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Using generalized advanced data validation and reconciliation in steam power unit energy balancing

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Abstract

There are advantages to be gained by using a generalized method of data validation and reconciliation in energy conversion processes in terms of decreasing the uncertainty of measurements data. This method was used to complete the validation model of the process (conditional equations of optimization task) including substance and energy conservation principles with additional equations describing energy conversion processes. The methodology developed was used for example for calculations of data reconciliation in the selected steam power unit. The equations of steam flow capacity, adiabatic internal efficiency and equations resulting from the form of an isobaric line on the h-s diagram for a group of turbine stages were applied. Also applied as additional equations in the validation model were: Darcy's equation of steam pressure drop in the pipeline into heat exchangers and *Peclet's* equations of heat transfer and equations of over-cooling of condensate in regenerative heat exchangers. The criterion of an assessment of the decrease of measurements uncertainty in the form of global decrease of measurements variance after measurement data reconciliation is proposed. Derivation of the analyzed coefficient was based on the characteristic property of the measurements variance, coming from the variance-covariance matrix of measurements before and after data reconciliation. The criterion for selection of the mathematical form of additional equations in the validation model in reconciliation calculation was formulated. Professor Jan Szargut introduced and developed the advanced data validation and reconciliation method in Poland for thermodynamic analysis of energy conversion processes. The author of this paper engaged in further research on the development and application of this method in thermodynamic analyses.

Keywords: advanced data validation and reconciliation, steam power unit energy balancing, conditional equations

1 Introduction

Scientific research was undertaken on application of the advanced data validation and reconciliation (DVR) method in the 1950s at the Institute of Thermal Technology of the Silesian University of Technology in Gliwice, Poland. ProfessorJan Szargut introduced this method to thermodynamic analysis of energy conversion processes, under the name of the justification calculations used in Poland. Other researchers have made significant contributions to developing the method theory and expanding its scope of application. Matters considered concerned primarily issues of reconciliation of mass and energy balances [1]; [2]; [3], solving problems of reverse heat conduction, multistage and multi-group reconciliation, and determining the coefficients of empirical equations. The results of these works were published in [4]; [5]; [6]; [7]; [8]. Further works concern the application of the advanced DVR method in the analysis of thermal processes in metallurgy and combustion processes [9]; [10]; [11].

At that time, scientific research on the application of the advanced DVR method was also carried out in other countries [12]; [13]; [14]; [15]; [16]; [17]; [18], mainly in connection with the chemical industry. The mathematical principles of the advanced DVR method have been described in many publications [19]; [20]; [14]; [15]; [21]; [22]; [23]; [18]. Polish contributions include: [6]; [7]; [24].

The advanced DVR method is the procedure of optimally adjusting measured variables in such a way that the adjusted values of these measurements satisfy the laws of conservation and other constraints. In general, it is formulated by the constrained weighted least-squares optimization problem resulting from maximizing the Gauss likelihood function:

$$\min\left\{\sum_{i=1}^{m}\left(\frac{\hat{x}_i - x_i}{\sigma_i}\right)^2\right\} \tag{1}$$

subject to

$$g_1\left(\hat{x}_i, \hat{y}_i\right) = 0 \quad for \quad l = 1, ..., r \tag{2}$$

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Eq. 2 defines the set of model constraints – the so-called conditional equations. These constraints in thermal analyses are generally mass and energy balances. Application of the advanced DVR method in energy conversion processes achieves the following aims:

- calculation of the most reliable thermal measurements values,
- unique solution of the most probable unknown quantities in thermal processes,
- assessment of the accuracy of the validated results of measurements and of calculated unknown quantities,
- reduced uncertainty of measured quantities,
- control on fulfilling the assumed measurements uncertainty.

The advanced DVR method that validates only real measurements is called the classical method. In this method estimated not-measured and non-measurable values adopted for reconciliation calculations have an infinite value of uncertainty estimation. Application of a finite value of uncertainty of the estimated unknown quantities leads to the so-called generalized (unified) advanced DVR method [14]; [22]. This generalized method makes it possible to increase the number of conditional equations (equality restrictions) of the advanced DVR method through utilizing equations in which unknown (not measured or non-measurable) quantities occur, which are not present in the other equations. These unknown values in the generalized advanced DVR method are so-called pseudo-measurements. Application of the generalized advanced DVR method delivers a greater reduction in uncertainty of measured quantities than the classical method, without preliminary estimation of the range of uncertainty of unknown values. In Poland, the method was presented in [7]; [25]; [8]. It was applied in the solving of boundary inverse problems of heat conduction process for the preliminary estimation of unknown temperature values and their uncertainty interval. The difference between the classical and generalized advanced DVR methods can be shown by comparing the form of a variance-covariance matrix of unknown variables after DVR calculations for both methods. The variance-covariance matrix of a solution of the generalized advanced DVR task including only unknown variables takes the following form [25]:

$$\hat{S}_Y = \left[S_Y^{-1} + a_Y \left(A_X S_X A_X^T\right)^{-1} A_Y\right]^{-1} \qquad (3)$$

As is apparent from Eq. 3, the generalized advanced DVR method always gives a better estimation of elements of the unknown variables variance-covariance matrix than the classical method. For the finite values of elements of matrix SY, elements of matrix resulting from Eq. 3 are always lower than elements of the variance-covariance matrix resulting from the classical method. If elements of variance-covariance matrix SY of a preliminary assessment of not-measured and non-measurable variables uncertainties approach infinity (i.e. we have a lack of information about the accuracy of assessment of these variables), a solution of Eq. 3 will approach to the solution of the classical method (without matrix SY in Eq. 3). This generalized method was also presented in other scientific works [24]; [22]; [23]. Further research has been undertaken by various scientists on the development and application of the generalized advanced DVR method in thermodynamics analyses of energy conversion processes by the author of this paper. They are presented in the next sections of the paper.

2 Advantages of applying generalized advanced DVR in thermal engineering

The benefits of applying the generalized advanced DVR method can also be shown by comparing the form of a variance-covariance matrix of the measurements after data validation calculations for both methods. For this purpose, to make a global assessment of the reduction in the measurements of data uncertainty, the value of a sum of weighted variances of measurement variables after DVR calculations was proposed. As weights of these variances, the inverses of measurements of variances adopted for DVR calculations were assumed. Utilizing a definition of the trace of a square matrix for the diagonal form of the variance-covariance matrix of measurement data $\mathbf{S_X}$ (i.e. in the case without stochastic dependences between measured variables), the equation can be written:

$$\sum_{i=1}^{m} \frac{\hat{\sigma}_i^2}{\sigma_i^2} = Tr\left(\hat{S}_X S_X^{-1}\right) \qquad (4)$$

On the other hand, utilizing the equation for calculating the variance-covariance matrix of measurement data after the classical advanced DVR calculations, obtained by the undetermined Lagrange multipliers method and the notion of the trace of a square matrix, it can be proven that [1]; [7]:

$$Tr\left(\hat{S}_X S_X^{-1}\right) = m + u - r \qquad (5)$$

It follows from eq. 4 and 5 that for a generalized advanced DVR task with a constant number of conditional equation r, changing the unmeasured variable u nature in the classical method to a pseudo-measurement one does not change the value of the sum of weighted variances of measurement variables after data validation calculations. It results from here that the value of the trace of the matrix is constant, but it includes a larger number of variables with a finite value of statistical weight – uncertainties of measurements. For the generalized advanced DVR method, eq. 5 takes the following form [24]:

$$Tr\left(\hat{S}_X S_X^{-1}\right) + Tr\left(\hat{S}_Y S_Y^{-1}\right) = m + u - r \quad (6)$$

As shown by eq. 6 for finite values of elements of matrix $\mathbf{S_Y}$, elements of the matrix $\hat{S}_X$ will always be smaller than elements of this matrix resulting from the classical method (i.e. for $S_Y^{-1} \to 0$).

Eq. 6 can be written as the equation of a straight line in two intercepts from:

$$\frac{Tr\left(\hat{S}_X S_X^{-1}\right)}{m+u-r} + \frac{Tr\left(\hat{S}_Y S_Y^{-1}\right)}{m+u-r} = 1 \quad (7)$$

where the value *m+u-r* is the *x*-intercept and *y*-intercept as well.

The reduction in the uncertainty of measurements obtained through the DVR method – interpreted as a whole – can then be presented graphically (Fig. 1).

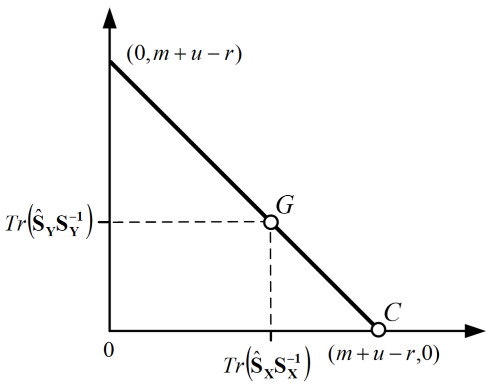

Figure 1: Graphical presentation of the reduction in measurements uncertainty through use of the generalized DVR method interpreted as a whole

For an advanced DVR task solved by the classical method, the point characterizing the uncertainty of measurements after their credibility interpreted as a whole will always be at the intersection of the line described by relationship 7, with the abscissa describing the values of the sum of the relative variances of the measurements after their validity – point *C* in Fig. 1). For the DVR task solved by the generalized method, however, this point will always be located on the straight line described by eq. 7 – exemplary point G in Fig. 1. Its location on the straight line will depend, among others, on the mutual relations of the assumed uncertainties of measurements and the uncertainties of the preliminary estimation of the unknown values. As is apparent from Fig. 1, every preliminary estimation of the uncertainty of the not-measured quantity in the generalized DVR method reduces the uncertainty of measurements.

The sum of weighted variances of measurements data and pseudo-measurements after reconciliation resulting from Eq. 6 can be used to evaluate the advantages of using the generalized advanced DVR method. After dividing the sum of the weighted variance including only real measurements in the generalized advanced DVR method resulting from Eq. 6 by the sum of variances in the classical method – Eq. 5 – the indicator can be obtained which describes the global reduction in the variances of real measurements after application of the generalized advanced DVR method:

$$\phi = 1 - \frac{Tr\left(\hat{S}_Y S_Y^{-1}\right)}{m+u-r} \quad (8)$$

It follows from Eq. 8 that the generalized advanced DVR method always leads to a greater reduction in the uncertainty of measured quantities (interpreted a global way) than the classical method. It is evident that the indicator resulting from eq. 8 for the classical method is always equal to one, whereas for the generalized advanced DVR method the indicator is always less than one. The advanced DVR method was developed further in [26], especially in its application in energy conversion processes in thermal power plants, mainly by applying the generalized (unified) method. The use of modern distributed control systems in industrial thermal processes has opened up brand new options for applying the advanced DVR method in computer systems decision support in the field of technical control and supervision of operation. These possibilities mainly concern energy conversion processes in thermal power and CHP plants [26]; [24]; [27]. However, use of the generalized advanced DVR method was not straightforward in this case. It required a solution to numerous new problems regarding both the basis of this method as well as application problems. The usefulness of control methods for assumed measurement accuracy as well as detection and identification of gross measurement errors from distributed control systems, using statistical inference methods, was tested [24]; [28]; [29]; [30]; [31]. Moreover, methods for preliminary estimation of not-measured and non-measurable values were developed. Methods of uncertainty assessment of this estimation in the generalized advanced DVR method have been established[24]; [29]. The methodology for developing the conditional equations set was elaborated with additional equations describing the ongoing energy conversion processes in order to obtain physically correct results of the advanced DVR task to be solved. Methodology and criteria were elaborated for developing the set of conditional equations for solving the DVR task with additional equations not having the exact status of physical laws [28]. For this purpose, the DVR task solution property described in Eq. 6 was used. It is presented in the next item of this article.

3 Requirements for applying additional conditional equations

In the generalized advanced DVR method, as described above, we can apply as conditional equations (equality constraints) additional equations which describe the physics of the processes that take place. Development of the conditional equations system in the task of the DVR method yields additional benefits, as presented in [28]; [24]. Additional conditional equations satisfying not only the substance and energy laws of conservation, but also the principle of entropy increasing in thermodynamics. Development of the conditional equations system also yields additional benefit for reducing measurement uncertainty after DVR. Evaluation of the effect of reducing uncertainty through the use of additional equations in generalized advanced DVR method in [24] is presented. To evaluate this effect, an indicator was implemented which results from dividing the sum of the weighted variance measurements and estimated values described by formula 6 by the number of all variables taken into consideration – measurements and estimated values. This indicator was defined as the global weighted average variance of all variables in the generalized advanced DVR method [24]:

$$\overline{D}^2 = \frac{Tr\left(\hat{S}_X S_X^{-1}\right) + Tr\left(\hat{S}_Y S_Y^{-1}\right)}{m+u} = 1 - \frac{r}{m+u} \tag{9}$$

On the base of eq. 9, the value of the global variance is uniquely determined by the number of equations, a number of measurements and estimated values in the DVR task. The value of the global variance $\overline{D}^2$ resulting from eq. 9 in the generalized advanced DVR method is always contained in the interval (0,1), because of the condition:$r<m+u$. Introduction to the advanced DVR task of additional conditional equations, measurements or estimated values would be beneficial if the global variance resulting from eq. 9 is less than the variance of the analyses without this development. The condition of an advantage considering the reduction of uncertainty in measurement after data reconciliation, in this case, is as follows [24]; [26]:

$$\frac{z-q}{z} < \overline{D}^2, \quad for \quad z,q \geq 0 \tag{10}$$

As is clear from eq. 10, the suitable effect of introducing additional conditional equations depends on the number of these equations z and the number of estimated values q , which will usually be the empirical coefficients present in these equations. For example, implementing one additional conditional equation with one estimated, an empirical coefficient (eq. z = 1 and q = 1) will always result in reducing the uncertainty of the reconciled variables. For these values of z and q the left-hand side of eq. 10 equals zero, in fact, this case. Thus, for a positive value of the global variance $\overline{D}^2$ resulting from Eq. 9, condition 10 is always satisfied.

4 The problem of estimating the specific enthalpy value of outlet steam from a condensing turbine

The outlet steam from the condensing turbine is wet saturated steam. The values of its specific enthalpy, as well as vapor mass fraction, are usually calculated from the energy balance of the turbo-generator. In the generalized advanced DVR method, the specific enthalpy value of the outlet steam from the condensing turbine must be treated as a pseudo-measurement.

Hence there is a need to estimate its value and the uncertainty of this estimation. This estimation should have physical justification. Hence, the limiting values of the specific enthalpy of the outlet steam from the turbine were adopted based on the operating conditions of the last stages of the steam condensing turbines. Experience shows that these stages work in the wet steam area.

In order to determine the quantitative impact of initial estimation of the specific enthalpy value of the outlet steam and the uncertainty of this estimation per specific heat consumption by the turbine set, simulation calculations were performed for the adopted steam specific enthalpy limiting values and the uncertainty of this estimate. Calculations were performed for the operating state of the analyzed power unit near the nominal parameters. The results of the calculations are presented in Fig. 2.

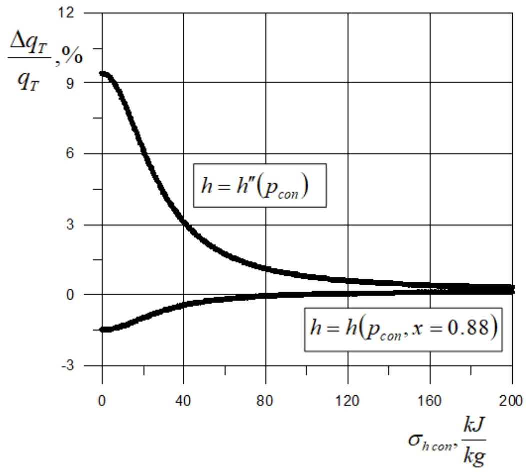

Figure 2: Influence of uncertainty estimation of the outlet enthalpy from the turbine to condenser on the relative change in the specific consumption of heat

On the abscissa axis, the assumed uncertainty estimation of specific enthalpy of the outlet steam is presented. The ordinate shows the relative change of the specific heat consumption by the turbine set calculated in relation to the value of this indicator calculated without the DVR method. In calculations, it was assumed that the upper limit value of this enthalpy is the enthalpy value of the dry saturated steam for a given pressure in the condenser, while the lower limit value is the wet steam enthalpy value for the adopted minimum vapor mass fraction value $x = 0.88$ [32].

The simulation calculations showed that the main factor that has an impact on the value of specific heat consumption by the turbine set is the correction of the live steam flow to the high-pressure part of the turbine, resulting from the reconciliation of the turbogenerator's energy balance. As shown in Fig. 2, for the uncertainty of the estimate of the outlet steam specific enthalpy, above a certain characteristic value, for the solved advanced DVR task (it is about 200 kJ/kg in the analyzed problem) this uncertainty has practically no effect on the specific heat consumption by the steam turbine set. The mutual ratio of measurement uncertainties (mainly live steam) and the uncertainty of the estimate of the outlet steam specific enthalpy mean that this enthalpy can vary across a wide range in DVR calculations.

It follows that in the case of assuming the uncertainty interval of an estimated variable with a high value, this interval is not very useful from the point of view of the generalized advanced DVR method. For general estimation of uncertainties, as presented, the initially estimated variable is treated as an unknown quantity.

For the assumed value of the specific enthalpy of saturated dry steam in the uncertainty range from 200 kJ/kg to approx. 120 kJ/kg, accepted for the calculation, there is a slight change in the specific heat consumption by the turbine set. This is due to the small correction of the live steam flow and the large specific enthalpy correction of the exhaust steam in order to fulfill the turbo-generator energy balance. For the assumed value of the specific enthalpy of wet saturated steam with the vapor mass fraction (steam quality) value $x = 0.88$, the small change in the specific heat consumption translates to the value of the uncertainty of the estimate of about 40 kJ/kg. After exceeding the uncertainty of specific enthalpy estimation of some characteristic values of both cases, there is a significant change in the specific heat consumption by the turbine set. This results from the impossibility of further significant correction of the specific enthalpy of the outlet steam to the condenser by the DVR calculation algorithm due to the value of the assumed uncertainty of the estimation. In order to fulfill the energy balance of the turbo-generator, in both cases the correction of the live steam stream before the turbine takes place – the stream is increased for the assumed saturated dry steam enthalpy and the decrease of this stream for the wet saturated steam enthalpy for the assumed vapor mass fraction limit of the exhaust steam $x = 0.88$.

It follows from the presented results of simulation calculations that, for the purpose of balancing the thermal system of the condensation power unit with the use of generalized advanced DVR method, unambiguous assumptions must be adopted for the initial estimation of the outlet steam enthalpy value and the

Table 1: Results of statistical analysis

No.	Designation	Value
1	Empirical coefficients:	
	c0	1.3957 ± 0.2778
	c1, 1/MW	0.0019 ± 0.0008
	c2	-51.16 ± 17.26
	c3, 1/MW2	-0,000008 ± 0.000003
	c4	959.2 ± 300.3
2	Multiple coefficient of determination R2, %	99.56
3	Estimated standard deviation of model error s	0.0025

accuracy of this estimation. Accordingly, for the initial estimation of the specific enthalpy value of the exhaust steam from the turbine, the empirical characteristic of adiabatic internal efficiency of the last group of stages of low-pressure turbine part was used. This empirical characteristic with use values of the special guarantee measurements was elaborated. After calculations using the stepwise regression method, the following mathematical form of the empirical characteristic was obtained:

$$\eta_i = c_0 + c_1 P_{el\ g} + c_2 \Pi + c_3 P_{el\ g}^2 + c_4 \Pi^2 \tag{11}$$

The values of the empirical coefficients of equation 11 and presented parameters of statistical analysis are given in Table 1.

In order to determine the uncertainty estimation interval of the outlet steam enthalpy, it was assumed that the final state of steam after expansion in the last group of turbine stages will not be in the area of superheated steam, and the upper limit value will be the enthalpy of dry saturated steam. As the lower limit of the uncertainty of the enthalpy estimation, the enthalpy for the limit vapor mass fraction $x = 0.88$ was assumed. Hence, the uncertainty interval results from the dependence:

$$\Delta h_{max}^0 = \tag{12}$$

$$= min \left[h_{out}^0 - h\left(p_{con}, x = 0.88\right); h"\left(p_{con}\right) - h_{out}^0 \right]$$

Assuming that the turbine exhaust steam enthalpy value is a random variable having a normal distribution, the maximum uncertainty interval resulting from dependence **??** is defined as expanded uncertainty for the confidence interval covering the value of the estimated parameter with probability (confidence coefficient) equal to $p = 0.95$.

From here, it can be assumed that the interval $\left[h_{out}^0 - \Delta h_{max}^0; h_{out}^0 + \Delta h_{max}^0\right]$ covers the value of the estimated enthalpy of the exhaust steam with the given probability. The value of the quantile $u_{\alpha/2}$ (where $\alpha = 1\text{-}p = 0.05$) resulting from the normal standardized distribution N (0,1) then takes the value $u_{\alpha/2} = 1.96$. Hence the standard uncertainty of the enthalpy of the outlet steam to the condenser results from the dependence:

$$\sigma_{hcon} = \frac{\Delta h_{max}^0}{u_{\alpha/2}} = \frac{\Delta d_{max}^0}{1.96} \tag{13}$$

5 Computational example

A computational example of the thermal cycle of a steam power unit of 150 MW electric power was developed. Fig. 3 shows a schematic chart of the investigated thermal cycle. According to the measurements system of this analyzed steam power unit, mass and energy balances equations in the steady state of operation of equipment – mainly turbine parts and heat exchangers have been formulated [26]; [33]. These balances constituted the conditional equations of the classical advanced DVR method.

Selected mass and energy balances in the steady-state take the following mathematical form:

- mass balance of the medium-pressure (MP) part of the steam turbine (Fig. **??**):

$$\dot{m}_{MPin} = \dot{m}_{MPvs} + \dot{m}_{R8} + \dot{m}_{SB5} + \tag{14}$$

$$+\dot{m}_{R4} + \dot{m}_{MPout} + \dot{m}_{MPexg}$$

- the energy balance of the (MP) part of the steam turbine (Fig. **??**):

$$\left(\dot{m}_{MPin} - \dot{m}_{MPsv}\right)\left(h_{MP/L} + h_{MP/R}\right)/2 = \tag{15}$$

$$\dot{m}_{R8}h_{R8} + \dot{m}_{SB5}h_{SB5} + \dot{m}_{R4}h_{R4} +$$

$$+ \left(\dot{m}_{MPout} + \dot{m}_{MPexg}\right) h_{MPout} + P_{iMP}$$

- the energy balance of the regenerative heat exchanger R6 (Fig. **??**):

$$\dot{m}_{R6}\left(h_{R6} - h_{condR6}\right)\eta_{R6} + \tag{16}$$

$$+\dot{m}_{condR7}\left(h_{condR7} - h_{condR6}\right)\eta_{R6} =$$

$$\dot{m}_{fw}\left(h_{fwout} - h_{fwin}\right)$$

A system of conditional equations can be developed that can deliver a reduction in measurements uncertainties after performing advanced DVR calculations, thereby reducing the uncertainties of calculated indicators of energy conversion processed. The reconciliation of measurements in steam power units to these equations can include equations of steam flow capacity in turbine stages and its adiabatic internal efficiency, the equations of pressure drop in pipelines, heat transfer in regenerative heat exchangers and equation resulting from the course of the isobars on the h-s diagram. In these equations to the values of not-measured, initially estimated in the generalized advanced DVR method, will also belong to the empirical coefficients. These coefficients are determined mostly on the basis of special or operation measurements of the considered power units. Mathematical forms of those equations are as follows:

the equation of adiabatic internal efficiency:

$$\eta_i = \frac{h_{in} - h_{out}}{h_{in} - h_{outs}} \tag{17}$$

the equation of steam flow capacity of a turbine:

$$\dot{m}_{st}\frac{\sqrt{T_{in}}}{p_{in}} = c_{sf}\sqrt{1 - \left(\frac{p_{out}}{p_{in}}\right)^2} \tag{18}$$

the equation of pressure drop in a flowing pipeline:

$$\Delta p = p_{in} - p_{out} = c_{pd}\dot{m}^2 \tag{19}$$

the equation of heat flow in regenerative heat exchangers (Peclet's equations):

$$\dot{m}_{fw}\left(h_{fwout} - h_{fwin}\right) = UA\Delta T_m \tag{20}$$

where the average temperature difference has been calculated as a thermodynamic temperature difference of fluid streams flow through the heat exchanger:

$$\Delta T_m = \left(\frac{\Delta h)}{\Delta s}\right)_{st} - \left(\frac{\Delta h}{\Delta s}\right)_{fw} \tag{21}$$

the equation of overcooling the condensate from the heat exchanger

$$\Delta T_{cond} = t_{sat}(p_{hes}) - t_{cond} \tag{22}$$

From the dependence describing the course of the isobars in the h-s diagram for water vapor

$$\left(\frac{\partial h}{\partial s}\right)_p = T \tag{23}$$

for finite values of increases of specific enthalpy $\Delta h = h_{out} - h_{outs}$ and entropy $\Delta s = s_{out} - s_{in}$ in an irreversible adiabatic process, the following form of the conditional equation was adopted in the reconciliation procedure:

$$\frac{h_{out} - h_{outs}}{s_{out} - s_{in}} = c_T\sqrt{T_{out}T_{outs}} \tag{24}$$

The right side of eq. 24 is the product of the geometric mean of the actual temperature T_{out} of the steam after expansion and the temperature after the reversible adiabatic expansion $T_{out\ s}$ and the empirical coefficient c_T .

Each of the presented equations contains a value (pseudo-measurement) which requires initial estimation. They are adiabatic internal efficiency, empirical coefficients c_{sf} ,c_{pd} , $c_{T,}$ and average overall heat transfer coefficient U . Similarly to the empirical characteristic of the adiabatic internal efficiency of the last group of stages of the low-pressure turbine part, for initial estimation of these quantities, values of the special guarantee measurements of the power unit were used.

Table 2: Results of statistical analysis

Estimated quantity y	Empirical coefficients of the equation (25)	Coefficient of determination R2, %
Adiabatic internal efficiency	c0 = 0.9894	93.43
	c1 = -0.0006686, 1/MW	
Coefficient in the equation of steam flow	c0 = 81,41	82.83
capacity of a turbine csf	c1 = -0,00415, 1/MW	

Using the methods of regression analysis, the estimation was made of linear empirical functions describing the variability of the analyzed quantity having the status of a pseudo-measurement in the solved generalized advanced DVR task. The analyses performed showed a significant dependence of the mentioned values on the real electric power of the generator. As a basic measure of characteristic fitting to the measured values, a coefficient of determination was calculated for each of them. The determined empirical functions take the general form:

$$y = c_0 + c_1 P_{elg} \tag{25}$$

where y is an estimated quantity.

The value of the uncertainty of the estimated quantity having the status of a pseudo-measurement was determined based on the confidence area for the predicted value obtained from linear relationship 25. For example, Table 1 presents the values of the empirical coefficients of equation 25 for estimating values of adiabatic internal efficiency η_i (Eq. 17) and coefficient in the equation of steam flow capacity of a turbine c_{zf} (Eq. 18), for the first group of stages of the medium part of turbine (inlet to turbine medium-pressure part – steam extraction to desuperheater R8), (Fig. **??**).

Similarly, Table 2 presents values of the empirical coefficients of equation 16 for estimating values of heat transfer coefficient U (Eq. 20) and overcooling of condensate (Eq. 22) from the low-pressure heat exchanger R4 (Fig.**??**).

Introduction of the abovementioned additional equations for the advanced DVR task, as the additional conditional equations, which includes initially only mass and energy balances will reduce the uncertainty of measured and estimated variables after data reconciliation. Each of equations 17-20, 22, 24 as shown contains only one variable, having the status of the pseudo-measurement. Equations 17, 17 and 24 were written for groups of turbine stages, whereas equations 19, 20 and 22 for regenerative heat exchangers. According to eq. 10, implementation of these equations to the generalized advanced DVR task will reduce uncertainties of measurements.

Table **??** presents a list of selected measured and not-measured quantities having the status of a pseudo-measurement of the thermal system of the investigated power unit (Fig. **??**).

Results of reconciliation calculation using the generalized advanced DVR method for these quantities for variant C are presented in Table **??**. Column no. 2 of this table shows the raw measurements and preliminary estimated values of the quantities presented in Table **??**. Column no. 3 contains the adopted standard uncertainties of the quantities. Columns no. 4 and 5 present the results of DVR calculation. Column no. 6 of Table **??** shows the results of calculations of the statistical test of control of the assumed uncertainty of measurements and preliminary estimated values using a variance-covariance matrix of measurements corrections according to VDI 2048 [22] . In the applied statistical method for each measured and preliminary estimated quantity the formulated null hypothesis H_{0i} (i -th quantity fulfills assumed accuracy) is not rejected, if the value of the test of statistics Z meets the following inequality:

$$Z_i = \frac{|\hat{x}_i - x_i|}{\sqrt{max\left(\sigma_{Vi}^2, \frac{\sigma_i^2}{10}\right)}} \tag{26}$$

where the value of the quantile $u_{\alpha/2}$ (where $\alpha = 0.05$) resulting from the normal standardized distribution N (0,1) takes the value $u_{\alpha/2} = 1.96$.

As can be seen from Table **??**, the use of the advanced DVR method (variant B) brings measurable benefits in terms of improving the reliability of the calculated specific consumption of heat. The relative standard uncertainty of this indicator decreased by 61.2% in relation to the uncertainty calculated without using the advanced DVR method. Further reduction in the uncertainty of the indicator q_T is obtained by extending the system of conditional equations with equations resulting from the course of the energy conversion processes, i.e., using the generalized advanced DVR method. In this case (variant C) the relative standard uncertainty of this indicator decreased by 72.1% in relation to the uncertainty calculated without using the advanced DVR method. As a result of applying the advanced DVR method, it is ensured that the values of quantities used to calculate the indicator q_T satisfy mass and energy balances and that, in the case of the generalized DVR task with additional conditional equations, these values also satisfy the principle of increasing entropy for the physical phenomena that take place.

6 Summary and final conclusions

This paper presents the benefits of using the generalized advanced DVR method in studies of exemplary energy conversion processes. Properties and indicators showing the advantages of this method, derived on the theoretical path, are presented. These include, first of all, the global weighted average variance of all variables and indicator of global reduction of real measurements variances. A computational example was developed of a thermal system of a steam power unit of 150 MW electric power using the classical and generalized advanced DVR methods. The advantages of both methods are shown in the example of decreasing uncertainty of measurements used to calculate the uncertainty of the specific consumption of heat q_T by the turbo-generator (Table **??**). It is shown that the generalized DVR method brings more benefits than the classic approach. In accordance with the theoretical predictions, for the classical method, the relative standard uncertainty of the indicator q_T decreased by 61.2% in relation to the uncertainty calculated without using the advanced DVR method, whereas in the case of the generalized method this decrease was 72.1%. The indicators presented in Table **??**, characterizing the DVR task in a comprehensive manner, show the favorable changes flowing from the development of the conditional equations system resulting from the course of energy conversion processes. Both the global weighted average variance of all variables and indicator of global reduction of real measurements variances are reduced.

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Table 3: List of selected measured quantities having the status of a pseudomeasurement

No.	Measured quantities	Unit
1	Live steam flow from the boiler	t/h
2	Pressure of the live steam at the inlet of the turbine HP part/L	MPa
3	Temperature of the live steam at the inlet of the turbine HP part/L	°C
4	Pressure of the live steam at the inlet of the turbine HP part/R	MPa
5	Temperature of the live steam at the inlet of the turbine HP part/R	°C
6	Water injection flow to the live steam	t/h
7	Flow of the boiler blowdown	t/h
8	Water injection flow to the re-superheated steam	t/h
9	Pressure of the steam at the inlet of the turbine MP part/L	MPa
10	Temperature of the steam at the inlet of the turbine MP part/L	°C
11	Pressure of the steam at the inlet of the turbine MP part/R	MPa
12	Temperature of the steam at the inlet of the turbine MP part/R	°C
13	Feed water flow to the boiler	t/h
14	Temperature of the feed water flow to the boiler	°C
15	Pressure of the feed water flow to the boiler	MPa
16	Flow of the condensate from heat exchanger R6	t/h
17	Temperature of the condensate from heat exchanger R6	°C
18	Pressure in steam bleeding no. 5	MPa
19	Temperature in steam bleeding no. 5	°C
20	Flow from steam bleeding no. 5	t/h
21	Flow from steam bleeding no. 3	t/h
22	Pressure in steam bleeding no. 3	MPa
23	Temperature in steam bleeding no. 5	°C
24	Flow of the condensate from heat exchanger R2	t/h
25	Temperature of the condensate from heat exchanger R2	°C
26	Pressure in steam bleeding no. 1	MPa
27	Temperature in steam bleeding no. 1	°C
28	Flow of the condensate from heat exchanger R1	t/h
29	Temperature of the condensate from heat exchanger R1	°C
30	Flow of the main condensate from the condenser	t/h
31	Flow of the condensate from the condensate tank of exchanger R1	t/h
32	Temperature of the condensate from the condensate tank	°C
33	Pressure of the exhaust steam from the turbine LP part	MPa
34	Temperature of the exhaust steam from the turbine LP part	°C
35	Real power of the turbo-generator	MW

Table 4: Empirical coefficients of equation (25) for low-pressure heat exchanger R4

Quantity	Empirical coefficients of the equation (25)	Coefficient of determination R2, %
heat transfer coefficient U, W/(m2 K),	c0 = -111.36 c1 = 11.91, 1/MW	99.91
Overcooling of the condensate from the exchanger, tcond, K	c0 = 1.65 c1 = 0.0353, 1/MW	86.88

Table 5: List of selected not measured quantities having the status of a pseudomeasurement

No.	Not-measured quantities having the status of a pseudo-measurement	Unit
36	Specific enthalpy of the exhaust steam from the turbine LP part	kJ/kg
37	Flow of the re-superheated steam at the outlet of the boiler	t/h
38	Flow of steam from the MP part of the turbine	t/h
39	Flow of steam from the LP part of the turbine	t/h
40	Steam flow from steam bleeding no. 6 to heat exchanger R8	t/h
41	Steam flow from steam bleeding no. 7 to heat exchanger R7	t/h
42	Steam flow from steam bleeding no. 4 to heat exchanger R4	t/h
43	Steam flow from steam bleeding no. 2 to heat exchanger R2	t/h
44	Flow of the main condensate to the feed water tank	t/h
45	Internal power of the HP part of the turbine	MW
46	Internal power of the MP part of the turbine	MW
47	Internal power of the LP part of the turbine	MW
48	Adiabatic internal efficiency of the HP part of the turbine	-
49	Adiabatic internal efficiency of the first group of stages MP turbine	-
50	Adiabatic internal efficiency of the second group of stages MP turbine	-
51	Adiabatic internal efficiency of the third group of stages MP turbine	-
52	Adiabatic internal efficiency of the first group of stages LP turbine	-
53	Heat transfer coefficient in heat exchanger R8	W/(m2K)
54	Heat transfer coefficient in heat exchanger R7	W/(m2K)
55	Heat transfer coefficient in heat exchanger R6	W/(m2K)
56	Heat transfer coefficient in heat exchanger R4	W/(m2K)
57	Heat transfer coefficient in heat exchanger R3	W/(m2K)
58	Heat transfer coefficient in heat exchanger R2	W/(m2K)
59	Heat transfer coefficient in heat exchanger R1	W/(m2K)

Table 6: Input data and results of the generalized advanced DVR task calculation

No.	The value of the measurement or the estimated quantity	The uncertainty of the measurement or the estimated quantity	The value of the measurement or the estimated quantity after DVR calculation	The uncertainty of the measurement or the estimated quantity after DVR calculation	Test statistic Zi of the corrections (Inequality no. 26)
1	417.30	10.00	409.02	2.63	0.86
2	15.12	0.10	15.11	0.10	0.21
3	556.20	2.00	557.11	1.95	1.45
4	15.10	0.10	15.09	0.10	0.21
5	556.80	2.00	557.71	1.95	1.44
6	27.30	1.00	27.17	1.00	0.40
7	0.50	0.30	0.50	0.30	0.03
8	13.10	0.50	13.01	0.50	0.59
9	3.62	0.02	3.63	0.02	0.99
10	554.90	2.00	554.69	1.84	0.27
11	3.62	0.02	3.63	0.02	0.99
12	554.20	2.00	553.99	1.84	0.27
13	395.60	10.00	383.06	2.80	1.31
14	253.70	2.00	255.02	0.85	0.73
15	16.90	0.10	16.90	0.10	0.00
16	63.50	2.00	62.25	0.93	0.71
17	191.30	2.00	189.56	0.70	0.93
18	0.85	0.01	0.85	0.01	0.76
19	341.20	2.00	342.71	1.36	1.03
20	15.20	0.20	15.23	0.20	0.53
21	22.70	1.60	24.93	0.46	1.46
22	241.21	2.00	241.60	1.90	0.61
23	202.80	2.00	202.19	1.81	0.73
24	47.30	1.50	47.38	0.41	0.06
25	74.50	1.00	74.99	0.51	0.57
26	22.45	1.00	22.59	0.49	0.16
27	63.20	1.00	62.74	0.48	0.52
28	11.20	0.20	11.26	0.19	0.89
29	62.10	1.00	60.89	0.58	1.48
30	325.80	10.00	326.38	3.37	0.06
31	57.90	1.50	58.64	0.43	0.52
32	71.60	1.00	71.92	0.42	0.36
33	0.00430	0.00010	0.00427	0.00009	0.83
34	29.40	1.00	30.11	0.38	0.77
35	148.30	0.50	148.07	0.47	1.37

Table 7: Input data and results of the generalized advanced DVR task calculation

No.	The value of the measurement or the estimated quantity	The uncertainty of the measurement or the estimated quantity	The value of the measurement or the estimated quantity after DVR calculation	The uncertainty of the measurement or the estimated quantity after DVR calculation	Test statistic Zi of the corrections (Inequality no. 26)
36	2320.23	28.09	2345.36	12.44	1.00
37	377.08	5.51	380.84	2.41	0.76
38	331.21	10.43	327.99	2.08	0.32
39	283.80	16.03	285.80	1.91	0.13
40	26.22	3.34	27.49	0.71	0.39
41	31.51	7.29	34.75	1.06	0.45
42	11.96	2.06	13.93	0.37	0.97
43	9.95	1.62	7.12	0.34	1.78
44	341.80	8.31	343.98	2.15	0.27
45	40.10	1.14	37.67	0.55	1.20
46	59.73	1.25	59.58	0.44	0.14
47	52.41	0.81	54.21	0.73	1.96
48	0.8389	0.08	0.8312	0.01	0.03
49	0.8934	0.02	0.8746	0.01	1.20
50	0.9578	0.02	0.9478	0.01	0.81
51	0.9809	0.0049	0.9803	0.0048	0.40
52	0.9245	0.05	0.8898	0.04	0.95
53	149.41	9.65	246.06	8.63	1.78
54	1479.43	73.59	1522.16	58.79	0.97
55	1965.67	60.77	1962.18	58.04	0.18
56	1655.62	39.60	1646.18	36.54	0.62
57	2104.58	68.39	2084.72	57.61	0.54
58	2072.13	81.42	2101.17	74.64	0.89
59	2357.30	111.62	2227.34	82.99	1.74

Table 8: Results of calculations of the advanced DVR tasks of the thermal cycle of the steam power unit

Variant of calculations		Evaluation of the specific consumption of heat by the turbo-generator		
	Parameters of the advanced DVR task			
A	Mathematical balance model with the minimum resources of measurement information (calculations without DVR method: u = r)	qT = 7971.8 a 292.1 kJ/kWh		
		RSD(qT) = 3.66 B	Number of measurements: m = 63	qT = 7822.7 a 111.0 kJ/kWh
	Number of not measured quantities: u = 22	RSD(qT) = 1.42	Number of conditional equations: r = 25	?qT = 0.6120
	D2= 0,7059	?= 0.8258		
C	Number of measurements: m = 63	qT = 7845.6 a 80.4 kJ/kWh		
	Number of not measured quantities (pseudo-measurements): u = 59	RSD(qT) = 1.02	Number of conditional equations: r = 57	?qT= 0.7213
	D2 = 0,5328	? = 0.5351		

8 Nomenclature

A area, m^{2,}

A_X Jacobian matrix of conditional equations for measurements data,

A_Y Jacobian matrix of conditional equations for preliminarily estimated variables,

c_{pd} empirical coefficient in the equation of pressure drop in a flowing pipeline,

c_{sf} empirical coefficient in the equation of steam flow capacity of a turbine,

c_T empirical coefficient in dependence describing the course of the isobars in the h-s diagram,

g_l *l* -th conditional equation of data validation task,

h specific enthalpy, kJ/kg,

$h_{MP/L}, h_{MP/R}$ specific enthalpies of steam on the inlet of the turbine,

h_{R8} specific enthalpy of steam to desuperheater R8,

h_{R4} specific enthalpy of steam to heat exchanger R4,

h_{SB5} specific enthalpy of steam from steam bleeding no.5,

h_{MPout} specific enthalpy of steam on the outlet of the MP part of the turbine,

h_{R6} specific enthalpy of steam to heat exchanger R6,

h_{condR6} specific enthalpy of condensate from heat exchanger R6

$h_{condrR7}$ specific enthalpy of condensate from heat exchanger R7

h_{fwin}, h_{fwout} specific enthalpies of feed water on inlet and outlet heat exchanger R6,

m number of measurements,

$\dot{m}$ mass flow, kg/s,

$\dot{m}_{MPin}$ steam flow to the MP part of the turbine,

$\dot{m}_{MPsv}$ flow from the valve seal before MP part of the turbine,

$\dot{m}_{R8}$ flow from the steam bleeding to desuperheater R8,

$\dot{m}_{SB5}$ flow from steam bleeding no. 5,

$\dot{m}_{R4}$ flow from the steam bleeding to heat exchanger R4,

$\dot{m}_{MPout}$ steam flow on the outlet of the MP part of the turbine,

$\dot{m}_{MPexg}$ steam flow from the external gland of the MP part of the turbine.

$\dot{m}_{R6}$ the mass flow of steam to heat exchanger R6,

$\dot{m}_{condR7}$ the mass flow of condensate from heat exchanger R7

$\dot{m}_{fw}$ the mass flow of feed water to heat exchanger R6,

p pressure, Pa,

p_{iMP} internal power of the MP part of the turbine,

P_{elg} the electric real power of the turbo-generator,

q number of estimated a pseudo-measurements,

r number of conditional equations,

s entropy, kJ/(kg K),

S_X variance-covariance matrix of measurements data before DVR,

$\hat{S}_X$ variance-covariance matrix of measurements data after DVR,

S_Y variance-covariance matrix of preliminarily estimated variables before DVR,

$\hat{S}_Y$ variance-covariance matrix of preliminary estimated not measured variable after DVR.

T absolute temperature, K

$Tr()$ trace of an square matrix.

U heat transfer coefficient, W/(m^2 K),

u number of not measured variables,

x raw measurement data,

$\hat{x}$ reconciled measurement data,

$\hat{y}$ reconciled not measured variable,

z number of additional conditional equations,

σ standard uncertainty of raw measurement data,

$\hat{\sigma}$ standard uncertainty of a reconciled measurement data,

Π the ratio of pressures before and behind the group of the last group of stages.

η_i adiabatic internal efficiency,

η_{R6} energy efficiency of heat exchanger R6 describing heat losses to the environment

ΔT_m average temperature difference, K,

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