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# MULTIVARIANT MODELING AS A TOOL IN DESIGNING ENERGY AND TECHNOLOGICAL SYSTEMS*)

An idea of description of many system structure variants with a one mathematical model is presented in the paper. This multivariant model is constructed on the basis of so called „superstructure" which combines structures of all considered variants. Particular variants of the system can be however judge separately with any accepted criterion as the multivariant model can be automatically reduced to a model of any chosen system variant. A set of logical relations defined for the superstructure and for the multivariant model itself is applied for these transformations. The presented method of modeling is discussed on an example of a steady state model of an energy system.

## INTRODUCTION

An energy system is to generate assumed quantity of products – energy carriers of defined quality, in an assumed period of time. A designed system must usually fulfil some additional, local limitations, e.g. on the system localization or system supply conditions. Apart from this, it is possible to design the system in different ways, considering a set of the system variants. These variants can differ at least in:

- process structure and topological structure, i.e. kind, number, size and arrangement of devices of equipment elements with their mutual relations; energy systems can be combined with different heat sources, transmission systems, heat transformations, and other components,
- operation parameters (pressures, rates of flows, specific enthalpies, temperatures, etc.).

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If the variants have different structures or operation parameters, they have different technical and economical characteristics – specific capital cost, specific operation and maintenance cost, thermodynamical efficiency, influence on the surrounding natural environment etc. The basic task of pre-designing a new energy system or modernizing an old one is the selection of its structure and determining nominal operation parameters for which the system is the cheapest, or/and the less energy-consuming, the most environmentally sound, the most reliable. So more than one criterion of the system quality estimation is usually taken into account, and it is necessary to judge all considered system variants separately. Mathematical modeling is an effective and the cheap tool for a variant selection. Steady-state optimization models can be used to select the system variant. This problem is discussed further on in the paper. One cannot forget, however, that a dynamic behavior of the system should be examined as well.

## 1. REMARKS ON THE STEADY-STATE MODELS

A mathematical model of an energy system is constructed on the basis of a system structure. There are several conventions for graphical representation of energy system structures. The structure can be depicted as a graph with nodes (where all processes take place) that are connected with links of constant parameter flows. The mathematical model of a system is a sum of sub-models of particular nodes. The steady-state models of energy systems can consist of:

- equations derived from mass and energy conservation principles,
- equations characterizing type and quality of operation of devices, a plant's components which are the system elements,
- equations and inequalities describing imposed technical, economical, ecological or other limitations,
- equations or inequalities describing qualitative relationships between thermodynamics parameters (pressures, temperatures, specific enthalpies, etc.),
- logical or Boolean relations,
- equations of thermodynamics parameters functions,
- criterion function,
- others – e.g. determining system reliability.

An energy system mathematical model is a set of equations and inequalities characterizing the system structure nodes (devices hidden in the nodes). Model variables are parameters of the links (mass or energy flows connecting these devices) or parameters of the nodes themselves (surfaces of heat exchangers, pipe diameters, boiler capacities, etc.). A characteristic feature of the steady-state optimization model is that a great part of it is composed of algebraic linear constraints. Logical or Boolean relations have been applied in mathematical modeling of energy systems for many years, e.g. [1–5]; they play a special role in this model, which will be discussed further.

Model dimensions depend on a number of elements considered in the system. This number is usually large. Therefore, preparation of an energy system model is a laborious and time-consuming process. This disadvantage increases when several models for particular variants of a system structure are created.

## 2. SUPERSTRUCTURE AND A CONCEPT OF MULTIVARIANT MODEL

Structures of all considered system variants can be added. The resulting structure of an abstract multivariant system is called a superstructure [2] or a multivariant structure [6].

An example of a relatively simple superstructure is shown in Fig. 1. It is a multivariant system for hot water generation. The heat source is a water boiler, a steam boiler or a simplified heat and power generating plant. The water-boiler house and the steam-boiler house can be located at a distance. Hot water can be generated directly in water boilers, or heated in surface water/water or steam/water heat exchangers, or heated in water-steam jet pumps. Both water and steam can be used as a heat carrier.

One „multivariant mathematical model" can be constructed on the basis of a given superstructure. Preparing such a model is much less labor-consuming than modeling a dozen or so system variants apart from the fact that a multivariant model is much bigger than any model of particular variant. This is an important feature of multivariant modeling. If the variants do not differ too much, the multivariant model can be used in computations directly as in [2]. However, there are usually some limitations in using this model, which may be huge and hard to control. In some cases it can contain equations (or inequalities) which are correct for particular variants but are mutually inconsistent. This is because particular nodes can play different roles at different system variants. Furthermore, as it was mentioned earlier, the variants of the system design should be estimated individually. To take the advantage of the positive feature of the multivariant modeling (reduction of time and labor for model elaboration), it is necessary to convert the multivariant model to models of particular variants in possibly the least labor-consuming way, so automatically. Definite equations and inequalities of the multivariant model must be removed or changed and definite variables which are not variables of the chosen variant model must be eliminated. For this purpose, logical or Boolean relations can be used.

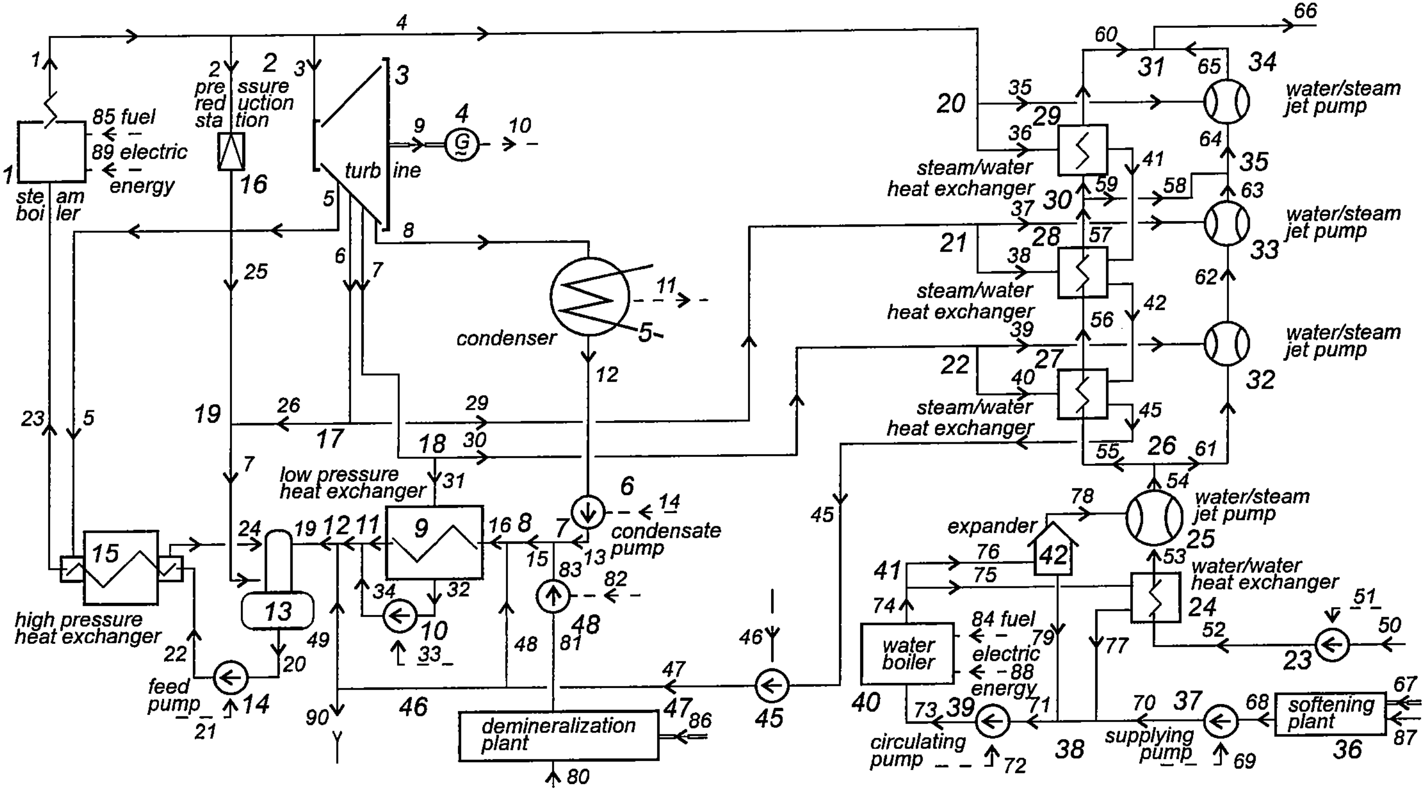


Fig. 1. A superstructure of a hot water generation plant

## 3. LOGICAL MODEL OF THE SYSTEM

The basic information for statement of the fact that a given variable or a given equation (inequality) are elements of a model of a chosen system variant is the fact of existence or non-existence of particular nodes and links of a superstructure in a structure of the chosen and considered variant. The superstructure can be described by a set of logical relations which associate mutual presence of links and nodes [6]. A logical variable $l_i$ can be assigned to each superstructure link $i$, logical variable $q_j$ can be assigned to a superstructure node $j$ (which parameters are variables of the mathematical model), and logical variables $v_i$ can be assigned to chosen equations of inequalities of the multivariant model. The value of a logical variable indicates if a link, a node or an equation (inequality), respectively, is an element of the structure or of the model of the chosen and considered system variant.

For each node of the system structure at least one logical relation derived from conservation principles can be defined. If any mass flow enters the node, another mass flow which leaves the node must exist and vice versa. If any energy (non-mass) flow enters the node, another mass or non-mass flow must leave it. This fact can be written in a logical relation form called here a conservation relation.

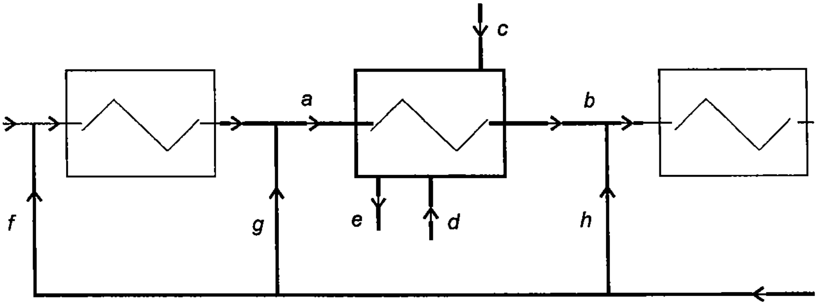


Fig. 2. Part of a superstructure – part of a regeneration system of a power unit: $a$ – cold water, $b$ – warmed water, $c$ – main heating agent (steam), $d$ – hot condensate from adjoining heat exchangers, $e$ – cold condensate, $f, g, h$ – condensate from other technological subsystems

Figure 2 presents a set of three surface steam/water heat exchangers which can be a part of a regeneration system at a steam power unit. For the second (central) heat exchanger the two following conservation relations can be defined:

$$l_a \Leftrightarrow l_b \tag{1}$$

$$l_c \vee l_d \Leftrightarrow l_e \tag{2}$$

Mutual existence of flows depends as well on technological features of modeled devices. This can be illustrated on the example of the water expander, shown in Fig. 3, where outflow of steam $c$ exists only if outflow of expanded water $b$ exists. It is described in the relation (3) called here a technological relation

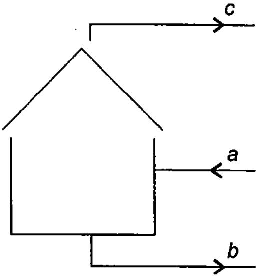


Fig. 3. Water expander: *a* – high-pressure water, *b* – low-pressure water, *c* – steam

$$l_c \Leftrightarrow l_b \wedge l_c \tag{3}$$

A good example of another technological relation is

$$l_a \Leftrightarrow l_a \wedge l_e \tag{4}$$

written for the node containing a steam turbine (Fig. 4). The relation (4) states that if the turbine is supplied with steam, some amount of steam must outflow behind the last group of the stages for cooling them.

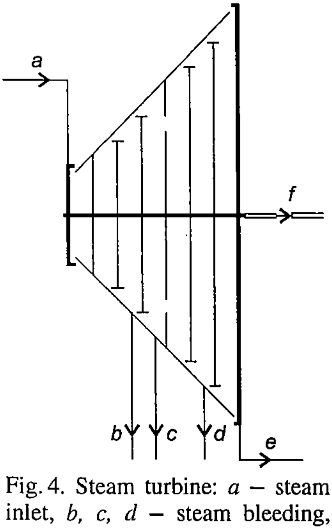


Fig. 4. Steam turbine: *a* – steam inlet, *b, c, d* – steam bleeding, *e* – steam outlet, *f* – mechanical energy

Other logical relations which associate mutual presence of links result from special features of given superstructures. More precisely, these features are a consequence of accepted mutual connections of nodes. One such structural relation can be defined for the part of superstructure given in Fig. 2. Three points of a condensate input to the regeneration system (flows *f, g, h*) are possible. Only one of these three solutions can be accepted in particular system variant. No two of the flows *f, g, h*, can exists simultaneously. This statement is written in the form of the logical structural relation

$$(l_f \wedge l_g) \vee (l_g \wedge l_h) \vee (l_f \wedge l_h) \Leftrightarrow \mathit{false} \tag{5}$$

The above-listed relations stated the mutual existence of links (flows). The existence of nodes or rather existence of particular devices in the nodes depends on the existence of flows assigned to them. This can also be written in the form of logical relations called here nodal relations. An example of a nodal relation for the central heat exchanger in Fig. 2 is

$$q_j \Leftrightarrow l_c \wedge l_a \tag{6}$$

If steam and water do not supply the node simultaneously, the heat exchange surface does not exist. The node converts to an „empty node” in which no processes occur.

The knowledge of existence of particular links and nodes in a given structure of a system variant is sufficient for removing needless variables from the multivariant model. These variables, as it was mentioned, are parameters of the flows represented by the links, or of devices hidden in the nodes. Removing a part of the variables transforms equations and inequalities of the multivariant model. Some of them disappear. However, this conversion is not always correct. Some equations or inequalities must be skipped besides the variables removed [8]. It is also possible to determine the existence of particular equations or inequalities of the multivariant model in the model of the chosen variant, using logical relations. These relations were called additional relations. For typical nodes of energy system structures, it is easy to determine for which equations (inequalities) additional relations should be defined.

For example, in the case of the water expander (Fig. 3) one of the model equations determines the specific enthalpy of the out-flowing saturated water as a function of pressure in the expander

$$i)\quad h_b - h'(p_b) = 0 \tag{7}$$

where: $h$ – specific enthalpy, $p$ – pressure, index $'$ – state of water saturation. The equation (7) – marked as equation $i)$ of the multivariant model – can be false if the steam flow $c$ is neglected in the superstructure and the water expander is replaced by an „empty node”. The water flow $a$ enters the node and flow $b$ of the same parameters leaves it. The variables $h_b$ and $p_b$ are actually variables of the chosen variant model as the link $b$ (flow $b$) exists in the structure of this variant but the water $b$ need not be saturated. Although $h_b$, $p_b$ are not removed from the multivariant model, the equation $i)$ should be skipped. If a logical variable $v_i$ is assigned to the equation $i)$, the following simple additional relation determines its value

$$v_i \Leftrightarrow q_j \tag{8a}$$

The additional relation has the equivalent form

$$v_i \Leftrightarrow l_c \tag{8b}$$

The above-mentioned five categories of the logical relations create a kind of a logical model of the multivariant system

$$\begin{array}{lll} \text{– conservation relations} & \boldsymbol{lC}(\boldsymbol{l}) \Leftrightarrow \boldsymbol{rC}(\boldsymbol{l}) & \\ \text{– technological relations} & \boldsymbol{lT}(\boldsymbol{l}) \Leftrightarrow \boldsymbol{rT}(\boldsymbol{l}) & \\ \text{– structural relations} & \boldsymbol{lS}(\boldsymbol{l}) \Leftrightarrow \boldsymbol{rS}(\boldsymbol{l}) & (9) \\ \text{– nodal relations} & \boldsymbol{q} \Leftrightarrow \boldsymbol{N}(\boldsymbol{l}) & \\ \text{– additional relations} & \boldsymbol{v} \Leftrightarrow \boldsymbol{A}(\boldsymbol{l},\boldsymbol{q}) & \end{array}$$

where $\boldsymbol{lC}$, $\boldsymbol{rC}$, $\boldsymbol{lT}$, $\boldsymbol{rT}$, $\boldsymbol{lS}$, $\boldsymbol{rS}$, $\boldsymbol{N}$, $\boldsymbol{A}$ are sets of left or right sides of specified groups of logical relations, respectively.

## Superstructure and multivariant model reduction

A structure of any system variant can be defined by neglecting a limited number of links of a superstructure

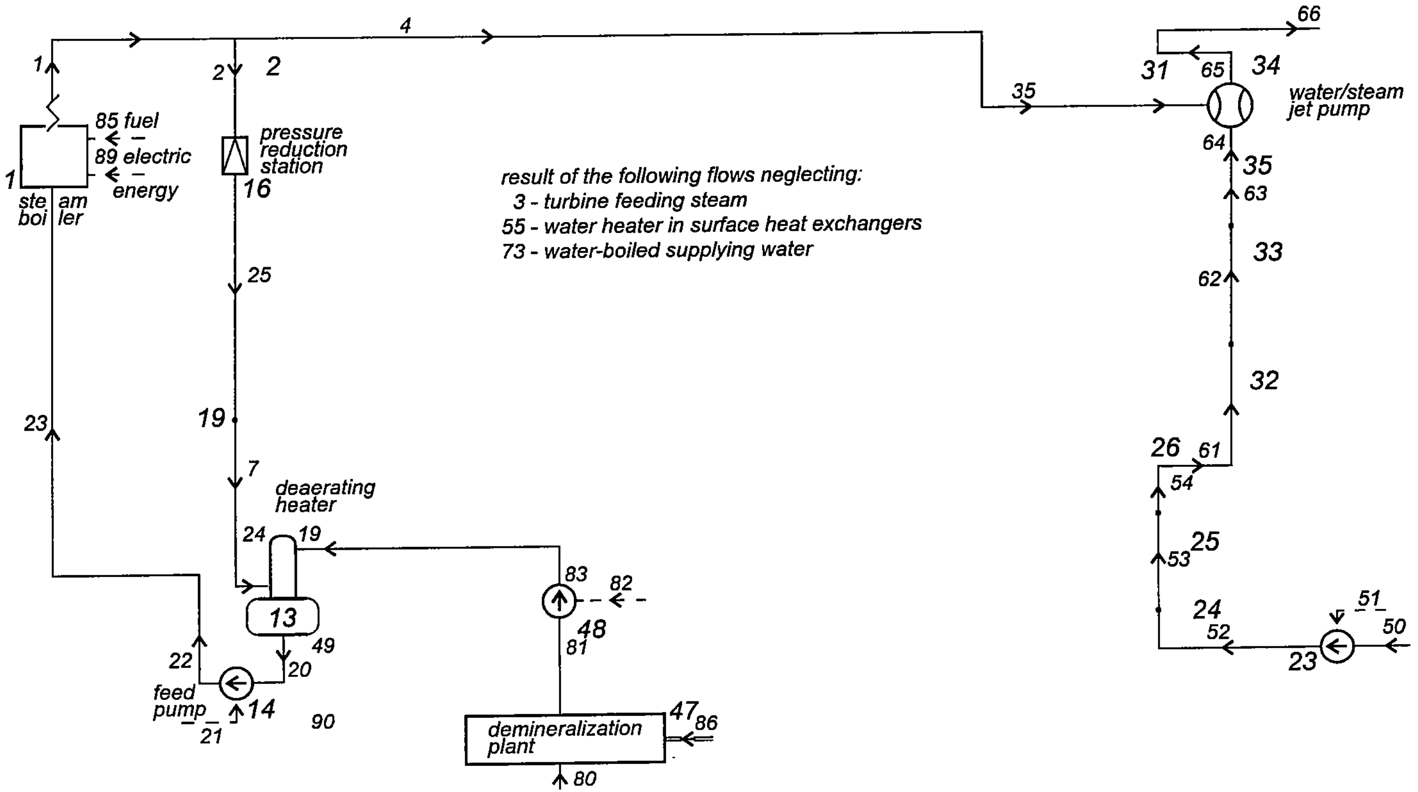
$$\bigwedge_{i \in I_N} l_i \equiv \mathit{false} \qquad (10)$$

where $I_N$ is a set of these neglected bonds.

Solution of a set of conservation, technological and structural relations with the assumption (10) obtains a value of $\boldsymbol{l}$ – a full information about existence or non-existence of any superstructure bond at the structure of the chosen variant [6]. Solving a set of nodal relations with this value of $\boldsymbol{l}$, one can determine $\boldsymbol{q}$ – so the information about the existence or non-existence of particular nodes of the superstructure, at a considered variant. Finally, the solution of additional relations with the earlier determined values of $\boldsymbol{l}$ and $\boldsymbol{q}$ provides the necessary information for eliminating some equations (inequalities) from the multivariant model.

Contrary to the multivariant model reduction, it is relatively easy to illustrate the superstructure reduction. In the case of the superstructure shown in Fig. 1, if one neglects only three following flows of: turbine feeding steam 3, water heated in surface heat exchangers 55, water-boiler feeding water 73, and solves a set of appropriate conservation, technological, structural and nodal relations, the structure of a system variant shown in Fig. 5 results. It is a simplified scheme of a real system of technological water heating in Sulfur Mine „Basznia” in the South-East Poland. In this system, a steam boiler house is a heat source, and contaminated technological water is heated in a one-stage water/steam jet pump.

The values of $\boldsymbol{l}$, $\boldsymbol{q}$ and $\boldsymbol{v}$, determined for the chosen variant of the system, can be used for the multivariant model conversion to a model of the considered variant. The conversion is a two step process. At first variables – parameters of flows (links) or devices (nodes) for which $l_i = \mathit{false}$ or $q_i = \mathit{false}$ respectively, are removed from the multivariant model equations and inequalities. Then model constraints (equation and inequalities besides an objective function) are skipped if all of their variables were removed or the value of $v_i$ assigned to the constraint is *false*. The automatic process of the multivariant model conversion is relatively easy in the case of model consisting only of linear or linearized constraints.



result of the following flows neglecting:
3 - turbine feeding steam
55 - water heater in surface heat exchangers
73 - water-boiled supplying water

Fig. 5. A chosen variant of a hot water generating system – technological water heating system in Sulfur Mine „Basznia"

An example of a multivariant model optimization is a model of a hot technological water supplying system for a sulfur mine, with a nonlinear objective function, presented in [7]. The model has been elaborated and successfully used for testing and verification of the described method of a linearized multivariant model reduction. It consists of more than 1360 constraints with more than 1120 variables. The accepted superstructure of the system has 213 nodes and 443 links.

## 4. FINAL REMARKS

The implementation of multivariant modeling of energy system would make „pre-designing" analyses quicker, cheaper and easier. Up to now, in frames of research work, done in the Institute of Heat Engineering – Warsaw University of Technology, several numerical programs were prepared for automatical generation of some parts of a linearized multivariant model and a logical model of an energy system, for solving a logical model with assumptions such as (10), and for the reduction of a multivariant optimization model with a linear or linearized constraints, and linear or nonlinear objective function. These programs were supplemented with appropriate optimization procedures. However, as optimization with linearized models is not efficient enough, if the multivariant modeling with logical relations is to be implemented widely, it seems that a method and numerical programs for reducing a multivariant model with nonlinear constraints (of defined forms) should be elaborated.

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# MODELOWANIE WIELOWARIANTOWE JAKO NARZĘDZIE W PROJEKTOWANIU SYSTEMÓW ENERGO-TECHNOLOGICZNYCH

## Streszczenie

W artykule przedstawiono koncepcję opisu wielu wariantów systemu za pomocą jednego modelu matematycznego. Ten wielowariantowy model jest tworzony na bazie tzw. „superstruktury", która łączy w sobie struktury wszystkich rozważanych odmian. Poszczególne warianty systemu mogą być jednak oceniane oddzielnie, względem przyjętego kryterium, ponieważ model wielowariantowy może być automatycznie redukowany do modelu dowolnego wybranego wariantu. Do przeprowadzenia tej transformacji wykorzystywany jest zbiór relacji logicznych opisujących superstrukturę i sam model wielowariantowy. Prezentowana metoda modelowania została omówiona na przykładzie modelu stanu ustalonego systemu energetycznego.

# МНОГОВАРИАНТНОЕ МОДЕЛИРОВАНИЕ КАК ИНСТРУМЕНТ ПРОЕКТИРОВАНИЯ ЭНЕРГО-ТЕХНОЛОГИЧЕСКИХ СИСТЕМ

## Краткое содержание

В статье представлена концепция описания множества вариантов структур системы одной математической моделью. Эта многовариантная модель конструируется на базе т.н. „суперструктуры", которая соединяет в себе структуры всех рассматриваемых разновидностей. Отдельные варианты могут оцениваться отдельно, относительно принятого критерия, так как мгновенная модель может автоматически ограничиваться моделью произвольно выбранного варианта. Для осуществления этой трансформации используется множество логических связей, которые описывают суперструктуры и саму многовариантную модель. Представленный метод описан на примере модели энергетической системы в стабильном состоянии.