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A novel and efficient power system state estimation algorithm based on Weighted Least Square (WLS) approach service

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Abstract

This paper presents a very fast power system state estimating algorithm to solve the power system state estimation problem. Conventional techniques of state estimation, which are based on the Weighted Least Square (WLS) method, face many issues, including lack of observability, high sensitivity to model parameters and long calculation time in large power systems. The main objective of conventional WLS methods is to minimize a linear objective function, while the aim of the presented method is to improve the results of conventional algorithms and obtain the least minimum possible value of the linear objective function alongside solving the problems mentioned above, by means of an iterative method. The proposed approach is tested on IEEE 14, 30 and 57 bus test systems using MATLAB software. The results reflect the considerable performance of the proposed method.

Keywords: power system state estimating; Iterative Weighted Least Square (WLS) method

1. Introduction

State estimation (SE) in power systems has become an essential tool for providing the real-time data required for reliable and secure operation of transmission networks. The supervisory control and data acquisition (SCADA) system gathers power system information such as active and reactive loads at substations, power flow of the transmission lines, voltage magnitudes, active power generated by the conventional units, circuit breaker status, etc. from remote terminal units (RTUs). SE uses this information to estimate the system states, including phase angles and bus voltage magnitudes [1].

With an overview, power system SE methods have been developed to overcome issues with monitoring transmission systems. These issues arise from the nature of measurement transducers and from communications problems in sending measured values to the control center. Measurement transducers may contain small measurement errors caused by noise or inaccuracy of the equipment. In addition, transducers may suffer from larger errors which may emerge from biases, wrong connection of devices, telecommunication system failures or interference from certain devices. If the errors are small, they may go undetected and could cause poor interpretation by those reading the measured value. Gross errors can leave the system operator deprived of information about part of the system. A state estimator can fix small measurement errors, detect and identify gross measurement errors and information lost due to communications failures and replace them with appropriate values [2]. SE literature concentrates on measurement device placement, observability analysis, poor data detection and identification, use of new technologies and algorithm studies [3–6].

The major foundations of the field of algorithm studies include: Kalman filter, winner filter, set membership filter, particle filter, etc., but the most widely used algorithm for solving a SE problem is the weighted least squares (WLS) method and most of the other solution methods for SE are based on this approach. Reference [7] presents a new method based on WLS state estimation to detect false data injection attacks. The idea of this method is to track variations in measurement and calculate the distance indices between adjacent steps, by using historical measurements. Authors in [8] implement a least absolute value state estimator employing the new format of equations which employ first- and second-order derivative functions and it could benefit from scaling techniques. In reference [9] SE problem is formulated for a stochastic hybrid system (SHS). SHS estimates both discrete and continuous states with continuous time observation process information, called hybrid SE problem. The main goal of reference [10] is to introduce a two-step method for estimating the state of large scale distribution networks. In this method, the network is divided into sub-areas based

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on topological or geographical constraints and the presence of the measurement system. Then, by using a local estimator for each area, all the measurements on the field are utilized. In the next step, data provided by the local estimator are used to obtain the network operation conditions. A novel algorithm is presented in [11] which combines the best of both the 'unscented' Kalman filter (UKF) and the weighted least squares methods. Authors in [12] have proposed a power system dynamic SE problem based on the extended Kalman filter (EKF) combined with load forecasting to enhance estimation accuracy. In this way missing load data could be predicted from the available measurement data by using a forecasting method. Then, through power flow analysis, these data are converted to system states. In the next step EKF combines the forecasted states and measurement data to access an accurate estimation. In reference [13] the effect of correlation between measurements and pseudo measurements in distribution networks for quality of the WLS estimation method is discussed using traditional and synchronized measurements. Zhao et al. [14] have introduced a new method using phasor measurement units (PMU) to estimate the power system states under different operation conditions. In this procedure, to enhance the robustness of the suggested algorithm, measurement weights are adjusted based on the distance of the biggest disturbance from the PMU. Shahidehpour and Marwalian [15] have proposed an enhancement to the least median of squares (LMS) estimation in the power system state estimation problem by using the applications of fuzzy sets. Reference [16] presents an approach based on Kalman filter to estimate the static states of the voltage phasors and dynamic states of the rotor angles and speeds of the generators. Authors in [17] propose a power system SE model in the presence of PMU. In this work, parallel Kalman filter and alternating minimization method are employed for SE by using the static and dynamic model. Reference [18] presents an algorithm for power system SE in which at each generation unit PUMs are utilized for measuring local signals. Then an unscented Kalman filter (UKF) is used for SE. The advantage of this algorithm is the independence of SE on one generation unit to SE in another generation unit. Reference [19] proposes an effective SE procedure which does not need Jacobian matrix calculation and linearization. In this method, first means and covariance of a random vector a nonlinear transformation calculated by using unscented transformation and then UKF is used in power system dynamic SE. In some references, iterative methods are used for solving the problem of SE. Authors in [20] have presented a new formulation for the state estimation of the power systems by means of a linear programming approach. Reference [21] presents the iteratively reweighted least square procedure. In this method, the least square method and iterative dynamic rescaling are combined, which results in enhancing the robustness of the SE.

This paper aims to propose a new weighted least square algorithm for solving the SE problem. Using the iterative procedure in the proposed method, a significant improvement is achieved in estimated values.

This paper is organized as follows. Sections II and III describe the state estimation formulation and the proposed algorithm for solving the SE problem, respectively. Also simulation results are illustrated in section IV. Section V gives the conclusion of this paper.

2. WLS Formulation

In this section, formulation of a conventional SE problem is proposed. Equation 1 shows the relation of the state variables and the measurement errors in the measurement of the system:

$$z = h(x) + e \tag{1}$$

In the above equation z stands for measurement vector $(m \times 1)$ and x defines the vector $(n \times 1)$ of system state variables which includes the voltage magnitude and phase angles of all the buses except the reference bus angle. The nonlinear function h(x) relates the measurements to the system states and $e$stands for the vector of the measurement errors with zero means and covariance matrix R. Also n and m are the number of state variables and measurements, respectively. Accordingly, by minimizing the following objective function the WLS estimation for x can be found.

$$minJ(x) = \sum_{i=1}^{N_m} \omega_i^2 [z_i - f_i(x)]^2 \tag{2}$$

Where:
f_i The function used to calculate the true value measured by the ith measurement.
ω_i Measurement weight of ith measurement.
N_m Number of the measurement.
z_i ith measured value.
$J(x)$ Objective function.

Equation (2) can be written as a matrix of the coefficients, f_i , which results in the equations below:

$$J(x) = \sum_{i=1}^{N_m} \omega_i^2 [z_i - h_i^T x]^2 \tag{3}$$

Equation (3) can be rewritten in a very compact form, as below:

$$J(x) = [z - [H]x]^T [R^{-1}][z - [H]x] \tag{4}$$

In which R is the covariance matrix of the measurement errors. To minimize $J(x)$, the equivalent gradient of $J(x)$ must be zero. The gradient of $J(x)$ is:

$$\nabla J(x) = -2[H]^T [R^{-1}]z + 2[H]^T [R^{-1}][H]x \tag{5}$$

Where H is $N_m \times N_S$ matrix including the coefficient of the functionf_i. Then considering$\nabla J(x) = 0$, this leads to the following equation:

$$x^{est} = [[H]^T [R^{-1}][H]]^{-1} [H]^T [R^{-1}]z \tag{6}$$

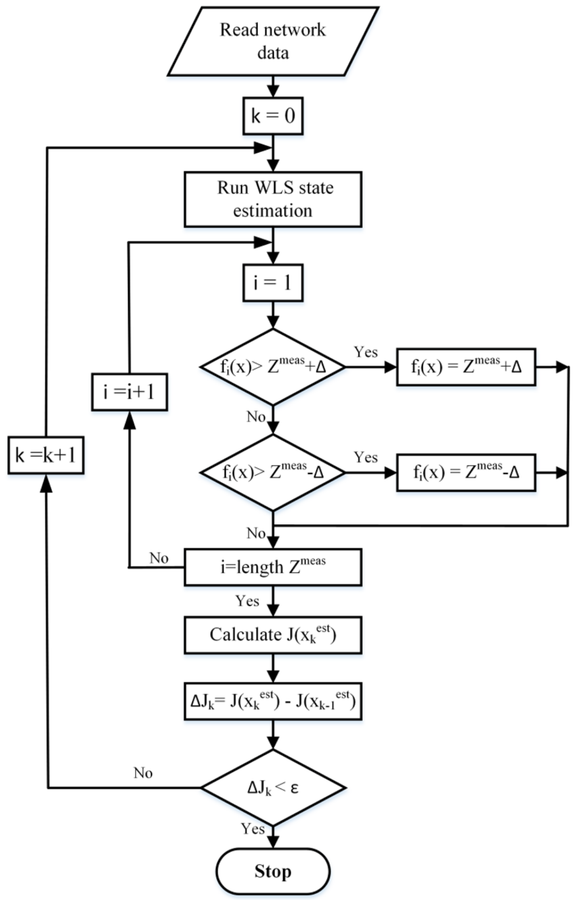

Figure 1: Flowchart of the proposed state estimation method

Consequently, f(x) can be obtained as follows:

$$f(x) = [H]x^{est} \tag{7}$$

In the next section the proposed algorithm for solving the SE problem is presented.

3. Proposed WLS method

To determine the accuracy of a measuring device, an index known as limiting error is used. The limiting error is used to define the maximum limit of the error in measuring instruments and it is determined by the manufacturer. For instance, if the accuracy of an ammeter with the range of 100A is specified as 2% of the scale of the device, this means that for any reading the error is limited within ±2 A. In this paper, the limiting error is specified by Δ. The flowchart of the proposed algorithm is shown in Fig. (2). The steps of the proposed iterative SE method are as follows:

- Step 1: read network data and calculate the coefficient matrix [H] and measurement covariance matrix. Then run the WLS state estimation and use equation (6), (7) and (2) to obtain state of the system, f(x) and J(x). In this step k which is the counter for iteration number is zero.

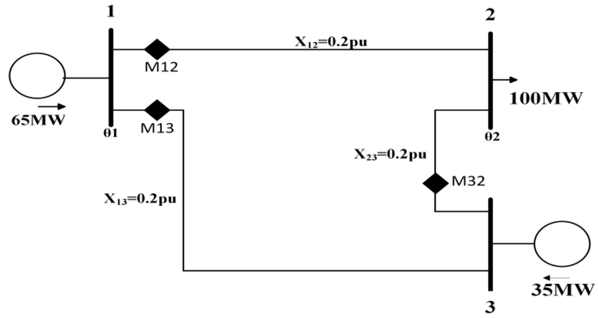

Figure 2: Three bus test system

- Step 2: Calculate the values of ($Z^{meas}+\Delta$) and ($Z^{meas}-\Delta$) for all the elements of Z^{meas} matrix. In this statement Δ is the limiting error.
- Step 3: Compute the matrix, $f(x^{est})$, and compare all elements of this matrix with the corresponding elements of ($Z^{meas}+\Delta$) and ($Z^{meas}-\Delta$) matrices. If $f(x_i^{est})$ is greater than ($Z_i^{meas}+\Delta$), set the $f(x_i^{est})$= ($Z_i^{meas}+\Delta$) and if $f\left(x_i^{est}\right)$ is less than ($Z_i^{meas}-\Delta$), set the $f(x_i^{est})$ = ($Z_i^{meas}-\Delta$). In this step i show the length of the measurements matrix.
- Step 4: Put the updated $f(x^{est})$, in Z^{meas} and run new WLS state estimation. This step gives the new values of x^{est}, $f(x^{est})$ and J(x).
- Step 5: Compute the maximum absolute difference between the J(x) obtained from step 4 and step 1. If both values are less than a specifed threshold, print the solution and stop. Otherwise, put k=k+1 and go to Step 2.

4. Implementation of the proposed algorithm on a test system

To prove the effectiveness of the proposed method a 3-bus test system is used, which is illustrated in Fig. (2). In this system, bus 3 is considered as a reference bus and the phase angle for this bus is assumed to be zero. The power flow results and Z^{meas} for this system are as follows:

$$f^{pf}(x) = \begin{bmatrix} 0.60 \\ 0.06 \\ 0.40 \end{bmatrix}, z^{meas} = \begin{bmatrix} 0.62 \\ 0.06 \\ 0.37 \end{bmatrix}, \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0.02 \\ -0.1 \\ 0 \end{bmatrix} \tag{8}$$

In the first stage, the state estimation results are as follows:

$$\begin{gathered} f(x) = \begin{bmatrix} 0.6143 \\ 0.0714 \\ 0.3771 \end{bmatrix} \\ x^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.0286 \\ -0.0943 \end{bmatrix} \\ J(x_1^{est}) = 2.1429 \end{gathered} \tag{9}$$

To calculate $(Z^{meas} + \varepsilon)$ and $(Z^{meas} - \varepsilon)$, σequals 0.01. As a result:

$$z^{meas} + \Delta = \begin{bmatrix} 0.6262 \\ 0.0606 \\ 0.3737 \end{bmatrix}, z^{meas} - \Delta = \begin{bmatrix} 0.6138 \\ 0.0594 \\ 0.3663 \end{bmatrix} \quad (10)$$

It can be seen that the second and third elements of f(x) are greater than the corresponding elements $(Z^{meas} + \Delta)$ and should be changed. On the other hand, the first element of the vector should not be changed. Accordingly, f(x) equals:

$$f(x) = \begin{bmatrix} 0.6143 \\ 0.0606 \\ 0.3737 \end{bmatrix} \quad (11)$$

Now, by substituting the updated f(x) instead in equation (6) can be obtained:

$$x^{estnew} = \begin{bmatrix} 0.02741 \\ -0.0947 \end{bmatrix} \quad (12)$$

Consequently, the new matrix f(x) can be derived as equations (7), (11):

$$f(x)^{new} = \begin{bmatrix} 0.6152 \\ 0.0703 \\ 0.3797 \end{bmatrix} \quad (13)$$

Also, the value of J(x) can be obtained using a new estimation, as follows:

$$J(x) = sum[(z^{meas} - f(x)^{new})^2] = 1.5288 \quad (14)$$

It is seen that J(x) is decreased in the first iteration, but the difference between this value and the value of the previous stage does not converge and this means iteration must proceed. So, by repeating this process, the following results can be obtained:

$$x^{estFinal} = \begin{bmatrix} 0.0272 \\ -0.0962 \end{bmatrix} \quad (15)$$

$$f(x)^{final} = \begin{bmatrix} 0.6172 \\ 0.0681 \\ 0.3848 \end{bmatrix}, J(x) = 0.52 \quad (16)$$

5. Simulation Results

The proposed state estimation procedure is implemented and tested on IEEE 14, 30 and 57 bus test systems. Bus 1 is assumed to be slack bus. SE algorithm convergence tolerance equals 0.01. Also Gaussian error in the ith measurement can be derived as the following equation [22]:

$$Z_i^{meas} = Z_i^{true} + rand * \sigma_i \quad (17)$$

Where Z^{true} is the true value of the measurements which could be obtained from line flow solution using matpower in MATLAB software, Z^{meas} is the measured value, rand is the normal distribution with zero means and variance one and σ_i is the standard deviation of measurement error. Different cases are simulated on four different measurement accuracies. The state estimation algorithm has been developed using MATLAB software. The performance of the proposed method is shown in this section.

Table 1: Actual and measured data for the 14-bus test system

Type	Z_{Actual}	Z_{meas}
P_{1-2}	0.1569	0.1588
P_{1-5}	0.0755	0.0672
P_{2-3}	0.0732	0.0730
P_{2-4}	0.0561	0.0537
P_{2-5}	0.0415	0.0357
P_{3-4}	-0.0233	-0.0297
P_{4-5}	-0.0612	-0.0692
P_{4-7}	0.0281	0.0522
P_{4-9}	0.0161	0.0123
P_{5-6}	0.0441	0.0468
P_{6-11}	0.0073	0.0218
P_{6-12}	0.0078	0.0330
P_{6-13}	0.0178	0.0196
P_{7-8}	0	-0.0079
P_{7-9}	0.0281	0.0340
P_{9-10}	0.0052	-0.0091
P_{9-14}	0.0094	0.0063
P_{10-11}	-0.0038	-0.0037
P_{12-13}	0.0016	0.0155
P_{13-14}	0.0056	0.0188

5.1. IEEE 14-bus test system

Power system information is presented in table (1). This system is formed of 20 lines and 20 power flow measurement devices. The results of state estimation in the first and last iteration are shown in table (2). Also table (3) 3 illustrates the f(x) value in the first iteration and several iterations. The reduction of the value of the objective function for measurement accuracy 0.01, is shown in fig. (3)(a). After four iterations, the convergence condition is established and the final value of J(x) is obtained. Also, these values for measurement accuracies of 0.009, 0.008 and 0.005 are shown in fig. (3) (b), (c) and (d), respectively.

5.2. IEEE 30-bus test system

The system information is obtained from matpower in MATLAB software. This system is formed of 41 lines and for this system 41 power flow measurement devices are considered. The results of state estimation in the first and last iteration are shown in table (4). Also table (5) illustrates the f(x) value in the first iteration and several iterations. The reduction of the value of the objective function is shown in fig. (4) (a), (b), (c) and (d) for measurement accuracies of 0.01, 0.009, 0.008 and 0.005 respectively.

Table 2: Estimated states of IEEE 14- bus test system in first and final iterations

Type	X_{est_First}	X_{est_Final}
$\delta(1)$	0	0
$\delta(2)$	-0.0093	-0.0094
$\delta(3)$	-0.0222	-0.0237
$\delta(4)$	-0.0195	-0.0185
$\delta(5)$	-0.0169	-0.0156
$\delta(6)$	-0.0286	-0.0266
$\delta(7)$	-0.0306	-0.0290
$\delta(8)$	-0.0292	-0.0276
$\delta(9)$	-0.0344	-0.0326
$\delta(10)$	-0.0337	-0.0318
$\delta(11)$	-0.0334	-0.0310
$\delta(12)$	-0.0323	-0.0301
$\delta(13)$	-0.0324	-0.0302
$\delta(14)$	-0.0374	-0.0352

5.3. IEEE 57-bus test system

The system information is obtained from matpower in MATLAB software. This system is formed of 80 lines and for this system, 80 power flow measurement devices are considered. The results of state estimation in the first and last iteration are shown in table (6) (Part 1 and 2). The reduction of the value of the objective function is shown in fig. (5) (a), (b), (c) and (d) for measurement accuracies of 0.01, 0.009, 0.008 and 0.005 respectively.

The quality of the state estimation is correlated to the accuracy of the measurement device. If the accuracy of the measurement device is high, then the measured data would be so close that the actual data and the objective function would have the least possible value. As is clear from the simulation results, accurate measurement devices deliver better state estimation. At lower accuracies, the objective function is greater than that at higher accuracies. Although, at lower accuracies, the difference between the objective function in the first and the last iteration is remarkable, which demonstrates the effectiveness of the proposed algorithm.

6. Conclusion

A new weighted least square algorithm for the state estimation problem is proposed in this paper. In this method, an iterative procedure is used to estimate the state of the system. To demonstrate the performance and efficiency of the presented method, test results for IEEE 14, 30 and 57 bus test systems are given. Simulation results show a significant improvement in the weighted sum of the measurement residual values.

Table 3: Estimated value of the measurements for IEEE 14- bus test system

Type	$F_{First}(x)$	$F_{Final}(x)$
P_{1-2}	0.1588	0.1581
P_{1-5}	0.0672	0.0699
P_{2-3}	0.0730	0.0724
P_{2-4}	0.0537	0.0521
P_{2-5}	0.0357	0.0358
P_{3-4}	-0.0297	-0.0302
P_{4-5}	-0.0692	-0.0702
P_{4-7}	0.0522	0.0498
P_{4-9}	0.0123	0.0252
P_{5-6}	0.0468	0.0438
P_{6-11}	0.0218	0.0221
P_{6-12}	0.0330	0.0138
P_{6-13}	0.0196	0.0277
P_{7-8}	-0.0079	-0.0079
P_{7-9}	0.0340	0.0327
P_{9-10}	-0.0091	-0.0092
P_{9-14}	0.0063	0.0098
P_{10-11}	-0.0037	-0.0039
P_{12-13}	0.0155	0.0005
P_{13-14}	0.0188	0.0143

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Table 4: Estimated states of IEEE 30-bus test system in first and final iterations

Type	X_{est_First}	X_{est_Final}
$\delta(1)$	0	0
$\delta(2)$	-0.0008	-0.0007
$\delta(3)$	-0.0037	-0.0035
$\delta(4)$	-0.0041	-0.0039
$\delta(5)$	-0.0029	-0.0027
$\delta(6)$	-0.0051	-0.0047
$\delta(7)$	-0.0054	-0.0050
$\delta(8)$	-0.0059	-0.00563
$\delta(9)$	-0.0083	-0.0077
$\delta(10)$	-0.0087	-0.0081
$\delta(11)$	-0.0130	-0.0124
$\delta(12)$	-0.0064	-0.0062
$\delta(13)$	0.0010	0.0012
$\delta(14)$	-0.0089	-0.0068
$\delta(15)$	-0.0069	-0.0071
$\delta(16)$	-0.0085	-0.0080
$\delta(17)$	-0.0100	-0.0093
$\delta(18)$	-0.0077	-0.0075
$\delta(19)$	-0.0090	-0.0087
$\delta(20)$	-0.0094	-0.0090
$\delta(21)$	-0.0090	-0.0086
$\delta(22)$	-0.0086	-0.0082
$\delta(23)$	-0.0027	-0.0021
$\delta(24)$	-0.0062	-0.0062
$\delta(25)$	-0.0047	-0.0040
$\delta(26)$	-0.0058	-0.0051
$\delta(27)$	-0.0049	-0.0043
$\delta(28)$	-0.0047	-0.0043
$\delta(29)$	-0.0095	-0.0085
$\delta(30)$	-0.0158	-0.0144

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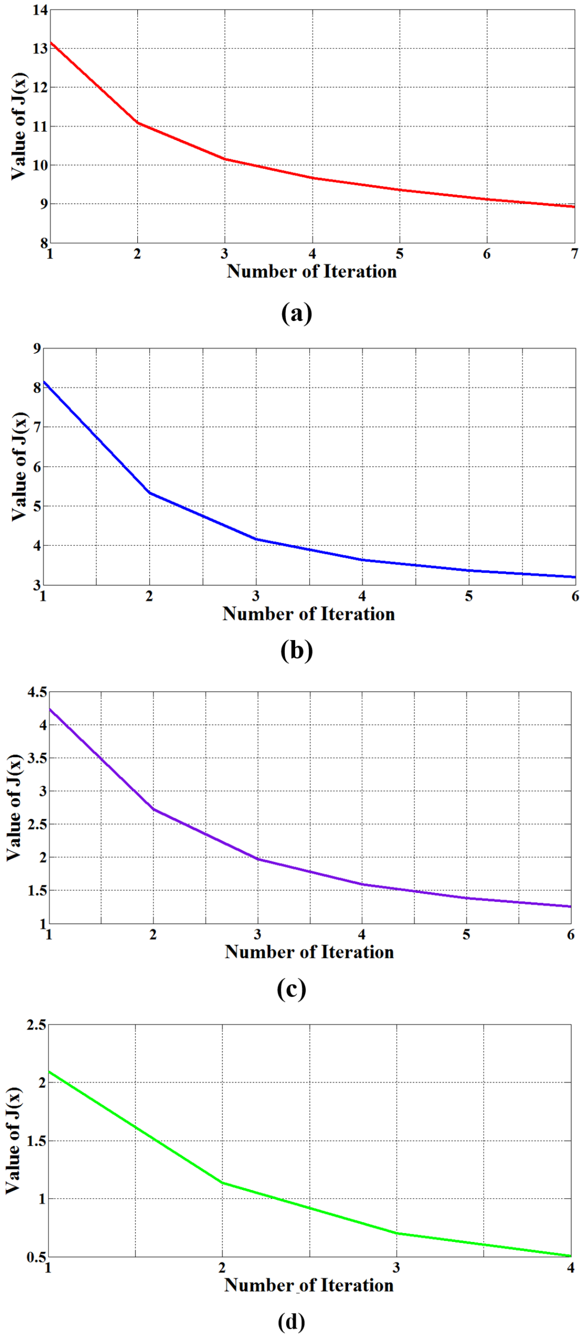

Figure 3: Objective function decrement for IEEE 14-bus test system in different measurement accuracies. (a) 0.01, (b) 0.009, (c) 0.008, (d) 0.005

Table 5: Estimated value of the measurements for IEEE 30-bus test system

Type	F_{First}(x)	F_{Final}(x)
P_{1-2}	0.0093	0.0126
P_{1-3}	0.0290	0.0185
P_{2-4}	0.0159	0.0185
P_{3-4}	0.0121	0.0099
P_{2-5}	0.0060	0.0100
P_{2-6}	0.0186	0.0222
P_{4-6}	0.0231	0.0213
P_{5-7}	0.0167	0.0191
P_{6-7}	0.0051	0.0035
P_{6-8}	0.0223	0.0223
P_{6-9}	0.0242	0.0143
P_{6-10}	-0.0180	0.0060
P_{9-11}	0.0223	0.0223
P_{9-10}	0.0086	0.0034
P_{4-12}	0.0077	0.0089
P_{12-13}	-0.0536	-0.0536
P_{12-14}	-0.0005	0.0026
P_{12-15}	0.0062	0.0074
P_{12-16}	0.0126	0.0093
P_{14-15}	-0.0010	0.0014
P_{16-17}	0.0097	0.0065
P_{15-18}	-0.0028	0.0018
P_{18-19}	0.0061	0.0088
P_{4-9}	0.0027	0.0042
P_{19-20}	0.0086	0.0041
P_{10-20}	0.0132	0.0146
P_{10-17}	0.0073	0.0068
P_{10-21}	-0.0005	0.0006
P_{10-22}	-0.0191	-0.0192
P_{21-22}	-0.0249	-0.0249
P_{15-23}	-0.0110	-0.0109
P_{22-24}	0.0152	0.0151
P_{23-24}	-0.0068	-0.0068
P_{24-25}	0.0030	0.0030
P_{25-26}	0.0016	0.0016
P_{25-27}	-0.0000	-0.0001
P_{28-27}	0.0090	0.0100
P_{27-29}	0.0183	0.0169
P_{29-30}	0.0121	0.0132
P_{8-28}	-0.0063	-0.0063
P_{6-28}	-0.0061	-0.0062

Table 6: Estimated states of IEEE 14- bus test system in first and final iterations (Part 1)

Type	Xest first	Xest final	Type	F(x) first	F(x) final
$\delta(1)$	0	0	P_{1-2}	0.0831	0.0812
$\delta(2)$	-0.0030	-0.0027	P_{2-3}	0.0953	0.0898
$\delta(3)$	-0.0120	-0.0107	P_{3-4}	0.0383	0.0389
$\delta(4)$	-0.0144	-0.0131	P_{4-5}	0.0118	0.0101
$\delta(5)$	-0.0161	-0.0152	P_{4-6}	-0.0046	-0.0003
$\delta(6)$	-0.0164	-0.0156	P_{6-7}	-0.0015	0.0000
$\delta(7)$	-0.0137	-0.0136	P_{6-8}	-0.0207	-0.0204
$\delta(8)$	-0.0096	-0.0098	P_{8-9}	0.1762	0.1769
$\delta(9)$	-0.0185	-0.0188	P_{9-10}	0.0148	0.0146
$\delta(10)$	-0.0212	-0.0205	P_{9-11}	-0.0005	0.0002
$\delta(11)$	-0.0194	-0.0190	P_{9-12}	-0.0005	0.0017
$\delta(12)$	-0.0183	-0.0183	P_{9-13}	-0.0001	0.0004
$\delta(13)$	-0.0180	-0.0169	P_{13-14}	-0.0406	-0.0414
$\delta(14)$	-0.0173	-0.0161	P_{13-15}	-0.0590	-0.0592
$\delta(15)$	-0.0138	-0.0120	P_{1-15}	0.1212	0.1278
$\delta(16)$	-0.0155	-0.0179	P_{1-16}	0.0747	0.0738
$\delta(17)$	-0.0107	-0.0109	P_{1-17}	0.0991	0.0987
$\delta(18)$	-0.0234	-0.0214	P_{3-15}	0.0367	0.0324
$\delta(19)$	-0.0270	-0.0251	P_{4-18}	0.0061	0.0117
$\delta(20)$	-0.0281	-0.0262	P_{4-18}	0.0195	0.0151
$\delta(21)$	-0.0266	-0.0254	P_{5-6}	-0.0207	-0.0215
$\delta(22)$	-0.0263	-0.0246	P_{7-8}	-0.0503	-0.0495
$\delta(23)$	-0.0264	-0.0248	P_{10-12}	-0.0156	-0.0155
$\delta(24)$	-0.0233	-0.0211	P_{11-13}	-0.0000	0.0006
$\delta(25)$	-0.0343	-0.0348	P_{12-13}	-0.0075	-0.0075
$\delta(26)$	-0.0223	-0.0201	P_{12-16}	-0.0249	-0.0246
$\delta(27)$	-0.0193	-0.0190	P_{12-17}	-0.0371	-0.0365
$\delta(28)$	-0.0185	-0.0184	P_{14-15}	-0.0615	-0.0611
$\delta(29)$	-0.0163	-0.0164	P_{18-19}	-0.0001	-0.0001
$\delta(30)$	-0.0371	-0.0375	P_{19-20}	-0.0078	-0.0078
$\delta(31)$	-0.0339	-0.0253	P_{21-20}	-0.0093	-0.0093
$\delta(32)$	-0.0374	-0.0313	P_{21-22}	0.0052	0.0052
$\delta(33)$	-0.0371	-0.0310	P_{22-23}	0.0179	0.0179
$\delta(34)$	-0.0293	-0.0283	P_{23-24}	-0.0052	-0.0052
$\delta(35)$	-0.0288	-0.0277	P_{24-25}	0.0021	0.0047
$\delta(36)$	-0.0284	-0.0271	P_{24-25}	0.0071	0.0045
$\delta(37)$	-0.0279	-0.0266	P_{24-26}	-0.0136	-0.0136
$\delta(38)$	-0.0261	-0.0244	P_{26-27}	-0.0079	-0.0078
$\delta(39)$	-0.0280	-0.0267	P_{27-28}	-0.0161	-0.0161
$\delta(40)$	-0.0283	-0.0270	P_{28-29}	-0.0343	-0.0343
$\delta(41)$	-0.0237	-0.0227	P_{7-29}	0.0571	0.0573
$\delta(42)$	-0.0259	-0.0239	P_{25-30}	-0.0123	-0.0123
$\delta(43)$	-0.0212	-0.0204	P_{30-31}	0.0140	0.0140
$\delta(44)$	-0.0234	-0.0217	P_{31-32}	-0.0026	-0.0026
$\delta(45)$	-0.0194	-0.0177	P_{32-33}	-0.0082	-0.0082
$\delta(46)$	-0.0204	-0.0190	P_{34-32}	0.0043	0.0042
$\delta(47)$	-0.0247	-0.0229	P_{34-35}	-0.0211	-0.0211
$\delta(48)$	-0.0255	-0.0236	P_{35-36}	-0.0133	-0.0133
$\delta(49)$	-0.0230	-0.0225	P_{36-37}	-0.0255	-0.0257
$\delta(50)$	-0.0230	-0.0183	P_{37-38}	0.0006	0.0006

Table 8: Estimated states of IEEE 14- bus test system in first and final iterations (Part 2)

Type	Xest first	Xest final	Type	F(x) first	F(x) final
$\delta(51)$	-0.0237	-0.0224	P_{37-39}	0.0132	0.0130
$\delta(52)$	-0.0207	-0.0200	P_{36-40}	-0.0176	-0.0174
$\delta(53)$	-0.0233	-0.0224	P_{22-38}	-0.0062	-0.0062
$\delta(54)$	-0.0247	-0.0232	P_{11-41}	-0.0042	0.0020
$\delta(55)$	-0.0204	-0.0206	P_{41-42}	0.0055	0.0054
$\delta(56)$	-0.0226	-0.0200	P_{41-43}	-0.0015	0.0021
$\delta(57)$	-0.0260	-0.0235	P_{38-44}	-0.0226	-0.0224
			P_{15-45}	0.0316	0.0313
			P_{14-46}	0.0553	0.0534
			P_{46-47}	0.0374	0.0357
			P_{47-48}	0.0182	0.0175
			P_{48-49}	0.0058	0.0020
			P_{49-50}	0.0097	0.0099
			P_{50-51}	-0.0086	-0.0082
			P_{10-51}	0.0322	0.0321
			P_{13-49}	0.0214	0.0273
			P_{29-52}	-0.0001	0.0005
			P_{52-53}	0.0302	0.0305
			P_{53-54}	-0.0002	0.0006
			P_{54-55}	-0.0001	0.0006
			P_{11-43}	0.0167	0.0153
			P_{44-45}	-0.0475	-0.0471
			P_{40-56}	-0.0049	0.0018
			P_{56-41}	-0.0112	-0.0111
			P_{56-42}	-0.0119	-0.0118
			P_{39-57}	0.0162	0.0093
			P_{57-56}	-0.0403	-0.0416
			P_{38-49}	-0.0004	-0.0004
			P_{38-48}	-0.0066	-0.0068
			P_{9-55}	0.0144	0.0140

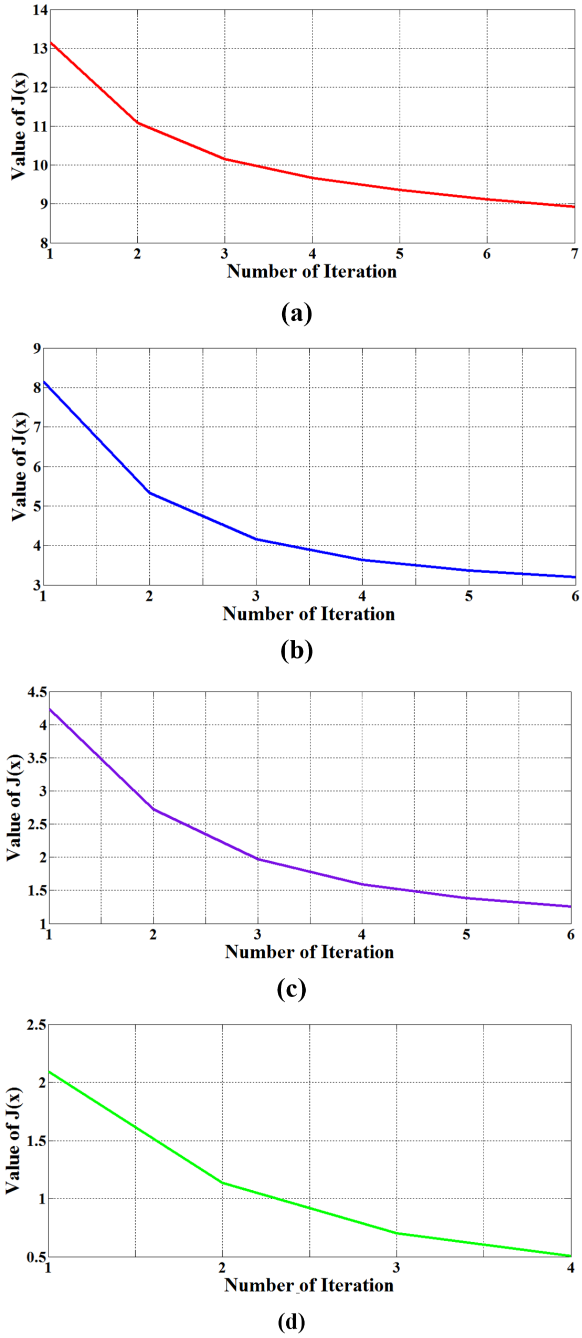

Figure 4: Objective function decrement for IEEE 30-bus test system in different measurement accuracies. (a) 0.01, (b) 0.009, (c) 0.008, (d) 0.005

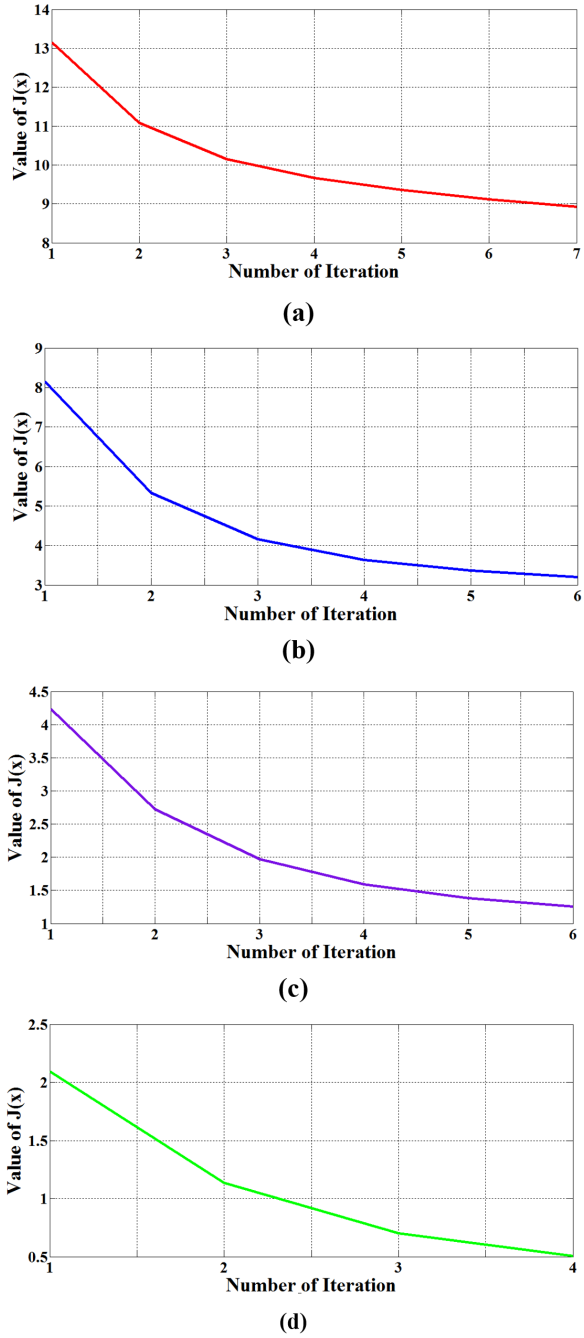

Figure 5: Objective function decrement for IEEE 57-bus test system in different measurement accuracies. (a) 0.01, (b) 0.009, (c) 0.008, (d) 0.005