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Using data reconciliation to improve the reliability of the energy evaluation of a gas-and-steam CHP unit

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Abstract

In this paper the thermal system of an example gas-and-steam CHP unit and its measurement system were determined and data reconciliation calculations performed. The measurement data in the distributed control system of the CHP unit was found to be redundant from the data reconciliation method point of view. A relative information entropy was applied to produce a global assessment of the results of data reconciliation calculations.

Keywords: gas-and-steam CHP unit, evaluation of the operation, data reconciliation, information entropy

1. Introduction

In CHP plants, as in power stations, common distributed control systems are used. The measurement data from this systems are then processed by mathematical models to determine indicators that express the level of operation of the energy machines and equipment of power or CHP units. Modern computer-aided systems of control in power or CHP units pay special attention to the need to verify the results of measurements. The results of measurements from the distributed control systems contain errors due to inaccuracy in the applied measurement method, failures of the device or in signal processing. Such errors are then transferred to calculations of the indicators expressing the parameters of operation. In many cases a redundancy of measurement information in the distributed control systems of power or CHP units occurs. In that case it's possible to apply an advanced method—data reconciliation—to increase the reliability of measurement data.

This paper deals with application of the data reconciliation method to improve the reliability of thermal measurements in a gas-and-steam CHP unit. The thermal system of the unit and the measurement system were determined. A mathematical model of investigated CHP unit equipped with a gas turbine, heat recovery steam generator, extraction-condensing steam turbine and heat exchangers is presented. As regards energy evaluation, mainly the balance of mass and energy was taken into account. Redundancy of measurement information in the distributed control system of the analyzed CHP unit was found from the data reconciliation method point of view. Mass and energy balances were applied as conditional equations in the data reconciliation algorithm. Data reconciliation calculations were carried out for the gas-and-steam CHP unit. Relative information etropy—Kullback-Leibler divergence—was applied for global assessment of the results of data reconciliation calculations. There is a demonstrated need for a data reconciliation method to increase reliability and reduce measurement data uncertainty in calculations of characteristic parameters of the process of energy conversion in the investigated gas-and-steam CHP unit. Simulation calculations were performed concerning installation of additional redundant measurements in the thermal system of the CHP unit. Their location in the thermal system structure was optimized. The minimum relative standard deviation of the energy utilization factor in the CHP unit was adopted as the optimization criterion.

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2. Principle of data reconciliation

Measurement results contain errors due to the inaccuracy of the applied measurement method, failures in the device or in signal processing. Such errors are then transferred to calculations of unknown values (quantities that are not measured). As the number of balance equations is larger than the number of unknown values, surplus equations are not satisfied because the substance and energy balance equations are not reconciled. Application of the data reconciliation method permits corrections of the measurement results and the determination of the most probable unknown values on the basic of which the most reliable values of technical and economical indicators can be calculated. Data reconciliation can be mathematically expressed as a constrained weighted least-squares optimization problem:

$$\min\left\{\sum_{i=1}^{m}\left(\frac{\hat{x}_i - x_i}{\sigma_i}\right)^2\right\} \tag{1}$$

subject to

$$g_l\left(\hat{x}_i, \hat{y}_j\right) = 0 \quad for \quad l = 1, ..., r \tag{2}$$

where: x—raw measurement data, $\hat{x}$—reconciled measurement data, m—number of measurement data, σ – standard uncertainty of raw measurement data, $\hat{y}$—reconciled not measured variable, r—number of a conditional equations.

The objective function (1) defines the total weighted sum of measurement corrections squares, whereas equation (2) defines the set of mathematical model constraints. In thermal engineering these constraints are generally mass and energy balances. Data reconciliation in thermal analysis helps achieve the following aims [1–5]:

- calculation of the most reliable thermal measurement values,
- unique solution of the most probable unknown quantities in thermal processes,
- an assessment of the accuracy of the corrected results of measurements and of calculated unknown quantities,
- a reduction in uncertainty of measured quantities,
- control of fulfilling the assumed measurement uncertainty.

3. Identification of the analyzed gas-and-steam CHP unit

Combining thermodynamic cycles of gas and steam turbines results in improved overall energy efficiency and lower fuel consumption. A heat recovery steam generator (HRSG) in the thermal system of the CHP unit constitutes a connection between the open cycle of the gas turbine and the steam cycle. This technical connection of both thermal cycles is simultaneously a thermodynamic connection due to the heat transfer from exhaust flue gases from the gas turbine to the thermodynamic media in the steam cycle. A schematic diagram of the analyzed gas-and-steam CHP unit is presented in Fig. 1. In this figure the measurement points in the thermal system of the gas-and steam CHP unit are presented. The number of measurements in the distributed control system of the analyzed gas-and-steam CHP unit exceeds the minimum number of measurements points that are indispensable for the mass and energy balances. The measurement data and their uncertainties for presented calculations were taken from [6–9] and from the distributed control system of the CHP unit.

The necessity of data reconciliation in the thermal system of the CHP unit is shown by the calculations of the HRSG thermal power. The number of measurements in the DCS of the gas-and-steam CHP unit exceeds the minimum number of measurement points required to calculate the thermal power of the HRSG. Hence, this thermal power can be calculated using a different set of measurement data. Connection of the two thermal cycles means that for measured fluxes of steam and flue gases and their thermodynamic parameters it is possible to calculate the thermal power of the HRSG based on calculations of the increase in steam enthalpy or decrease in the flue gases enthalpy. Due to the inevitability of errors in measurement, there are differences in the thermal power calculated in the two ways.

Table 1 presents the results of calculations of thermal powers of HRSG and their complex standard uncertainty for assumed variants of utilizing measurement information [6]. The maximum relative difference between calculated thermal powers of the HRSG is about 11%.

The data reconciliation algorithm must be applied to remove the incompatibility between all calculated thermal powers of the HRSG. The same concerns the whole thermal system of the investigated gas-and-steam CHP unit.

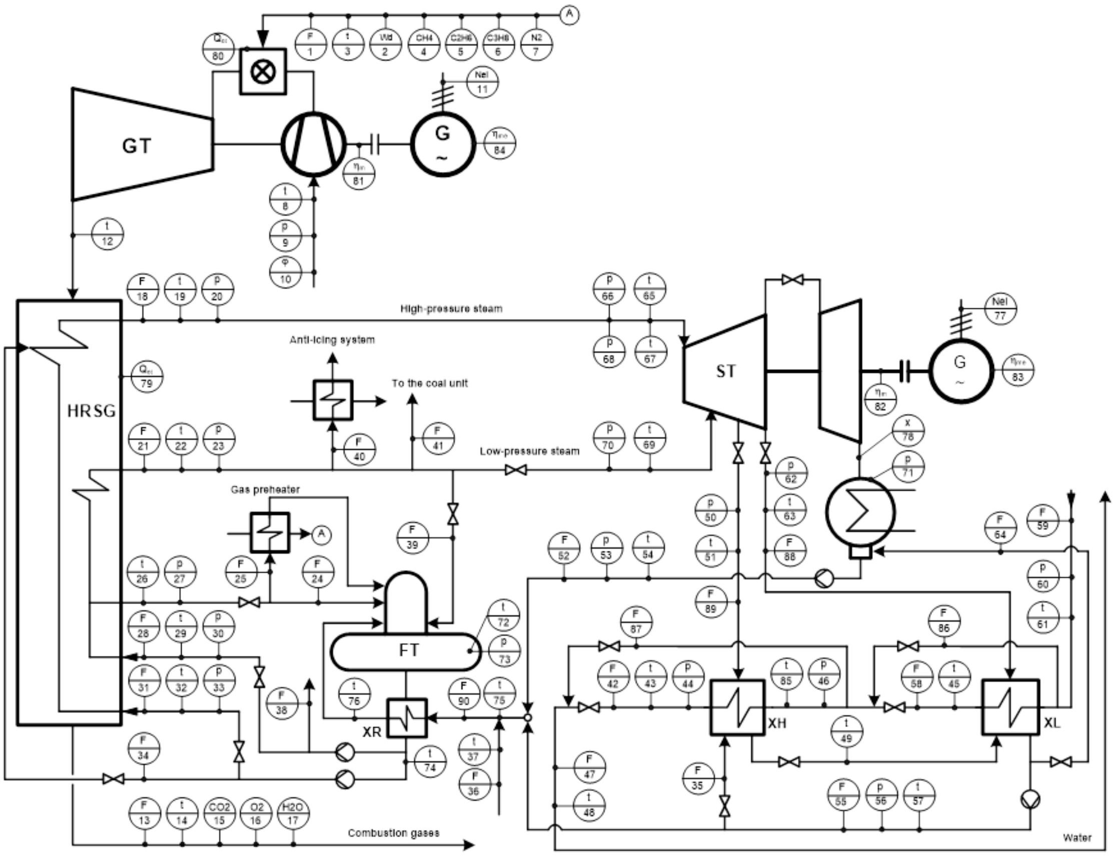

Figure 1: Scheme of the thermal system of the considered gas-and steam CHP unit

Table 1: Results of calculation of the thermal power of HRSG

No.	Variants of calculation	Thermal power of HRSG, MW	Complex standard uncertainty of thermal power, MW
Calculations based on measurements in the steam-water cycle			
1.	The measurements of thermal parameters and mass flows of high and low pressure steam	191,11	2,66
2.	The measurements of thermal parameters and mass flows of high and low pressure feed water	190,27	2,86
Calculations based on measurements in the gas turbine cycle			
3.	The measurements of thermal parameters of flue gases, molar fractions of components and flue gases volumetric flow	208,94	3,14
4.	The measurements of thermal parameters of flue gases, molar fraction of CO_2 in flue gases and natural gas volumetric flow	194,79	2,93
5.	The measurements of thermal parameters of flue gases, molar fraction of O_2 in flue gases and natural gas volumetric flow	188,08	2,83

4. Conditional equations of the data reconciliation task

According to the schematic for the analyzed system of the gas-and-steam CHP plant presented in the Fig. 1 the mass and energy balances equations in the steady state of CHP unit operation can be formulated. Total number of conditional equations is equal to $r = 22$. Selected mass and energy balances are illustrated below.

Energy balance of the gas turbo-generator takes the following form:

$$\dot{V}_{n(1)}\left[LHV_{g(2)}+\frac{\Delta(Mh)_g+n'_a\Delta(Mh)_a}{(Mv)_n}\right]\times \times\left(1-\varepsilon_{g(80)}\right)=\frac{P_{el(11)}}{\eta_{(81)}\eta_{(84)}}+\frac{\dot{V}_{n(13)}}{(Mv)_n}\Delta(Mh)\Bigg|_{t_n}^{t_{(12)}} \tag{3}$$

where: $\dot{V}_n$—normal volumetric flow, $LHVg$—net calorific value of the natural gas, (Mh)—molar specific enthalpy, $(Mv)_n$—normal molar specific volume, P_{el} – electric power of the gas turbo-generator, η—efficiency, ε—relative heat losses, t—temperature.

Mass balance of the flue gases flowing through the HRSG takes the form:

$$\dot{V}_{n(1)}n''_{dg}=\dot{V}_{n(13)} \tag{4}$$

where: n''_{dg}—specific amount of dry flue gases.
Thermal power of the HRSG based on calculation of the increase in steam enthalpy results from the following equation:

$$\begin{aligned}\dot{Q}^s_{SG}&=\dot{m}_{(18)}h\left(t_{(19)},p_{(20)}\right)+\dot{m}_{(21)}\times\\ &\times h\left(t_{(22)},p_{(23)}\right)+\left(\dot{m}_{(24)}+\dot{m}_{(25)}\right)\times\\ &\times h\left(t_{(26)},p_{(27)}\right)-\dot{m}_{(28)}h\left(t_{(29)},p_{(30)}\right)+\\ &-\left(\dot{m}_{(31)}+\dot{m}_{(34)}\right)h\left(t_{(32)},p_{(33)}\right)\end{aligned} \tag{5}$$

whereas thermal power of the HRSG results from the decrease in the flue gases enthalpy:

$$\dot{Q}^g_{SG}=\frac{\dot{V}_{n(13)}}{(Mv)_n}\Delta(Mh)\Bigg|_{t_{(14)}}^{t_{(12)}} \tag{6}$$

where: $\dot{Q}_{SG}$—thermal power of a HRSG, $\dot{m}$—mass flow, h—specific enthalpy, p—pressure, t—temperature. The relation between thermal powers of HRSG resulting from the equations (5) and (6) is expressed as follows:

$$\dot{Q}^s_{SG}=\dot{Q}^g_{SG}\left(1-\varepsilon_{SG(79)}\right) \tag{7}$$

Energy balance of the steam turbo-generator has the following form:

$$\begin{aligned}&\dot{m}_{(18)}0.5\left[h\left(t_{(65)},p_{(66)}\right)+h\left(t_{(67)},p_{(68)}\right)\right]+\\ &+\left(\dot{m}_{(21)}-\dot{m}_{(40)}-\dot{m}_{(41)}\right)h\left(t_{(69)},p_{(70)}\right)=\\ &=\dot{m}_{(89)}h\left(t_{(51)},p_{(50)}\right)+\dot{m}_{(88)}h\left(t_{(63)},p_{(62)}\right)+\\ &+\left(\dot{m}_{(18)}+\dot{m}_{(21)}-\dot{m}_{(40)}-\dot{m}_{(41)}-\dot{m}_{(89)}-\dot{m}_{(88)}\right)\times\\ &\times h\left(p_{(71)},x_{(78)}\right)+\frac{P_{el(77)}}{\eta_{(82)}\eta_{(83)}}\end{aligned} \tag{8}$$

Selected mass and energy balances for steam cycle of the investigated gas-and-steam CHP unit have the form:

- energy balance of the district heating water mixing point behind the high-pressure heat exchanger:

$$\begin{aligned}&\dot{m}_{(47)}h\left(t_{(48)},p_{(44)}\right)=\dot{m}_{(42)}h\left(t_{(43)},p_{(44)}\right)+\\ &+\dot{m}_{(87)}h\left(t_{(85)},p_{(46)}\right)\end{aligned} \tag{9}$$

- energy balance of the district heating water mixing point behind the low-pressure heat exchanger:

$$\begin{aligned}&\dot{m}_{(59)}h\left(t_{(85)},p_{(46)}\right)=\dot{m}_{(58)}h\left(t_{(45)},p_{(46)}\right)+\\ &+\dot{m}_{(86)}h\left(t_{(61)},p_{(60)}\right)\end{aligned} \tag{10}$$

- energy balance of the high-pressure heat exchanger XH:

$$\begin{aligned}&\left[\dot{m}_{(89)}\,h\left(t_{(51)},p_{(50)}\right)+\dot{m}_{(35)}h\left(t_{(57)},p_{(56)}\right)+\right.\\ &\left.-\left(\dot{m}_{(89)}+\dot{m}_{(35)}\right)h\left(t_{(49)},p_{(50)}\right)\right]\eta_{HE}=\\ &=\dot{m}_{(42)}h\left(t_{(43)},p_{(44)}\right)-\dot{m}_{(42)}h\left(t_{(85)},p_{(46)}\right)\end{aligned} \tag{11}$$

where: η_{HE}—efficiency of the heat exchanger taking into account the heat losses to the environment.

- energy balance of the low-pressure heat exchanger XL:

$$\begin{aligned}&\left[\dot{m}_{(88)}\,h\left(t_{(63)},p_{(62)}\right)+\dot{m}_{(89)}h\left(t_{(49)},p_{(50)}\right)+\right.\\ &\left.-\left(\dot{m}_{(55)}+\dot{m}_{(64)}\right)h\left(t_{(49)},p_{(50)}\right)\right]\eta_{HE}=\\ &=\dot{m}_{(58)}\left[h\left(t_{(45)},p_{(60)}\right)-h\left(t_{(61)},p_{(60)}\right)\right]\end{aligned} \tag{12}$$

- mass balance of the condensate from the low-pressure heat exchanger XL:

$$\dot{m}_{(89)}+\dot{m}_{(35)}+\dot{m}_{(88)}=\dot{m}_{(64)}+\dot{m}_{(55)} \tag{13}$$

- mass balance of the mixing point before the heat exchanger XR:

$$\dot{m}_{(52)}+\dot{m}_{(55)}-\dot{m}_{(35)}+\dot{m}_{(36)}=\dot{m}_{(90)} \tag{14}$$

- energy balance of the mixing point before the heat exchanger XR:

$$\begin{aligned}&\dot{m}_{(52)}h\left(t_{(54)},p_{(53)}\right)+\left(\dot{m}_{(55)}-\dot{m}_{(35)}\right)\times\\ &\times h\left(t_{(57)},p_{(56)}\right)+\dot{m}_{(36)}h\left(t_{(37)},p_{(56)}\right)=\\ &=\dot{m}_{(90)}h\left(t_{(75)},p_{(56)}\right)\end{aligned} \tag{15}$$

- energy balance of the heat exchanger XR:

$$\begin{aligned}&\dot{m}_{(90)}\left[h\left(t_{(76)},p_{(53)}\right)-h\left(t_{(75)},p_{(53)}\right)\right]=\\ &=\left(\dot{m}_{(31)}+\dot{m}_{(34)}+\dot{m}_{(28)}+\dot{m}_{(38)}\right)\times\\ &\times\left[h\left(t_{(72)},p_{(73)}\right)-h\left(t_{(74)},p_{(73)}\right)\right]\eta_{HE}\end{aligned} \tag{16}$$

Table 2: Results of calculation for minimum resource of measurement information

No.	Indicator	Value	Complex standard uncertainty	Relative standard deviation
1	Energy utilization factor (EUF)	0.5876	0.0098	1.67 %
2	Specific consumption of chemical energy of fuel by gas turbine set per unit electricity production	11 055 kJ/kWh	91 kJ/kWh	0.82%
3	Specific consumption of heat by steam turbine set per unit electricity production	12 055 kJ/kWh	168 kJ/kWh	1.40 %

Table 3: Results of calculation for reconciled measurement data

No.	Indicator	Value	Complex standard uncertainty	Relative standard deviation
1	Energy utilization factor (EUF)	0.5794	0.0027	0.46 %
2	Specific consumption of chemical energy of fuel by gas turbine set per unit electricity production	11 222 kJ/kWh	47 kJ/kWh	0.42 %
3	Specific consumption of heat by steam turbine set per unit electricity production	12 119 kJ/kWh	73 kJ/kWh	0.60 %

5. Results of calculations

Results of calculation of the main indicators characterizing operation of the investigated gas-and-steam CHP unit are presented in Table 2 and Table 3. The following indicators were calculated; energy utilization factor, specific consumption of chemical energy by gas turbine per unit of electricity production and specific consumption of heat by steam turbine per unit of electricity production. Table 2 shows the values of considered indicators for minimum resource of measurement information. Table 3 presents the results of calculations of considered indicators using reconciled measurement data.

Results of calculations concerning installation of additional redundant measurements in the thermal system of the investigated gas-and-steam CHP unit are presented in Table 3. The value of uncertainty reduction for selected measurements, or a reduction of complex uncertainty of indirect measurements depends on the number and location of redundant measurements in the considered thermal process [10]. Minimization of uncertainty of the energy utilization factor can be used as a criterion of optimal location of redundant measurements in the thermal system of a gas-and-steam CHP unit. Since in data reconciliation calculations all measurement data are corrected, it is convenient for the analysis to use the relative standard deviation of the energy utilization factor—$RSD(EUF)$ in the CHP unit. Minimum of the $RSD(EUF)$ can be a criterion of the optimal location of an additionally redundant measurements in the measurement system of the considered CHP unit. The objective function of the described optimization task can be defined as follows:

$$\left.\begin{array}{c} RSD(EUF)_k = \min\left\{RSD(EUF)_{kj}\right\} \\ for \\ k = 1,...,n \\ j = 1,...,L(k) \end{array}\right\} \tag{17}$$

where: k—current number of redundant measurement, n—number of considered additional redundant measurements, L—number of location configurations of additional measurement data.

The procedure of an optimization calculations requires first of all identification of a potential location for installation of an additional redundant measurement in a thermal system structure, considering the technical constraints. These potential places determine the maximum number of available redundant measurements in the system. The determined number of redundant measurements (lower than the possible maximum) can be installed in a measurement system in different configurations. Solution of the optimization task (17) requires determination of the $RSD(EUF)$ for all available configurations of installation of this measurement in the analyzed CHP unit. The number of this installation configurations results from the binomial coefficient. For any set containing n-elements, the number of considered k-elements subsets is given by the formula:

$$L(k) = C_n^k = \left(\begin{array}{c} n \\ k \end{array} \right) = \left(\frac{n!}{k!(n-k)!} \right), \quad \text{where } k \leq n, \quad n, k \in N^+ \tag{18}$$

The optimization calculations were performed for previously defined variants of configurations of additional measurements in the analyzed thermal system on the basis of the algorithm presented in [10]. Calculations were carried out for the following additionally redundant measurements in the thermal system of the CHP unit:

- temperature of the district heating water at the inlet of high-pressure heat exchanger (index No. 85 in the Fig. 1),
- flow of the district heating water in the low-pressure heat exchanger bypass (index No. 86 in the Fig. 1),
- flow of the district heating water in the high-pressure heat exchanger bypass (index No. 87 in the Fig. 1),
- flow of the extraction steam to the low-pressure heat exchanger (index No. 88 in the Fig. 1),
- flow of the extraction steam to the high-pressure heat exchanger (index No. 89 in the Fig. 1),
- flow of the main condensate at the inlet of the heat exchanger XR (index No. 90 in the Fig. 1).

The number of variants of measurement location for each number of redundant measurements is calculated by means of the formula (18). Column 2 of Table 3 presents the number of analyzed variants of redundant measurement locations. The optimization calculations were carried out using the computer program elaborated in Fortran language.

Kullback–Leibler divergence was applied to assess the measurement system of the CHP unit as a whole [11]. The quality of a redundant system of measurement after the introduction of a new additional measurement should be evaluated by applying criterion, which enables one to measure the increase in reliability of measurements—both the values of the measurements as well as the uncertainties. It was assumed that the measurement system of the thermal process represents a signals system of communication, which can be used to send information about the thermodynamic parameters of the thermal process. For this assumption the entropy of information can be applied as an assessment criterion of quality of the measurement system. In information theory, entropy is a measure of the uncertainty which is associated with a random variable [12]. In probability and information theory the concept of relative entropy called Kullback–Leibler divergence has also been introduced [13]. In an assessment of an increase in reliability of measurement data, in principle, we are not interested in the absolute value of information entropy, but only in the decrease in this entropy $H(X)$ from the state of raw measurement data to state $H\left(\hat{X}\right)$ of reconciled measurements. In this case the Kullback-Leibler divergence can constitute the criterion of an assessment of the increase in reliability of measurement data in a redundant system of measurements of the thermal process:

$$D_{KL}\left(\hat{N} \middle\| N\right) = H(X) - H\left(\hat{X}\right) \tag{19}$$

Introducing the variance-covariance matrices property in the data reconciliation method [3], the Kullback–Leibler divergence (in bits) has the following form [11]:

$$D_{KL}\left(\hat{N} \middle\| N\right) = \frac{1}{2 \cdot \ln(2)} \times \\ \times \left\{ \ln \left[\prod_{i=1}^{m} \left(\frac{\sigma_i}{\hat{\sigma}_i} \right)^2 \right] + \sum_{i=1}^{m} \left(\frac{\hat{x}_i - x_i}{\sigma_i} \right)^2 + u - r \right\} \tag{20}$$

From the equation (20) it results that Kullback-Leibler divergence includes both values of measurements $\hat{x}_i$ as well as its uncertainties $\hat{\sigma}_i$ after the application of data reconciliation.

6. Conclusions

The energy utilization factor and its complex standard uncertainty for minimum measurement information (without data reconciliation) is EUF = 0.5876 ± 0.0098. Relative standard deviation of energy utilization factor in that case is 1.67% (Table 2). After data reconciliation calculations this factor is EUF = 0.5794 ± 0.0027 and relative standard deviation is 0.46% (Table 3). Use of the data reconciliation method causes a significant reduction of nearly 73% in $RSD(EUF)$.

Column 2 of Table 4 shows the configuration number for considered additional redundant measurements. Column 3 of this table presents the optimal configurations of the additional redundant measurement location, whereas column 4 shows the optimized parameter $RSD(EUF)$and the other calculated indicators. From the $RSD(EUF)$ values shown in column 4, it follows that adding additional surplus measurements causes a reduction in $RSD(EUF)$.

However decreasing this relative standard deviation has limits for three additional redundant measurements. Value of the $RSD(EUF)$, comparing to the basic case, decreases to about 76%. Entering a larger number of redundant measurements to the thermal system of the investigated CHP unit no longer benefits from the point of view of the assumed optimization criterion. From column 3 in Table 4, the optimal configuration of additional redundant measurements which provide the minimization of $RSD(EUF)$ are measures number: 88—flow of the extraction steam to the low-pressure heat exchanger, 89—flow of the extraction steam to the high-pressure heat exchanger and 90—flow of the main condensate at the inlet of heat exchanger XR.

An information entropy was applied for a global assessment of results of data reconciliation calculations. The relative entropy—Kullback-Leibler divergence—was used. This divergence was calculated for two multivariate normal probability distributions, which represents the set of raw and reconciled measurements of the investigated gas-and-steam CHP unit for analyzed cases. Usefulness of application of the mentioned relative entropy for global assessment of the improvement in reliability of the set of measurements of the thermal system after data reconciliation calculations was presented.
It should be noted that the Kullback-Leibler divergence continuously increases with the number of additional redundant measurements in the thermal system of the investigated gas-and-steam CHP unit. In this case each additional redundant measurement in the thermal system of the CHP unit reduces the information entropy of its redundant measurement system.

Example calculations of the Kullback-Leibler divergence show (Table 4) that the biggest decrease in the entropy information of the redundant measurement system is in the case where all the measurement information about the analyzed thermal process is used.

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Table 4: Results of calculation for an additional redundant measurement data

Number of redundant measure-ments data, k	Number of configuration of installation of redundant measurement data, L(k)	Sets of the optimal configuration of additional redundant measurement data	Value of the indicator
1	2	3	4
k = 1	$L(1) = C_6^1 = 6$	$M_{k=1}^{opt} = \{88\}$	$RSD(EUF)$ = 0.44% EUF = 0.5802 ± 0.0025 $b_{ch\,GT}$ = 11228 ± 47kJ/kWh q_{ST} = 12130 ± 72kJ/kWh D_{KL} = 46.9 bit
k = 2	$L(2) = C_6^2 = 15$	$M_{k=2}^{opt} = \{88, 90\}$	$RSD(EUF)$ = 0.42% EUF = 0.5798 ± 0.0025 $b_{ch\,GT}$= 11237 ± 45 kJ/kWh q_{ST}= 12146 ± 67 kJ/kWh D_{KL} = 49.6 bit
k = 3	$L(3) = C_6^3 = 20$	$M_{k=3}^{opt} = \{88, 89, 90\}$	$RSD(EUF)$ = 0.41% EUF = 0.5794 ± 0.0024 $b_{ch\,GT}$= 11235 ± 45 kJ/kWh q_{ST}= 12142 ± 67 kJ/kWh D_{KL} = 50.4 bit EUF = 0.5794 ± 0.0024 $b_{ch\,GT}$= 11235 ± 45 kJ/kWh q_{ST}= 12142 ± 68 kJ/kWh D_{KL} = 53.1 bit

Number of redundant measure-ments data, k	Number of configuration of installation of redundant measurement data, L(k)	Sets of the optimal configuration of additional redundant measurement data	Value of the indicator
1	2	3	4
k = 4	$L(4) = C_6^4 = 15$	$M_{k=4}^{opt} = \{85, 88, 89, 90\}$	$RSD(EUF) =$ 0.41% EUF = 0.5795 ± 0.0024 $b_{ch\,GT}$= 11235 ± 45 kJ/kWh q_{ST}= 12142 ± 68 kJ/kWh D_{KL} = 51.5 bit
k = 5	$L(5) = C_6^5 = 6$	$M_{k=5}^{opt} = \{85, 86, 88, 89, 90\}$	$RSD(EUF) =$ 0.41% EUF = 0.5794 ± 0.0024 $b_{ch\,GT}$= 11235 ± 45 kJ/kWh q_{ST}= 12142 ± 68 kJ/kWh D_{KL} = 52.2 bit
k = 6	$L(6) = C_6^6 = 1$	$M_{k=6}^{opt} = \{85, 86, 87, 88, 89, 90\}$	$RSD(EUF) =$ 0.41% EUF = 0.5794 ± 0.0024 $b_{ch\,GT}$= 11235 ± 45 kJ/kWh q_{ST}= 12142 ± 68 kJ/kWh D_{KL} = 53.1 bit