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A mathematical model of a steam condenser in off-design operation☆

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Abstract

The paper presents a mathematical model of a steam condenser in changed conditions. A list of independent parameters on which the water temperature at the outlet from the steam condenser depends was selected and by means of the Buckingham Π theorem a functional relation between two dimensionless quantities was obtained. The exact form of the function was determined on the basis of data from the characteristics of the steam condenser for a 200 MW power plant and actual measurement data for a different condenser operating in a 200 MW power plant. A linear relation between two dimensionless quantities was obtained. The correctness of the proposed relation was examined by comparing the measured temperature and calculated temperature from the proposed relation.

Keywords: Heat exchange, condenser, off-design

1. Introduction

A steam condenser is a heat exchanger where the condensation of steam occurs. Wet steam (close to saturation) is directed from the turbine to the steam condenser, where it flows through the outer side of the tubes and gives off heat (condenses) to the cooling water flowing inside the tubes. The cooling water is usually drawn from a natural water source (river, lake, sea). The location of the steam condenser in a 200 MW power plant, with the accepted symbols for heat transfer fluids, is presented in Fig. 1.

The condenser as a "lower source of heat" plays a special role in a power plant, because the parameters of its work have a significant impact on the efficiency of the installation. Therefore, it is important to know the condenser operating parameters during both design and operation. For this purpose, mathematical models describing the work of the condenser in changed conditions are created.

The most popular mathematical model for a steam condenser in changed conditions is based on the energy balance equation and Peclet's law [1, 2], which is completed with the overall heat transfer coefficient as constant or as a function of the heat transfer coefficients for both heat transfer fluids

$$\dot{Q} = \dot{m}_h r = \dot{C}_c \left(T_{c2} - T_{c1}\right) \tag{1}$$

$$\dot{Q} = kA \triangle T_{ln} \tag{2}$$

Heat transfer coefficients depend on dimensionless quantities such as the Reynolds Number and the Prandtl Number. Approximation relations for heat transfer coefficients are used within relevant ranges

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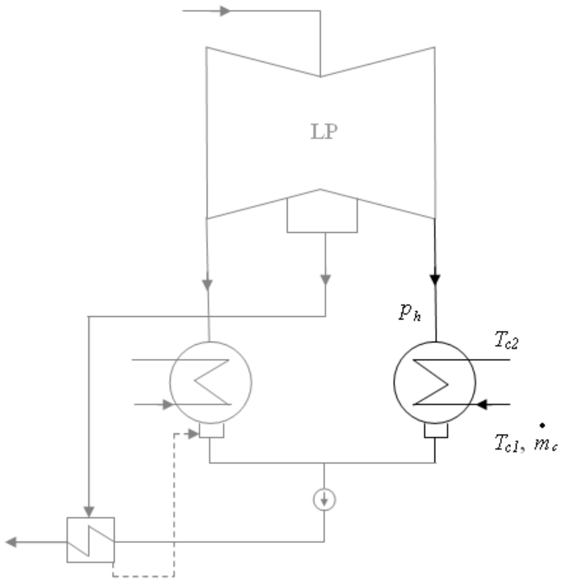

Figure 1: Location of the analyzed steam condenser in a 200 MW power plant with accepted symbols

of parameter changes [3] and are not always accurate [4].

By rearranging (1, 2) equations the work of the steam condenser in changed conditions can be described by using the heat transfer effectiveness [5, 6]

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = 1 - exp\left(-\frac{kA}{\dot{C}_c}\right) \tag{3}$$

Beckman, when analyzing the parameters on which the heat transfer coefficient depends in relation (3), proposed an approximate formula for heat transfer effectiveness for the steam condenser with reference conditions in the following form [7, 8]

$$\frac{\varepsilon}{\varepsilon_o} = \left(\frac{T_{h1}}{T_{h1o}}\right)^{\alpha_1} \left(\frac{T_{c1}}{T_{c1o}}\right)^{\alpha_2} \left(\frac{\dot{m}_h}{\dot{m}_{ho}}\right)^{\alpha_3} \left(\frac{\dot{m}_c}{\dot{m}_{co}}\right)^{\alpha_4} \tag{4}$$

The $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ coefficients are calculated based on the measured data. The approximated relation proposed by Beckman works well for the steam condenser in changed conditions when the parameters do not vary over a wide range. The aim of this paper is to create a simpler approximate relation, describing the work of the steam condenser in changed conditions based on measured parameters. For this purpose the Buckingham Π theorem was used.

2. A mathematical model

The paper analyzes the steam condenser for a 200 MW power plant. In the analyzed steam condenser the following parameters were measured: water temperature at the inlet (T_{c1}) and at the outlet (T_{c2}) of the steam condenser, the cooling water mass flow ($\dot{m}_c$) and vapor pressure (p_h). The paper attempts to obtain the functional relation describing the work of the steam condenser in changed conditions only using the measured parameters.

The analysis of the (1, 2) relations shows that the water temperature at the outlet of the steam condenser (T_{c2}) depends on temperatures at the inlet of both fluids (T_{h1}, T_{c1}), the heat capacity of cooling water $\left(\dot{C}_c\right)$, the overall heat transfer coefficient (k) and the heat transfer surface area (A)

$$T_{c2} = f\left(T_{h1}, T_{c1}, \dot{C}_c, k, A\right) \tag{5}$$

The heat capacity of the cooling water is equal to the product of specific heat at constant pressure and the mass flow. Assuming that physical properties of the cooling water, in the analyzed range of changes of parameters, are constant, the heat capacity is only a function of the mass flow

$$\dot{C}_c = c_{pc}\dot{m}_c = f(\dot{m}_c) \tag{6}$$

We turn now to the parameters on which the overall heat transfer coefficient depends in our case. The overall heat transfer coefficient (k) is a function of heat transfer coefficients for both fluids (α_1, α_2) and wall thickness (δ) and wall thermal conductivity (λ). In order to simplify the analysis, the overall heat transfer coefficient for the flat wall was adopted [9]

$$\frac{1}{k} = \frac{1}{\alpha_h} + \frac{\delta}{\lambda} + \frac{1}{\alpha_c} \tag{7}$$

The heat transfer coefficient from the water side is a function of the cooling water mass flow and can be written as follows [3, 10, 11]

$$\alpha_c = \frac{Nu_c\lambda_c}{d_i} = \frac{\lambda_c}{d_i}0.021Re_c^{0.8}Pr_c^{0.43} = C_c\dot{m}_c^{0.8} = f(\dot{m}_c) \tag{8}$$

where constant C_c is equal

$$C_c = \frac{\lambda_c}{d_i} 0.021 \frac{d_i}{\eta_c} Pr^{0.43} \tag{9}$$

The heat transfer coefficient from the steam side can be written as [6, 11]

$$\alpha_h = \left[\frac{(\rho_{con} - \rho_v)\, g \lambda_{con}^3 r}{n v_{con} (T_s - T_{wcon})\, d_o} \right]^{\frac{1}{4}} \tag{10}$$

The temperature of saturation (T_s) which is in the formula (10) is a function of the measured steam pressure $T_s = f(p_h)$. Likewise latent heat (r) is the steam pressure function $r = f(p_h)$. Assuming that the other physical parameters of the steam change over a small range, the heat transfer coefficient from the steam side can be written as

$$\alpha_h = f(p_h, g) \tag{11}$$

Hence, the overall heat transfer coefficient can be written as a function of the following parameters

$$k = f(\dot{m}_c, p_h, g) \tag{12}$$

Finally, the water temperature at the outlet of the steam condenser can be written as

$$T_{c2} - T_{c1} = f(T_{h1} - T_{c1}, \dot{m}_c, p_h, A, g) \tag{13}$$

A difference of temperatures was assumed, since if this is the case it is of no importance in which units the temperature is expressed.

A dimensional analysis may be used for the selected independent parameters. According to the dimensional analysis it can be written

$$T_{c2} - T_{c1} = C (T_{h1} - T_{c1})^a (\dot{m}_c)^b p_h^c A^d g^e \tag{14}$$

The relation (14) is true when the units on the left are equal to the units on the right [10]

$$K^1 = C \cdot K^a \left(kg \cdot s^{-1}\right)^b \left(kg \cdot s^{-2} \cdot m^{-1}\right)^c \left(m^2\right)^d \left(m \cdot s^{-2}\right)^e \tag{15}$$

A comparison of the exponents at the appropriate units gives the following system of equations

[K] $1 = a$

[kg] $0 = b + c$

[s] $0 = -b - 2c - 2e$

[m] $0 = -c + 2d + e$

In the analyzed case we have six independent variables ($n = 6$) and four equations ($r = 4$). According to Buckingham's Π theorem, the number of dimensionless quantities is two ($k = 2$).

The solution of the equation gives

$$a = 1,\ c = -b,\ d = -\frac{3}{4}b,\ e = \frac{b}{2}$$

The (14) solution takes the following form

$$T_{c2} - T_{c1} = C (T_{h1} - T_{c1})^1 (\dot{m}_c)^b (p_h)^{-b} A^{-\frac{3}{4}b} g^{\frac{b}{2}} \tag{16}$$

Arrangement of the expressions with the same exponents gives

$$\frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = C \left(\frac{\dot{m}_c}{p_h} \frac{\sqrt{g}}{A^{\frac{3}{4}}} \right)^b \tag{17}$$

After the introduction of the two dimensionless quantities

$$\Pi_1 = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} \tag{18}$$

$$\Pi_2 = \frac{\dot{m}_c}{p_h} \frac{\sqrt{g}}{A^{\frac{3}{4}}} \tag{19}$$

the (14) relation may be eventually written as follows

$$\Pi_1 = f(\Pi_2) \tag{20}$$

On the basis of the conducted analysis a relation was obtained between the two dimensionless quantities in which there are the following measured parameters: water temperature at the inlet to the steam condenser (T_{c1}), cooling water mass flow ($\dot{m}_c$) and steam pressure (p_h). The exact form of the function is determined on the basis of the characteristics and measured data for the steam condenser.

3. Results

At first, the relation (20) between the two dimensionless quantities was checked using data obtained from the characteristics of the steam condenser for a 200 MW power plant. Table 1 sets out the values of

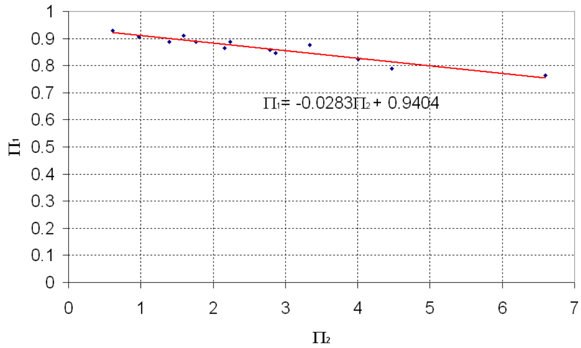

Figure 2: A comparison between two dimensionless quantities (data from the characteristics of the steam condenser)

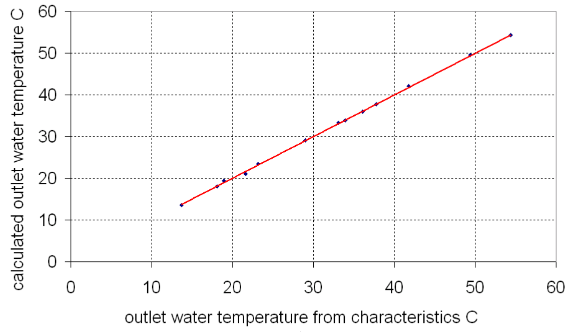

Figure 3: A comparison between water temperature at the outlet of the steam condenser, from the characteristics and calculated (21)

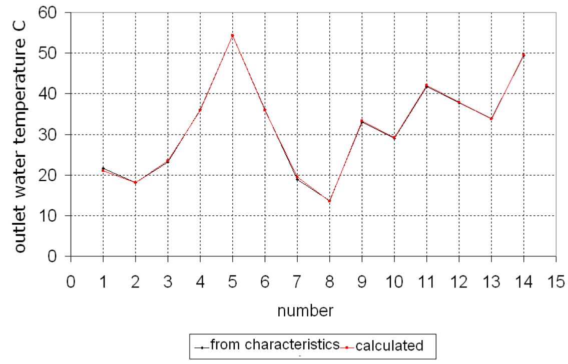

Figure 4: Change in water temperature at the outlet of the steam condenser, from characteristics and calculated

parameters (steam mass flow, steam pressure, water inlet temperature, difference between saturation temperature and water outlet temperature, cooling water mass flow) obtained from characteristics of the steam condenser.

Based on the data in Table 1 two dimensionless quantities were determined and their values are presented in Fig. 2. On the basis of the data obtained, a linear relation between the dimensionless quantities can be assumed with a good approximation.

The coefficients in the straight line were determined by the least squares method and the following values obtained: for the directional coefficient $a = -0.0283$ and for the intercept coefficient $b = 0.9404$.

The relation between the dimensionless quantities can be written as

$$\Pi_1 = -0.0283 \cdot \Pi_2 + 0.9404 \qquad (21)$$

A comparison between the water temperature at the outlet of the steam condenser obtained from the characteristics and calculated from (21) for 14 values is presented in Fig. 3.

The very good correlation between these two temperatures can be observed.

Fig. 4 sets out the change in water temperature at the outlet of the steam condenser from the characteristics, calculated for 14 values. Very good correspondence may be observed between these two temperatures in Fig. 4.

After determining the coefficients in (21) relation, the correctness of this formula was checked for the next 11 different values. In Table 2 the next 11 parameters from the characteristics of the steam condenser are presented.

The change in water temperature at the outlet of the steam condenser, from characteristics and calculated for the next 11 values, is presented in Fig. 5. There is very good correlation between these two temperatures.

The correctness of 20 relation was also checked on the basis of measured data for the steam condenser working in different 200MW power plants. Within the steam condenser the following measurements of parameters were made: water temperature at the inlet and outlet of the steam condenser, steam pressure and total mass of cooling water flowing through the two parts of the steam condenser. The mass of cooling water flowing through one part of the steam condenser was obtained by dividing the total mass flow of cooling water by two. The data in the control system were recorded every hour during the operation

Table 1: Values of parameters from characteristics of the steam condenser

Number	$\dot{m}_h$, kg/s	p_h, kPa	T_{c1}, °C	$T_s(p_h) - T_{c2}$, °C	$\dot{m}_c$, kg/s
1	60	3	4	2.5	3200
2	80	2.5	4	3	3200
3	110	3.5	4	3.5	3200
4	60	6.3	26	1	3200
5	120	16.4	35	1.5	3200
6	60	6.3	26	1	3200
7	110	2.8	4	4	4000
8	70	1.9	4	3	4000
9	120	9	17	2.5	4000
10	90	7.1	17	2	4000
11	120	9	26	2	4000
12	90	7.1	26	1.5	4000
13	60	5.6	26	1	4000
14	110	12.9	35	1.5	4000

Table 2: Values of parameters from characteristics of the steam condenser

Number	$\dot{m}_h$, kg/s	p_h, kPa	T_{c1}, °C	$T_s(p_h) - T_{c2}$, °C	$\dot{m}_c$, kg/s
15	110	4.75	17	2.5	4800
16	70	9	35	1	4800
17	80	1.7	4	4	6400
18	40	1.15	4	2	6400
19	120	4.3	17	3	6400
20	100	3.8	17	2.5	6400
21	60	2.95	17	1.5	6400
22	130	7.05	26	2.5	6400
23	80	6.6	26	1.5	6400
24	60	6	30	1	6400
25	130	11	35	2	6400

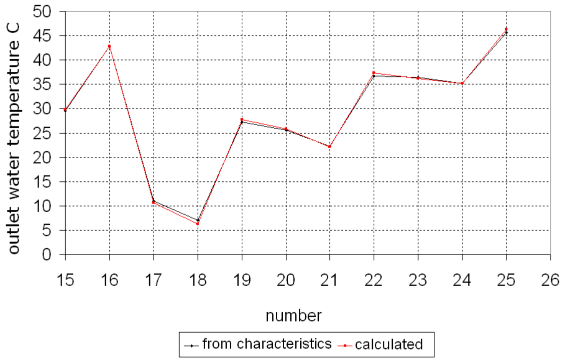

Figure 5: Change in water temperature at the outlet of the steam condenser, from characteristics and calculated for the next 11 values

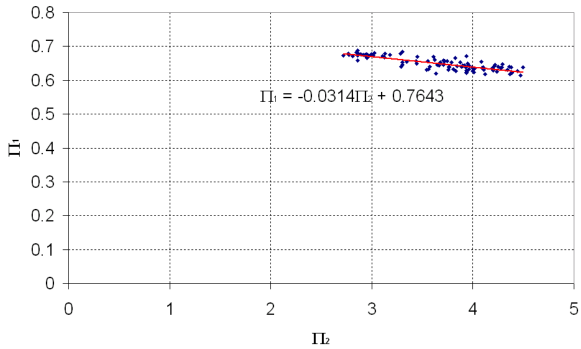

Figure 6: A comparison between two dimensionless quantities (measured data)

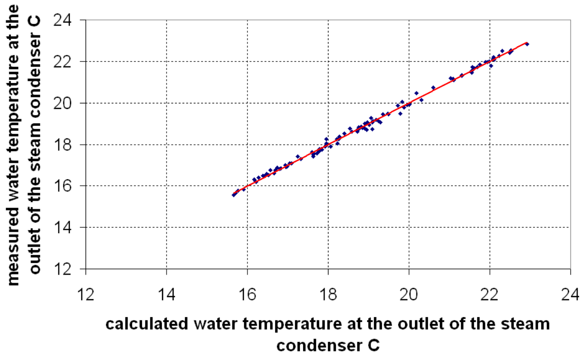

Figure 7: A comparison between water temperature at the outlet of the steam condenser, measured and calculated

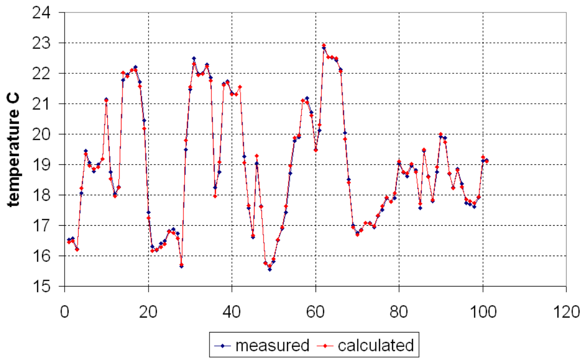

Figure 8: Change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the beginning of the year

of the power plant (average value per hour). Two dimensionless quantities based on 100 values at the beginning of the year were calculated. The comparison between these two dimensionless quantities is presented in Fig. 6. On the basis of the data obtained a linear relation between these two dimensionless quantities can be assumed with a good approximation. The coefficients in the straight line were determined by the least squares method for 100 measured values and the following values were obtained: for the directional coefficient $a = -0.0314$ and for the intercept coefficient $b = 0.7643$.

The relation between the dimensionless quantities can be written as

$$\Pi_1 = -0.0314 \cdot \Pi_2 + 0.7643 \qquad (22)$$

A comparison between the water temperature at the outlet of the steam condenser, measured and calculated from (22), is presented in Fig. 7.

The very good correlation between the measured and calculated temperature can be observed. The change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the beginning of the year, is presented in Fig. 8.

After determining the coefficients in (22) relation, measured and calculated temperature at the outlet of the steam condenser for the next 100 values were compared (Fig. 9).

The change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values in the middle of the year, is presented in Fig. 10.

The change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the end of the year, is presented in Fig. 11.

Based on the performed analysis, it appears that the proposed linear relation for the steam condenser

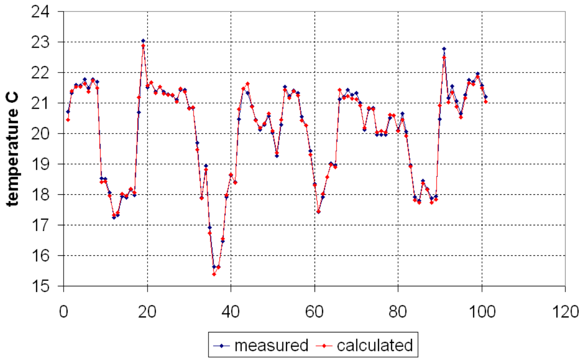

Figure 9: Change in water temperature at the outlet of the steam condenser, measured and calculated for the next 100 values

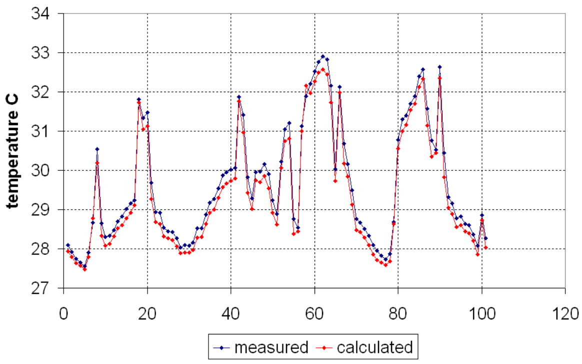

Figure 10: Change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values in the middle of the year

in changed conditions is correct. Introducing the concept of heat transfer effectiveness and knowing the constant value of the condenser heat transfer surface and gravity acceleration, the relation which describes the work of the steam condenser in changed conditions can be written as a linear function with two constants

$$\varepsilon = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}} = D \cdot \frac{\dot{m}_c}{p_h} + E \qquad (23)$$

4. Conclusion

This article presents a mathematical model of the steam condenser in changed conditions. A list of independent parameters was selected and by means of the Buckingham Π theorem two dimensionless quantities were obtained. Based on the data from the characteristics as well as the actual measurement data for a steam condenser in a 200 MW power plant,

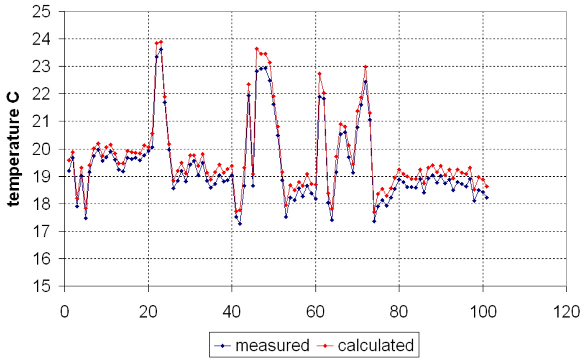

Figure 11: Change in water temperature at the outlet of the steam condenser, measured and calculated for 100 values at the end of the year

a linear relation between two dimensionless quantities was obtained. A simple linear relation was obtained which allows one to determine the work of the steam condenser in changed conditions. To calculate the water outlet temperature from the steam condenser in changed conditions, two coefficients in the linear relation must be determined and basic measured parameters must be known such as water temperature at the inlet to the steam condenser, cooling water mass flow and steam pressure. A comparison between measured and calculated water temperature from the proposed linear relation at the outlet of the steam condenser was performed. A very good correspondence was obtained between these two temperatures for both the data from the characteristics of the steam condenser as well as for measured data. The observed differences between the measured and calculated water temperature at the outlet of the steam condenser from the proposed relation (in the middle and at the end of the year) may result from a deterioration in working conditions of the steam condenser. Data from the characteristics and actual measurement data are for two different condensers working in different 200 MW power plants.

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Nomenclature

α heat transfer coefficient, W/m^2/K

α_1, α_2, α_3, α_4 coefficients of power series

δ coefficients of power series

$\dot{C}$ fluid heat capacity, W/K

$\dot{m}$ mass flow, kg/s

$\dot{Q}$ heat flow, W

η dynamic viscosity, kg/m/s

λ thermal conductivity, W/m/K

λ_{con} thermal conductivity of condensate, W/m/K

ν_{con} kinematic viscosity of condensate, m^2/s

Π_i (i=1,2) number of similarities, dimensionless

ρ_{con} condensate density, kg/m^3

ρ_v vapor density, kg/m^3

$\triangle T_{ln}$ logarithmic mean temperature difference, K

ε heat transfer effectiveness, dimensionless

$_1$ inlet to the heat exchanger

$_2$ outlet of the heat exchanger

$_c$ cold fluid

$_h$ hot fluid

$_o$ reference state

A heat transfer surface area, m^2

a, b, c, d, e constants, dimensionless

C constant, dimensionless

C_c constant, dimensionless

c_p specific heat at constant pressure, J/kg/K

D constant, 1/(m s)

d_i inside diameter, m

d_o outside diameter, m

E constant, dimensionless

g gravity acceleration, m/s^2

i enthalpy, J/kg

k overall heat transfer coefficient, W/(m^2 K)

n number of tubes in a row, dimensionless

Nu Nusselt number, dimensionless

Pr Prandtl number, dimensionless

r latent heat, J/kg

Re Reynolds number, dimensionless

T_{con} condensate temperature, K

T_s saturation temperature, K