken from Fig. 1 [8, 16].

2. Thermal Constants

For the steady state problem in two dimensions, the following properties are required for each different element materials: Thermal conductivity in radial direction V_r (W/°C). i.e 2.007 for Copper and Insulation Thermal conductivity in circumferential direction V_θ (W/°C). i.e. 1.062 for Copper and Insulation. A typical set of thermal constants are mentioned above for copper wires of the induction coil.

Table 1: Parameter to determine Eddy Current Loss

Parameter	Value
Constant Dependent upon the Material (K_e)	0.2
Maximum Flux Density (B_m^2), Wb/m^2	0.7832
Frequency (f), kHz	55
Thickness (t), m	0.1
Area (v), m^2	0.196

3. Calculation Of Eddy Current Loss

Calculating the eddy current loss in the induction coil in the following:

$$\begin{aligned} P_e &= K_e B_m^2 f^2 t^2 v \\ &= 0.2 \times (0.7832)^2 \times \left(55 \times 10^3\right)^2 \times (0.1)^2 \times 0.196 \\ &= 72737.238\,\mathrm{W} \end{aligned} \tag{1}$$

4. Convective Heat Transfer Coefficient

Two convective heat transfer coefficients are considered: forced convection and normal convection for turbulent and steady state flow in the cylindrical core of the conductor. These depend upon Reynolds number and the Prandtl number.

Force Convection

$$\mathrm{h} = \frac{0.026 \times (\mathrm{R_e})^{0.805} \times (\mathrm{P_r})^{1/3}}{\mathrm{d}} \times \mathrm{k} \tag{2}$$

$$\mathrm{R_e} = \mathrm{PVd}/\mu$$

where:k—fluid thermal conductivity, 70 W/m°C; P—fluid density, 1.022 kg/m^3; V—fluid velocity, 17.5 m/s; μ—fluid viscosity, 2.06 kg/m-s; Re—Reynolds's number

$$\mathrm{R_e} = 16964.70$$

$$\mathrm{R_e} = \frac{1.022 \times 17.5 \times 0.5}{2.06 \times 10^{-5}} = 434101.94$$

$$\mathrm{P_r} = \mathrm{C_p}\,\mu/\mathrm{K} = 2.96 \times 10^{-4}$$

where: Cp—fluid specific heat, 1008.34 J/kg°C; Pr—Prandtl number

$$\begin{aligned} h &= \frac{0.026 \times (434101.94)^{0.805} \times \left(2.96 \times 10^{-4}\right)^{0.33} \times 70}{0.5} \\ &= 30009.42\,\mathrm{W/m^2\,°C} \end{aligned} \tag{3}$$

Natural Convection

$$\mathrm{h} = \frac{0.53 \times (\mathrm{G_r P_r})^{0.4} \times \mathrm{k}}{\mathrm{d}} \tag{4}$$

Table 2: The heat transfer co-efficient for the cylindrical curved surface of the conductor

Item	Symbol	Value
Fluid thermal conductivity, W/m°C	K	70
Hydraulic Diameter, m	D	0.5
Fluid density, kg/m^3	P	1.022
Fluid velocity, m/s	V	17.5
Fluid viscosity, kg/m·s	μ	2.06
Reynolds number	Re	434101.94
Fluid specific heat, J/kg°C	Cp	1008.34
Prandtl number	Pr	2.96×10^{-4}
Heat transfer co-efficient, W/m^2 °C	H	30009.42

Grashhoff Number

$$(G_r) = \frac{g\beta(T_\omega - T_\infty)\times d^3}{v^2} = \frac{9.8\times2.89\times10^{-3}\times600\times0.5^3}{\left(20.03\times10^{-5}\right)^2} = 529444796.48$$

$$\beta = \frac{1}{T_\infty} = \frac{1}{345.5} = 2.89\times10^{-3}$$

So

$$h = \frac{0.53\times(G_rP_r)^{0.4}\times k}{d} = \frac{0.53\times\left(529444796.48\times2.96\times10^{-4}\right)^{0.4}\times70}{0.5} = 830.19\,\mathrm{W/m^2\,°C} \tag{5}$$

To compute the heat transfer coefficient for the rod-shaped curved surface of the conductor, the Reynolds number and therefore the Prandtl number calculated on the basis of fluid thermal conductivity, the hydraulic diameter, fluid density, fluid velocity, fluid viscosity and fluid heat energy were measured. Their values are presented in Table 2.

5. Steps Of Solving Finite Element Problem

Regardless of the approach used to find the element properties, the solution of a continuum problem by the finite element method always follows an orderly step-by-step process.

Step-1: Discretize the continuum: The continuum in the solution region is divided into elements.

Step-2: Select Interpolation Function: Each element is assigned nodes, and the interpolation function is chosen to represent the variation of the field variable over the element. A polynomial is often selected as an interpolation function as it is easy to integrate and differentiate, the degree of the polynomial depending on the number of nodes assigned to the element.

Step-3: Find the Element properties: The matrix equations expressing the properties of individual elements are determined. For this, we may use one of the four approaches mentioned earlier.

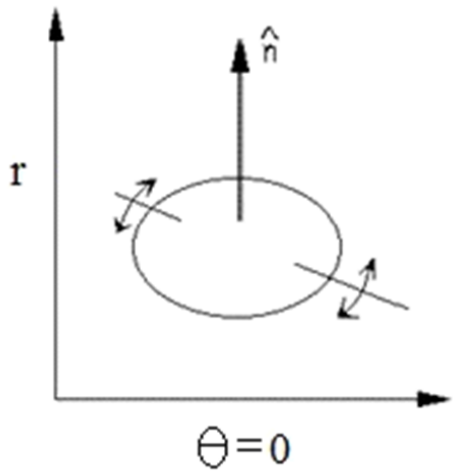

Figure 3: 2-Dimensional (2-D) heat conduction domain

Step-4: Assemble the Element properties to obtain the system equations: The matrix equations are combined to obtain the behavior of the entire solution region.

Step-5: Solving the System equation: The assembly process in step-4 needs a set of simultaneous equations which can be solved to obtain the unknown nodal values of the field variable, after assigning the boundary conditions.

Step-6: Make additional computation if desired: Sometimes it may require using the solution of the system to calculate other important parameters.

6. 2-D Steady State Heat Conduction In Cylindrical Polar Co-Ordinates

The problem concerns the steady state temperature distribution and heat conduction in a 2-Dimensional (2-D) domain as defined in Fig. 3. The governing differential equation for temperature distribution is expressed in general forms as,

$$q = -V\nabla T \tag{6}$$

$$\nabla \cdot q = Q \tag{7}$$

Here, T is the potential function (Temperature), °C; V is the medium permeability (Thermal conductivity), W/m°C; q is the flux (heat flux), W/mm^2; Q is the forcing function (Heat source).

Combining equations (6) and (7), one obtains the general partial differential equation describing the 3-Dimensional (3-D) heat conduction problems.

$$\nabla \cdot (V\nabla T) = -Q \tag{8}$$

In cylindrical polar co-ordinates, the equation above can be expressed as,

$$\frac{1}{r}\frac{\delta}{\delta r}\left(V_r r\frac{\delta T}{\delta r}\right) + \frac{V_\theta}{r^2}\frac{\delta^2 T}{\delta\theta^2} + Q = 0 \tag{9}$$

V_r, and V_θ are thermal conductivities in the radial and circumferential directions respectively.

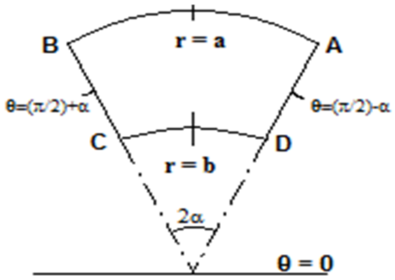

Figure 4: 2-D Arch Shaped Element

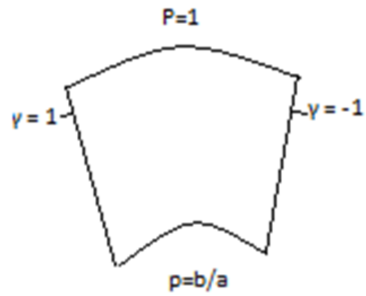

Figure 5: Non-Dimensional Arc element

6.1. Finite Element Equations (Galerkin's Method):

The solution of equation (9) can be obtained by assuming the general functional behavior of the dependent field variable in some way so as to approximately satisfy the given differential equation and boundary conditions. Substitution of this approximation into the original differential equation and boundary conditions then results in some error, called a residual. This residual is required to vanish in some average sense over the entire solution domain.
The approximate behavior of the potential function within each element is prescribed in terms of their nodal values and some weighted functions N_1, N_2 ... so that,

$$T = \int_{i=1,\ 2,\ \dots m} N_i T_i \tag{10}$$

The weighting functions are strictly functions of the geometry and are termed interpolation functions. The interpolation functions determine the order of the approximating polynomials for the heat conduction problem.
The methods of weighted residuals determine the m unknowns T_i in such a way that the error over the entire solution domain is small. This is accomplished by forming a weighted average of the error and specifying that this weighted average vanishes over the solution domain.
The required equations governing the behavior of an element are given by the expression,

$$\int_{D^{(e)}} N_i \left[\frac{\delta}{\delta r}\left(V_r \frac{\delta T^{(e)}}{\delta r} \right) + \frac{\delta}{\delta \theta}\left(\frac{V_\theta}{r^2} \frac{\delta T^{(e)}}{\delta \theta} \right) + Q \right] r dr d\theta = 0 \tag{11}$$

Equation (11) expresses the desired averaging to the error or residual within the element boundaries. But it does not admit the influence of the boundary. Since we have made no attempt to choose the N_i so as to satisfy the boundary conditions, we must use integration by parts to introduce the influence of the natural boundary condition.

6.2. Arch Element Interpolation Function

Consider the arch shaped element of Fig. 4, formed by circle arch radii a, b. Radii inclined at an angle of 2α.
The interpolation function can now be defined in terms of a set of non-dimensional co-ordinates by non-dimensional cylindrical polar co-ordinates r, θ using,

$$\rho = r/a; v = \theta - \pi/2/\alpha \tag{12}$$

The arch element with non-dimensional co-ordinates is shown in Fig. 4 and Fig. 5.
The temperature at any point within the element is given in terms of its nodal temperature by

$$T = TANA + TBNB + TCNC + TDND \tag{13}$$

Here the N's are the interpolation functions chosen as follows.

$$\begin{aligned} N_A &= \frac{\left(\rho - \frac{b}{a}\right)(v-1)}{-2\left(1-\frac{b}{a}\right)}; \quad N_B = \frac{\left(\rho - \frac{b}{a}\right)(v+1)}{2\left(1-\frac{b}{a}\right)}; \\ N_C &= \frac{(\rho-1)(v+1)}{-2\left(1-\frac{b}{a}\right)}; \quad N_D = \frac{(\rho-1)(v-1)}{2\left(1-\frac{b}{a}\right)} \end{aligned} \tag{14}$$

It is seen that the interpolation functions satisfy the following conditions.

1. At any given vertex 'A', the corresponding interpolation function N_A has a value of unity and other shape functions N_B, N_C have a zero value at this vertex thus at node j, $N_j = 1$ but $N_i = 0$; where $i = j$.
2. The value of potential varies linearly between any two adjacent nodes on the element edges.
3. The value of the potential function in each element is determined by the order of the finite element. The order of the element is the order of the polynomial of the spatial co-ordinates which describes the potential within the element. The potential varies as a quadratic function of the spatial co-ordinates on the faces and within the element.

6.3. Boundary Condition

We consider the portion of the conductor bounded by a plane passing through the center of a single core and another plane which is 90° displaced clockwise from the previous plane. The temperature distribution is assumed symmetrical across two planes, with the heat flux normal to the surface being zero. From the other two boundary surfaces heat is transferred by convection to the surface. The boundary conditions may be written in terms of $\delta T/\delta n$, the temperature gradient normal to the surface.
Mid core horizontal surface

$$\frac{\delta T}{\delta \eta_h} = 0 \tag{15}$$

Mid core vertical surface

Table 3: Nodal temperatures in induction coil

Node numbers	Free convection temp., °C	Forced convection temp., °C
1	195.64	155.78
2	174.98	136.73
3	149.22	114.06
4	125.13	93.71
5	104.06	76.64
6	86.03	62.46
7	70.21	50.06
8	195.46	155.63
9	182.28	143.26
10	161.79	124.42
11	138.79	103.68
12	115.33	83.19
13	93.22	64.80
14	74.01	50.17
15	194.94	155.18
16	199.96	159.39
17	200.56	158.43
18	192.54	148.52
19	175.10	129.06
20	148.21	100.23
21	112.23	62.54
22	194.62	154.89
23	210.32	168.97
24	226.69	182.20
25	232.26	183.33
26	223.52	168.33
27	195.92	133.28
28	146.60	74.78

$$\frac{\delta T}{\delta \eta_v} = 0 \tag{16}$$

Conductor surface

$$h\left(T - T_c\right) = -V_r \frac{\delta T}{\delta \eta_c} \tag{17}$$

where: T—Surface temperature and T_C—conductor surface temperature.

7. Statistical Analysis & Performance Of Solution

The steady state response of the coil of induction heater was scientifically determined with its surface temperature mounted at a potential temperature of 200.56°C. Heat was transferred from the surface by natural convection to the encircling space and therefore the resolution was compared. The temperature distribution is more realistic in the case of convection than in the case of the assumed extreme temperature on the boundary. The temperature distribution of the 2-Dimensional (2-D) core conductor of the coil at steady state, in each cases, was numerically obtained then the heat transfer co-efficient was calculated at the mean of the temperatures as tabulated below in Table 3. Fig. 6 shows temperature variations with completely different nodal points on the coil free convection and forced convection temperatures after the same current passes through the coil. The system of global equations, as determined by equation, should be resolved to determine the nodal temperatures. The answer to this set of linear equations is given by the Gauss methodology.

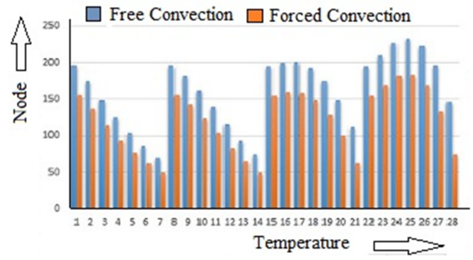

Figure 6: Node vs Temperature

8. Conclusion

The 2-Dimensional (2-D) steady state finite-element procedure for the thermal analysis of induction coil is a useful approach to conducting in-depth studies of coil heating issues, in particular with the new, expressly derived arch part, along with associated economic data and Gauss routine. A novel two-dimensional finite element procedure in cylindrical polar co-ordinates, with expressly derived answer matrices, was applied to resolve the heat conduction equation under steady state conditions. Though the results are approximate, the strategy is quick, cheap and leads itself to immediate visual imaging of the temperature pattern in an exceedingly 2-Dimensional (2-D) slice of the core conductor (single)bounded by two 90° inclined planes divided into arc shaped elements of the coil.

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