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# Optimal robust integrated power distribution network planning under load demand uncertainty

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## Abstract

Due to the new technologies introduced in smart grids, it is hard to forecast future load demands with deterministic values. This makes it essential to consider load demand uncertainty in power distribution planning (PDP) approaches. The purpose of this paper was to find an approach that can solve optimal integrated power distribution long-term planning under load demand uncertainty. A single objective function was used that considers costs of low and medium voltage feeders, distribution transformers (DT) and high voltage (HV) substations simultaneously. Imperialist competitive algorithm (ICA) was used to solve the optimization problem. The proposed approach was applied to a semi-real hypothetical test-case with geographical attributes. Normal distribution function was used to model load demand uncertainty and Monte Carlo simulation (MCS) technique was applied to solve optimal planning under uncertainty. MCS takes statistical data and gives statistical results. A technique was utilized to take a single solution from statistical results. Based on comparisons with deterministic approach, the proposed approach is capable of giving a robust solution.

*Keywords:* Distribution network planning, Load demand uncertainty, Monte Carlo simulation, Optimization

## 1. Introduction

Smart grids introduce new technologies, which make load demand data uncertain. Load demand data is the starting point for power distribution planning (PDP) studies Islam et al. [11]. Therefore, consideration of load demand uncertainty plays an important role in PDP approaches.

A review of recent PDP approaches was presented in Georgilakis and Hatziargyriou [10]. Typically, a distribution network contains two voltage levels: low voltage (LV)–as secondary network–and medium voltage (MV)–as primary network– after the high voltage (HV) substation. Integrated planning of these voltage levels has been studied in the literature Backlund and Bubenko [2], Fletcher and Strunz [5, 6], Ganjavi [9], Mendoza et al. [13], Nazar et al. [17], Paiva et al. [18], Ziari et al. [23]. Among the works, there is no evidence of inclusion of load demand uncertainty in integrated PDP approaches.

Various techniques have been proposed in the literature to deal with load demand uncertainty in PDP approaches: Monte Carlo simulation (MCS) Khatami and Ravadanegh [12], Samper and Vargas [21, 22], scenario-driven technique Bagheri et al. [3], mixed-integer second-order cone programming model Franco et al. [7] and etc. MCS as a basic uncertainty modelling technique is used in this paper.

The aim of this paper was to find a method that can solve optimal integrated power distribution long-term planning under load demand uncertainty in a semi-real hypothetical test-case. It was asked whether the method could give a robust solution compared to the deterministic approach. In this paper, an objective function is proposed that considers costs of low and medium voltage feeders, distribution transformers (DT) and HV substations simultaneously. The proposed objective function is solved by a meta-heuristic optimization technique, which is called imperialist competitive algorithm (ICA) Atashpaz-Gargari and Lucas [1], Najafi-Ravadanegh and Gholizadeh-Roshanagh [15, 16]. Using normal distribution function Khatami and Ravadanegh [12], load demand uncertainty is modelled and, applying MCS, statistical results are obtained. Finally, a technique is proposed to obtain an expected solution based on statistical results.

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## 2. Statement of optimal power distribution planning problem

Power distribution planning (PDP) is a complex mixed-integer non-linear optimization problem Saboori et al. [20]. The purpose of PDP is locating, sizing and timing HV substations, distribution transformers (DT), and MV and LV feeders. The PDP approaches, applied to primary and secondary networks, can be isolated or integrated. The approach presented in this paper integrates optimal planning of primary and secondary networks to locate and size HV substations, DTs and route MV feeders. The PDP is modeled as an objective function (1), which consists of costs related to LV feeders, DTs, MV feeders and HV substation.

$$OF_t = CF_{LVF} + CF_{DT} + CF_{MVF} + CF_{HVS}. \tag{1}$$

The planning problem takes deterministic or uncertain load demand data, and candidate locations of DTs, MV feeders and HV substations as input; and gives locations of DTs, MV feeders and HV substations and sizes of DTs and HV substations as output. The selection or not-selection of HV substations and DTs are modeled as decision variables of optimization problem. The area to be planned is assumed to be modeled as square load blocks with load demand values.

### 2.1. Low voltage feeder cost modeling

A process is used to model cost of low voltage feeders. At first, random $0/1$ values are generated for decision variables of DTs and HV substations as a member of initial population, where $1$ denotes selection and $0$ denotes not-selection. The load blocks are assigned to the selected DTs based on the technique presented in Najafi et al. [14]. The maximum distance and LV voltage drop constraints are considered in the load assignment algorithm. The generated member of initial population must afford the network load demand; if not, a new set of values must be generated. Then, the assigned load blocks to each DT are clustered using the well-known K-means algorithm. The cost of low voltage feeders can be given as (2) which consists of construction and loss costs.

$$\begin{aligned} CF_{LVF} = &\sum_{i=1}^{N_{DT}} \sum_{j=1}^{N_{clus,i}} \Bigg\{ CC(LVF_{j,i}) \cdot d_{ij} \cdot K_{C,ind} + \\ &\frac{R_{ij} \cdot LI_{ij} \cdot ELCF \cdot T_P \cdot 8760}{3000 V_{LV}^2 \cdot APF(Clus_{j,i})^2} + \\ &\sum_{k=1}^{N_{LB,j,i}} \Bigg[ CC(LVF_{k,j,i}) \cdot d_{jk} \cdot K_{C,ind} + \\ &\frac{R_{jk} \cdot LI_{jk} \cdot ELCF \cdot T_P \cdot 8760}{3000 V_{LV}^2 \cdot PF(LB_{k,j,i})^2} \Bigg] \Bigg\} \cdot \lambda_i. \end{aligned} \tag{2}$$

where,

$$LI_{ij} = \left[ \sum_{k=1}^{N_{LB,j,i}} P(LB_{k,j,i}) \right]^2 \cdot d_{ij} \cdot K_{C,ind}. \tag{3}$$

$$LI_{jk} = P(LB_{k,j})^2 \cdot d_{jk} \cdot K_{C,ind}. \tag{4}$$

$$APF(Clus_{j,i}) = \frac{\sum_{k=1}^{N_{LB,j,i}} P(LB_{k,j,i})}{\sum_{k=1}^{N_{LB,j,i}} \frac{P(LB_{k,j,i})}{PF(LB_{k,j,i})}} \tag{5}$$

Using the technique in Najafi et al. [14] a correction factor is obtained for indirect path of LV feeder as $K_{C,ind} = 1.13$. The cost of loss in feeders is modeled as loss index in (4)-(5)

### 2.2. Distribution transformer cost modeling

After assigning the load blocks to the selected DTs, the load of each DT is obtained. Using power factor ($PF$) and annual load demand factor ($ALDF$), actual supplied load ($SPL$) of each DT is obtained as given in (6). Size of each DT can be determined rounding the $SPL$ to the upper near standard ranges as: 25, 50, 100, 125, 200, 250, 315, 400, 500, 630, 800, 1000, 1250, 1600, 2000 kVA.

$$SPL(DT_i) = \frac{\sum_{j=1}^{N_{LB,i}} P(LB_{j,i})}{PF(DT_i) \cdot ALDF(DT_i)}. \tag{6}$$

Then, the cost of DTs can be calculated as (7) Najafi et al. [14].

$$\begin{aligned} CF_{DT} = \sum_{i=1}^{N_{DT}} [&CC(DT_i) \cdot Size(DT_i) + \\ &CL(DT_i) \cdot T_P \cdot 8760] \cdot \lambda_i. \end{aligned} \tag{7}$$

where,

$$\begin{aligned} CL(DT_i) = [&P_{NLL}(DT_i) + P_{SCL}(DT_i) \cdot \\ &TL^2(DT_i) \cdot ALSF(DT_i)\big] \cdot ELCF. \end{aligned} \tag{8}$$

$$TL(DT_i) = \frac{\sum_{j=1}^{N_{LB,i}} P(LB_{j,i})}{Size(DT_i) \cdot PF(DT_i)}. \tag{9}$$

$TL$ is current of DT at the operation time (in p.u.). The average annual loss factor ($ALSF$) is obtained as Gangel and Propst [8]:

$$\begin{aligned} ALSF(DT_i) = &0.15 \cdot ALDF(DT_i) + \\ &0.85 \cdot ALDF^2(DT_i). \end{aligned} \tag{10}$$

### 2.3. Medium voltage feeder cost modeling

The model presented in this section is known as feeder routing in the literature. After assigning loads and sizing selected DTs, a graph with vertices and branches is obtained. Vertices of the graph are HV and DT nodes and branches are candidate MV feeders. The graph is checked for connectivity using the technique presented in Biggs [4]. If not connected, a new set of decision variables are generated. If connected, the graph is solved for minimum spanning tree (MST) using Prim's technique Prim [19] to obtain a radial network structure. From MST solution the set of selected MV feeders (where $\kappa_i = 1$) is formed. The cost of MV feeders can be obtained as (11) where the constraints (12)–(13) must be satisfied.

$$CF_{MVF} = \sum_{i=1}^{N_{MVF}} \left[CC(MVF_i) \cdot d_i + I^2(MVF_i) \cdot R_i \cdot d_i \cdot ELCF \cdot T_P \cdot 8.760\right] \cdot \kappa_i. \quad (11)$$

*subject to,*

$$I(MVF_i) < I_{max}(MVF_i) \quad \forall i \in \{S_{MVF} | \kappa_i = 1\} \quad (12)$$

$$\sum_{j=1}^{N_{UMVF,i}} VD(MVF_j) < VD_{MV,max} \quad \forall i \in \{S_{DT} | \lambda_i = 1\}. \quad (13)$$

where,

$$VD(MVF_j) = \left[R_j \cdot PF(MVF_j) + X_j \cdot \sqrt{1 - PF^2(MVF_j)}\right] \cdot d_j \cdot \frac{I(MVF_j)}{1000 V_{MV}} \cdot 100. \quad (14)$$

$$S_{MVF} = \{1, 2, ..., N_{MVF}\} \quad (15)$$

$$S_{DT} = \{1, 2, ..., N_{DT}\} \quad (16)$$

In this paper, it is assumed that one HV substation is enough due to the demand of the test-case. Therefore, when generating decision variables for HV substations just one is selected randomly. If the demand is higher, a clustering technique can be utilized to make groups of DTs; then independent feeder routing can be applied for each group.

### 2.4. HV substation cost modeling

In order to calculate the cost of HV substations, actual supplied load ($SPL$) is obtained as given in (17). Size of HV substation is determined rounding the $SPL$ to the upper near standard ranges as: 3, 8, 15, 25, 30, 50 MVA.

$$SPL(HVS_k) = \sum_{i=1}^{N_{DT}} \frac{\lambda_i \cdot \sum_{j=1}^{N_{LB,i}} P(LB_{j,i})}{PF(HVS_k) \cdot ALDF(HVS_k)} \quad (17)$$

Then, the cost of HV substation is calculated as (18) Najafi et al. [14].

$$CF_{HVS} = \sum_{i=1}^{N_{HVS}} [CC(HVS_i) \cdot Size(HVS_i) + CL(HVS_i) \cdot T_P \cdot 8760] \cdot \gamma_i. \quad (18)$$

where,

$$CL(HVS_i) = \left[P_{NLL}(HVS_i) + P_{SCL}(HVS_i) \cdot TL^2(HVS_i) \cdot ALSF(HVS_i)\right] \cdot ELCF. \quad (19)$$

$$TL(HVS_k) = \frac{\sum_{i=1}^{N_{DT}} \sum_{j=1}^{N_{LB,i}} P(LB_{j,i})}{Size(HVS_k) \cdot PF(HVS_k)}. \quad (20)$$

### 2.5. Planning under deterministic load demand

A deterministic load demand is considered at this stage. The process is to solve an optimization problem with objective function as (1). The decision variables of HV substations and DTs are as initial population for the optimization technique. In order to optimize the objective function, a meta-heuristic algorithm, namely imperialist competitive algorithm (ICA), is used. Detailed explanation of ICA can be found in Atashpaz-Gargari and Lucas [1], Najafi-Ravadanegh and Gholizadeh-Roshanagh [15, 16].

### 2.6. Planning under uncertain load demand

A Monte Carlo simulation (MCS) technique is used to consider uncertainty in the optimal planning process. A normal probability distribution is considered for uncertainty modeling of load demand Khatami and Ravadanegh [12] as in (21).

$$PDF(P_k) = \frac{1}{\sqrt{2\pi}\sigma_L} \exp \frac{(P_k - \mu_L)^2}{2\sigma_L^2} \quad (21)$$

where, forecasted load demand of $k$-th load block is considered as mean value ($\mu_L$) and the load forecasting error is considered as standard deviation ($\sigma_L$). $P_k$ is a random value of $k$-th load block demand and $PDF$ gives its probability of occurrence. Using normal probability distribution function, a set of random values are generated for each load block. The $k$-th row of the matrix in (22) gives the random generated values of $k$-th load block.

$$\begin{bmatrix} P_1^1 & \cdots & P_1^j & \cdots & P_1^{N_{SC}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_k^1 & \cdots & P_k^j & \cdots & P_k^{N_{SC}} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_B^1 & \cdots & P_B^j & \cdots & P_B^{N_{SC}} \end{bmatrix} \tag{22}$$

Every column of the matrix in (22) is considered as a scenario; i.e. $N_{SC}$ number of scenarios are generated. It is assumed that the load demand of the load blocks are not correlated, which is a realistic assumption in future smart grids environment. Then, the deterministic planning approach is repeated for each scenario. In order to obtain a robust solution, an algorithm is proposed as follows:

*Step 1.* Take the results of planning for $N_{SC}$ scenarios;
*Step 2.* Sort HV substations and DTs based on higher frequency of selection;
*Step 3.* Choose the HV substation with higher selection;
*Step 4.* Choose a DT with higher selection from the remaining set of DTs;
*Step 5.* Form the graph of network with chosen HV substation and DTs;
*Step 6.* Check for connectivity: if not go to step 4;
*Step 7.* Do a load assignment;
*Step 8.* Check if the DTs are adequate: if not go to step 4;
*Step 9.* Size the DTs; form the graph; find MST; and calculate the costs.

## 3. Simulation results

Simulations are performed on a semi-real hypothetical test-case as shown in Fig. 1. It consists of 34 candidate nodes ($2$ HV substations and $32$ DTs) and $81$ candidate MV feeders. The test-case is an area of a city planned for $10$ years later. The load density has been illustrated with scatter color. It is assumed that the load forecasting studies have been performed. The area is divided into $50 \times 50\ m^2$ blocks with forecasted load values. The power factor ($PF$) is assumed as $0.8$ for all load blocks, DTs and HV substations. Average annual load factor ($ALDF$) is assumed as $0.7$. Total load is $3.2$ MW. No-load loss ($P_{NLL}$) and short-circuit loss ($P_{SCL}$) of transformers can be obtained from standards. Table 1 gives other parameter values used in the simulations.

### *3.1. Results of planning under deterministic load demand*

When planning under deterministic load demand, the values of the loads are average demands. As described in section 2, the objective function consists of four different costs. Table 2 gives the optimized costs. HV substation and LV network costs are the highest of the four costs.

Fig. 2 shows optimal network configuration planned under deterministic load demands. One HV substation, 22 DTs and 22 MV feeders are selected. The network is radial with three feeders outgoing from HV substation. Load blocks assigned to a DT are divided into one or multiple clusters and are connected to the DT through cluster centroids. Not selected HV substation and DTs are shown with white markers.

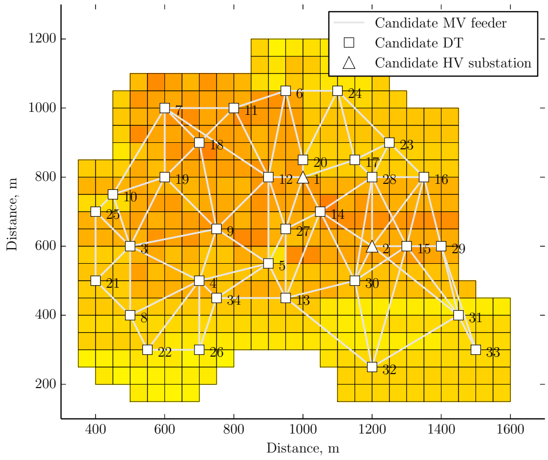

Figure 1: The 409 load blocks test-case with candidate HV substations, DTs and MV feeders

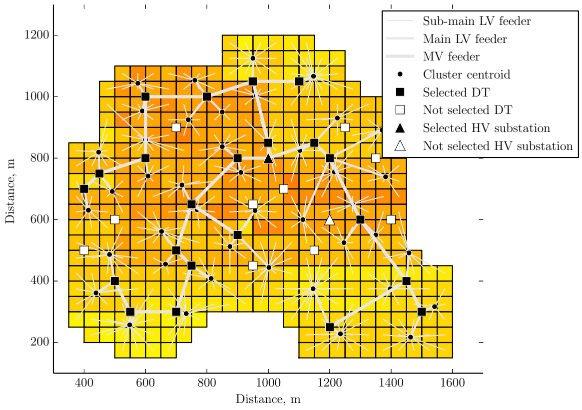

Figure 2: Optimal configuration of the network after planning under deterministic load demand

Sizes of selected HV substation and DTs are given in Table 3 on column two. DT sizes are obtained based on assigned load (on column three) and using the technique presented in section 2.2. Size of HV substation is based on the technique presented in section 2.4. Loading of DTs for average load demand and $1.1\times$ average load demand are given in Table 3. Columns five and ten show that 13 DTs exceed the 70% maximum loading constraint.

### *3.2. Results of planning under uncertain load demand*

In this section, the optimal distribution system planning is solved for uncertain load demands. Based on normal probability distribution function, for every load block, $1000$ random

Table 1: Parameters used in the simulations

| Parameter | value |
|---|---|
| Cost of construction of LV feeder, $CC(LVF)$ | 19.01, \$/m |
| Cost of construction of MV feeder, $CC(MVF)$ | 19.73, \$/m |
| Cost of construction of DTs from 25 up to 630 kVA, $CC(DT)$ | 55.97, \$/kVA |
| Cost of construction of DTs from 800 up to 2000 kVA, $CC(DT)$ | 69.97, \$/kVA |
| Cost of construction of HV substation of 8000 kVA, $CC(HVS)$ | 118.39, \$/kVA |
| LV feeder resistance, $R$ | 0.2116, Ω/km |
| MV feeder resistance, $R$ | 0.342, Ω/km |
| MV feeder reactance, $X$ | 0.127, Ω/km |
| Energy loss cost factor, $ELCF$ | 0.0287, \$/kWh |
| Planning period, $T_P$ | 10, years |
| LV voltage level, $V_{LV}$ | 0.4, kV |
| Maximum allowed current through MV feeder, $I_{max}$ | 386, A |
| Maximum allowed MV feeder voltage drop, $VD_{MV,max}$ | 2, % |

Table 2: Costs of planning under deterministic load demand

| Costs | Cost, \$ |
|---|---|
| LV cost | 690,299.03 |
| MV feeder cost | 118,947.58 |
| DT cost | 451,909.23 |
| HV substation cost | 974,679.83 |
| Total cost | 2,235,835.67 |

values are generated with mean value as the average demand and the standard deviation as $0.1\times$ average demand. As an example, Fig.3 shows the histogram and the probability distribution curve of the generated load values of load block number 160.

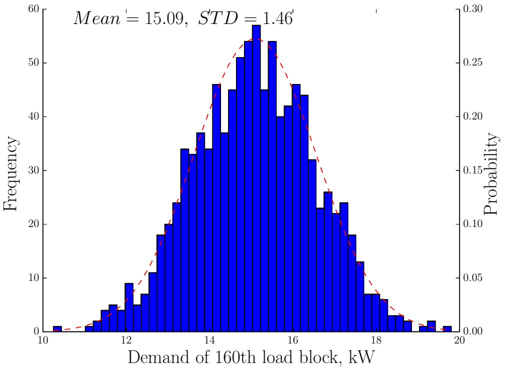

Figure 3: Uncertain modelling of $160^{th}$ load block: Histogram of generated load values (bars); Normal probability distribution curve of generated load values (dashed lines)

A Monte Carlo simulation (MCS) technique is used to solve the planning problem under uncertainty. Based on the technique presented in section 2.6, $1000$ scenarios are formed. The optimal deterministic planning approach is performed for $1000$ scenarios. Table 3.2 gives the results of HV substations and DTs over $1000$ scenarios. The rows of the table are sorted based on higher selection instances of nodes. Columns 2&7 give the number of selections; Columns 4&9 and 5&10 give the mean and the standard deviation values of assigned load to DTs, respectively. Surprisingly, HV node number one and one DT node are selected in all the scenarios. Number of selections of the DT on node 3 is the lowest ($226$ times). The histogram and probability distribution curves of the assigned load of some of the DTs are shown in Fig. 4. It shows that the DTs have different distribution based on their different locations on the test-case. Fig.5 gives the current histogram of two feeders. As it shows, HV substation outgoing feeder current is much more predictable than other far distant feeders.

Based on the proposed approach, the network configuration is obtained. Table 3.2 gives the costs; Fig. 6 shows the network configuration; and Table 3.2 gives the size of selected HV node and DT nodes besides loading of DT nodes under average and $1.1\times$ average demands. Compared to the deterministic approach, the overall network configurations and the total costs are slightly similar while the number of DTs is increased. Also, the number of DTs exceeding the 70% constraint is decreased, which shows the robustness of the proposed approach.

## 4. Conclusion

In this paper, an approach was proposed for optimal robust integrated distribution network planning under load demand uncertainty. Monte Carlo simulation (MCS) technique was utilized to consider uncertainty. The MCS technique results in statistical data which is suitable for further decisions. The proposed approach was applied to a semi-real hypothetical test-case. A technique was proposed to extract a single solution based on statistical data. The results showed that MCS as a basic uncertainty modeling technique can give a robust solution.

## 5. Acknowledgment

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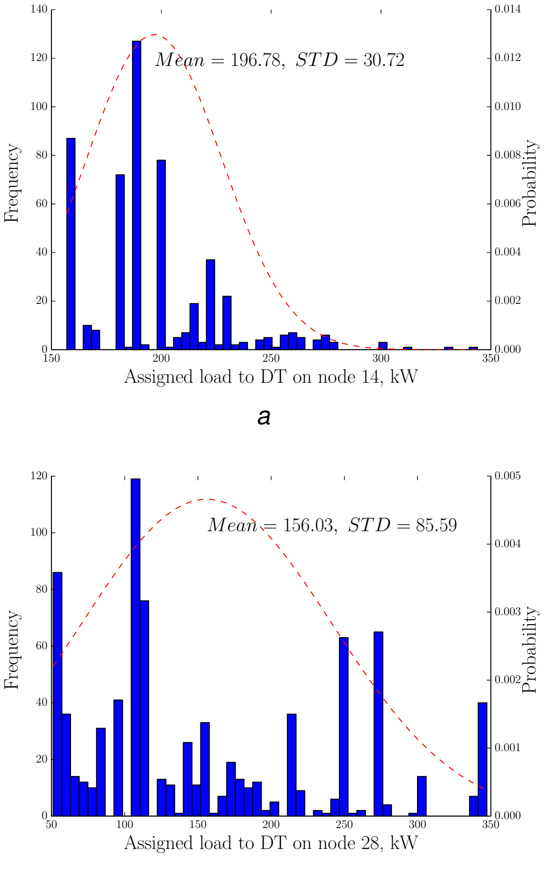
Table 3: Planned sizes of selected HV substation and DTs; and assigned load and loading of DTs under deterministic load demands

| DT | Size, | Load, | Loading, % | | DT | Size, | Load, | Loading, % | |
|---|---|---|---|---|---|---|---|---|---|
| No. | kVA | kW | AD$^*$ | $1.1\times$AD | No. | kVA | kW | AD | $1.1\times$AD |
| 1 (HV) | 8000 | - | - | - | | | | | |
| 4 | 315.0 | 145.6 | 57.8 | 63.6 | 19 | 315.0 | 167.5 | 66.5 | 73.1 |
| 5 | 500.0 | 278.8 | 69.7 | 76.7 | 20 | 200.0 | 92.2 | 57.6 | 63.4 |
| 6 | 250.0 | 134.5 | 67.2 | 74.0 | 22 | 100.0 | 48.5 | 60.6 | 66.7 |
| 7 | 315.0 | 175.5 | 69.6 | 76.6 | 24 | 200.0 | 89.4 | 55.9 | 61.5 |
| 8 | 315.0 | 162.0 | 64.3 | 70.7 | 25 | 200.0 | 74.3 | 46.4 | 51.1 |
| 9 | 315.0 | 168.3 | 66.8 | 73.5 | 26 | 100.0 | 37.4 | 46.8 | 51.4 |
| 10 | 250.0 | 120.8 | 60.4 | 66.4 | 28 | 630.0 | 347.3 | 68.9 | 75.8 |
| 11 | 400.0 | 214.3 | 67.0 | 73.7 | 31 | 200.0 | 103.6 | 64.8 | 71.2 |
| 12 | 315.0 | 172.5 | 68.5 | 75.3 | 32 | 315.0 | 157.1 | 62.3 | 68.6 |
| 15 | 400.0 | 219.1 | 68.5 | 75.3 | 33 | 200.0 | 99.0 | 61.9 | 68.1 |
| 17 | 250.0 | 139.9 | 70.0 | 76.9 | 34 | 100.0 | 55.8 | 69.8 | 76.7 |

Table 4: MCS results of HV and DT nodes, sorted based on higher selection instances

| Node No. | $f^*$ | $\mu_L$, kW | $\sigma_L$, kW | Node No. | $f$ | $\mu_L$, kW | $\sigma_L$ kW |
|---|---|---|---|---|---|---|---|
| 1 (HV) | 1000 | - | - | 26 | 781 | 39.87 | 9.35 |
| 32 | 1000 | 122.14 | 15.51 | 15 | 763 | 202.16 | 24.4 |
| 33 | 972 | 100.19 | 6.55 | 24 | 706 | 92.85 | 23.09 |
| 31 | 969 | 84.08 | 21.11 | 18 | 688 | 145.69 | 23.95 |
| 11 | 965 | 188.27 | 26.4 | 19 | 679 | 169.37 | 35.52 |
| 6 | 949 | 156.59 | 31.24 | 17 | 678 | 120.39 | 34.1 |
| 7 | 905 | 175.5 | 0.0 | 16 | 639 | 164.06 | 12.5 |
| 4 | 888 | 172.99 | 33.85 | 13 | 637 | 151.68 | 27.66 |
| 22 | 873 | 67.61 | 36.84 | 34 | 593 | 72.74 | 39.37 |
| 10 | 865 | 181.23 | 49.38 | 14 | 533 | 196.78 | 30.72 |
| 20 | 862 | 105.91 | 28.04 | 23 | 496 | 125.68 | 45.59 |
| 28 | 840 | 156.03 | 85.59 | 21 | 484 | 72.22 | 17.86 |
| 8 | 835 | 139.1 | 36.16 | 25 | 417 | 77.21 | 24.19 |
| 12 | 833 | 180.12 | 33.43 | 30 | 342 | 162.23 | 38.65 |
| 29 | 812 | 123.51 | 43.87 | 27 | 319 | 129.16 | 41.79 |
| 5 | 802 | 200.93 | 61.01 | 3 | 226 | 168.57 | 8.15 |
| 9 | 783 | 169.06 | 2.21 | 2 (HV) | 0 | - | - |

$^*f$ is frequency of selection



*a*

*b*

Figure 4: Number of selections (bars) and probability distribution curves (dashed) Versus assigned load to the DT on node (a) 14 and (b) 28.

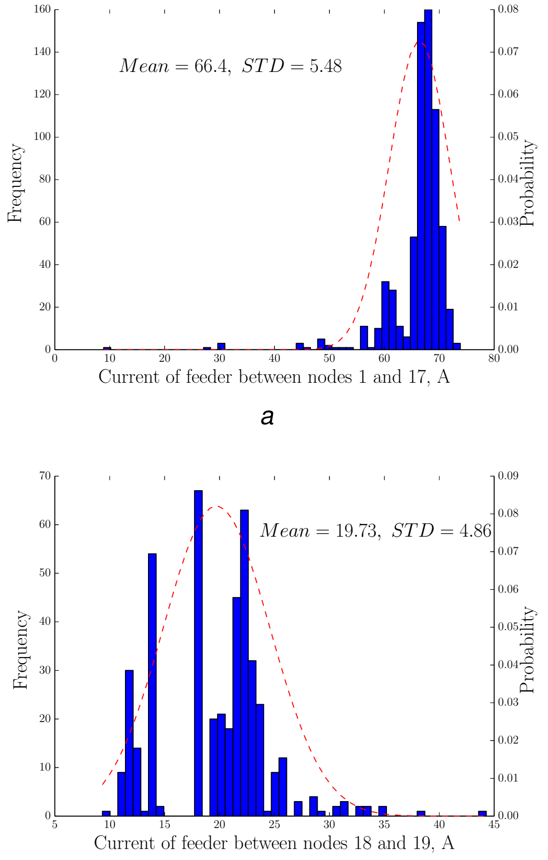


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Figure 5: Number of selections (bars) and probability distribution curves (dashed) versus current of feeder between nodes (a) 1 and 17, (b) 18 and 19.

## References

[1] E. Atashpaz-Gargari and C. Lucas. Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition. In *2007 IEEE Congress on Evolutionary Computation*, pages 4661–4667, Sep. 2007.

[2] Yngve Backlund and Janis A. Bubenko. Computer-aided distribution system planning. *International Journal of Electrical Power & Energy Systems*, 1(1):35–45, Apr. 1979. ISSN 0142-0615.

[3] A. Bagheri, H. Monsef, and H. Lesani. Integrated distribution network expansion planning incorporating distributed generation considering uncertainties, reliability, and operational conditions. *International Journal of Electrical Power & Energy Systems*, 73:56–70, Dec. 2015. ISSN 0142-0615.

[4] N. Biggs. *Algebraic Graph Theory*. Cambridge University Press, 1974.

[5] R. H. Fletcher and K. Strunz. Optimal distribution system horizon planning-Part I: Formulation. *IEEE Transactions on Power Systems*, 22(2):791–799, May 2007. ISSN 0885-8950.

[6] R. H. Fletcher and K. Strunz. Optimal distribution system horizon planning-Part II: Application. *IEEE Transactions on Power Systems*, 22(2):862–870, May 2007. ISSN 0885-8950.

[7] J. F. Franco, M. J. Rider, and R. Romero. Robust multi-stage substation expansion planning considering stochastic demand. *IEEE Transactions on Power Systems*, 31(3):2125–2134, May 2016. ISSN 0885-8950.

[8] M. W. Gangel and R. F. Propst. Distribution transformer load characteristics. *IEEE Transactions on Power Apparatus and Systems*, 84(8): 671–684, Aug. 1965. ISSN 0018-9510.

[9] M. R. Ganjavi. Coordinated comprehensive design of distribution systems at medium and low voltage levels. In *16th International Conference and Exhibition on Electricity Distribution, 2001. Part 1: Contributions. CIRED. (IEE Conf. Publ No. 482)*, volume 5.29, pages 1–6, June18–21, 2001.

[10] Pavlos S. Georgilakis and Nikos D. Hatziargyriou. A review of power distribution planning in the modern power systems era: Models, methods and future research. *Electric Power Systems Research*, 121:89–100, Apr. 2015. ISSN 0378-7796.

[11] A. Islam, S. R. Hasib, and Md. S. Islam. Short term electricity demand forecasting for an isolated area using two different approaches. *Journal of Power Technologies*, 93(4):185–193, 2013.

[12] H. Khatami and S. N. Ravadanegh. Probabilistic optimal robust multistage feeder routing under load forecasting uncertainty. *IET Generation, Transmission & Distribution*, 9(14):1977–1987, Nov. 2015.

[13] Jorge E. Mendoza, Miguel E. López, Héctor E. Peña, and David A. Labra. Low voltage distribution optimization: Site, quantity and size of distribution transformers. *Electric Power Systems Research*, 91:52–

Table 5: Costs of planning under uncertain load demand

| Costs | Cost, $ |
|---|---:|
| LV cost | 663,548.46 |
| MV feeder cost | 127,853.23 |
| DT cost | 472,174.00 |
| HV substation cost | 974,679.83 |
| Total cost | 2,238,255.52 |

Table 6: Planned sizes of selected HV substation and DTs; and assigned load and loading of DTs under uncertain load demands

| DT | Size, | Load, | Loading, % | | DT | Size, | Load, | Loading, % | |
|---|---|---|---|---|---|---|---|---|---|
| No. | kVA | kW | AD* | 1.1×AD | No. | kVA | kW | AD | 1.1×AD |
| 1 (HV) | 8000 | - | - | - | 17 | 250.0 | 125.1 | 62.6 | 68.8 |
| 4 | 315.0 | 145.6 | 57.8 | 63.6 | 18 | 250.0 | 113.8 | 56.9 | 62.6 |
| 5 | 400.0 | 201.2 | 62.9 | 69.2 | 19 | 250.0 | 118.1 | 59.0 | 65.0 |
| 6 | 250.0 | 134.5 | 67.2 | 74.0 | 20 | 200.0 | 92.2 | 57.6 | 63.4 |
| 7 | 315.0 | 175.5 | 69.6 | 76.6 | 22 | 100.0 | 48.5 | 60.6 | 66.7 |
| 8 | 315.0 | 168.4 | 66.8 | 73.5 | 24 | 200.0 | 89.4 | 55.9 | 61.5 |
| 9 | 315.0 | 168.3 | 66.8 | 73.5 | 26 | 100.0 | 34.3 | 42.9 | 47.2 |
| 10 | 400.0 | 188.7 | 59.0 | 64.9 | 28 | 250.0 | 112.1 | 56.1 | 61.7 |
| 11 | 315.0 | 162.1 | 64.3 | 70.8 | 29 | 200.0 | 87.5 | 54.7 | 60.2 |
| 12 | 315.0 | 160.3 | 63.6 | 70.0 | 31 | 200.0 | 74.9 | 46.8 | 51.5 |
| 13 | 315.0 | 141.5 | 56.2 | 61.8 | 32 | 250.0 | 117.6 | 58.8 | 64.7 |
| 15 | 400.0 | 221.3 | 69.2 | 76.1 | 33 | 200.0 | 99.0 | 61.9 | 68.1 |
| 16 | 315.0 | 176.0 | 69.8 | 76.8 | 34 | 100.0 | 47.5 | 59.4 | 65.3 |

*AD stands for average demand

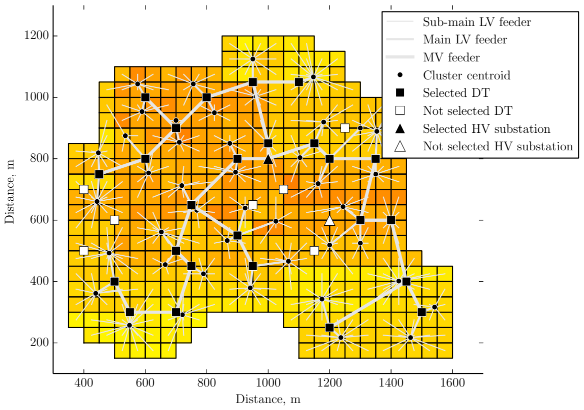

Figure 6: Optimal configuration of the network after planning under uncertain load demand

60, Oct. 2012. ISSN 0378-7796.

[14] S. Najafi, S. H. Hosseinian, M. Abedi, A. Vahidnia, and S. Abachezadeh. A framework for optimal planning in large distribution networks. *IEEE Transactions on Power Systems*, 24(2):1019–1028, May 2009. ISSN 0885-8950.

[15] S. Najafi-Ravadanegh and R. Gholizadeh-Roshanagh. A heuristic algorithm for optimal multistage sizing, siting and timing of MV distribution substations. *Electric Power Systems Research*, 105:134–141, Dec. 2013. ISSN 0378-7796.

[16] S. Najafi-Ravadanegh and R. Gholizadeh-Roshanagh. On optimal multistage electric power distribution networks expansion planning. *International Journal of Electrical Power & Energy Systems*, 54:487–497, Jan. 2014. ISSN 0142-0615.

[17] Mehrdad Setayesh Nazar, Mahmood R. Haghifam, and M. Nažar. A scenario driven multiobjective primary-secondary distribution system expansion planning algorithm in the presence of wholesale-retail market. *International Journal of Electrical Power & Energy Systems*, 40 (1):29–45, Sep. 2012. ISSN 0142-0615.

[18] P. C. Paiva, H. M. Khodr, J. A. Domínguez-Navarro, J. M. Yusta, and A. J. Urdaneta. Integral planning of primary-secondary distribution systems using mixed integer linear programming. *IEEE Transactions on Power Systems*, 20(2):1134–1143, May 2005. ISSN 0885-8950.

[19] R. C. Prim. Shortest connection networks and some generalizations. *Bell System Technical Journal*, 36(6):1389–1401, Nov. 1957. ISSN 1538-7305.

[20] Hedayat Saboori, Reza Hemmati, and Vahid Abbasi. Multistage distribution network expansion planning considering the emerging energy storage systems. *Energy Conversion and Management*, 105:938 – 945, 2015. ISSN 0196-8904.

[21] M. E. Samper and A. Vargas. Investment decisions in distribution networks under uncertainty with distributed generation-Part I: Model formulation. *IEEE Transactions on Power Systems*, 28(3):2331–2340, Aug. 2013.

[22] M. E. Samper and A. Vargas. Investment decisions in distribution networks under uncertainty with distributed generation-Part II: Implementation and results. *IEEE Transactions on Power Systems*, 28(3):2341–2351, Aug. 2013.

[23] Iman Ziari, Gerard Ledwich, and Arindam Ghosh. Optimal integrated planning of MV-LV distribution systems using DPSO. *Electric Power Systems Research*, 81(10):1905–1914, October 2011. ISSN 0378-7796.

## Nomenclature

| | |
|---|---|
| $\gamma_i$ | Binary decision variable for *i*-th HV substation |
| $\kappa_i$ | Binary decision variable for *i*-th MV feeder |
| $\lambda_i$ | Binary decision variable for *i*-th MV feeder |
| $\mu_L$ | Mean value of load demand, kW |
| $\sigma_L$ | Standard deviation of load demand, kW |
| $CF_{DT}$ | Cost function of DTs, $ |
| $CF_{HVS}$ | Cost function of HV substations, $ |
| $CF_{LVF}$ | Cost function of LV feeders, $ |
| $CF_{MVF}$ | Cost function of MV feeders, $ |
| $Clus_{j,i}$ | *j*-th cluster of *i*-th DT |
| $d_{i,j}$ | Distance of feeder from *i*-th to *j*-th node, m |
| $d_i$ | Length of *i*-th MV feeder |

| | |
|---|---|
| $DT_i$ | *i*-th DT |
| $HVS_i$ | *i*-th candidate HV substation |
| $I_{max}$ | Maximum allowed current through a feeder, A |
| $K_{c,ind}$ | Correction factor to calculate real length of an indirect path of LV feeder |
| $LB_{j,i}$ | *j*-th load block assigned to i-th DT |
| $LB_{k,j,i}$ | *k*-th load block of *j*-th cluster centroid related to *i*-th DT |
| $LB_{k,j}$ | *k*-th load block of *j*-th cluster centroid |
| $LI_{i,j}$ | Loss index of LV feeder from *i*-th DT to its *j*-th cluster centroid, m/kW$^2$ |
| $LI_{jk}$ | Loss index of LV feeder from *j*-th cluster centroid to *k*-th load block, m.kW$^2$ |
| $LVF_{j,i}$ | LV feeder from *i*-th DT to its *j*-th cluster centroid |
| $LVF_{k,j,i}$ | LV feeder from *k*-th load block to *j*-th cluster centroid of *i*-th DT |
| $MVF_i$ | *i*-th candidate MV feeder |
| $N_{Clus,i}$ | Number of clusters of *i*-th DT |
| $N_{DT}$ | Number of candidate DT nodes |
| $N_{HVS}$ | Number of candidate HV substations |
| $N_{LB,i}$ | Number of load blocks assigned to *i*-th DT |
| $N_{LB,j,i}$ | Number of load blocks in *h*-th cluster of *i*-th DT |
| $N_{LB}$ | Number of all load blocks |
| $N_{MVF}$ | Number of candidate MV feeders |
| $N_{SC}$ | Number of scenarios |
| $N_{UMVF,i}$ | MV feeders upper hand of *i*-th DT |
| $OF_t$ | Objective function |
| $P_k^j$ | Random load demand of *k*-th load block in *j*-th scenario, kW |
| $P_{NLL}$ | No-load loss of a DT or HV substation, kW |
| $P_{SCL}$ | Short-circuit loss of a DT or HV substation, kW |
| $R_{ij}$ | Resistance value of feeder from *i*-th to *j*-th node, Ω/m |
| $R_i$ | Resistance of *i*-th MV feeder, Ω/m |
| $S_{DT}$ | Set of candidate DTs |
| $S_{MVF}$ | Set of candidate MV feeders |
| $T_p$ | Planning period, years |
| $V_{LV}$ | Line-to-line low voltage level, kV |
| $V_{MV}$ | Line-to-line medium voltage level, kV |
| $VD_{MV,max}$ | Maximum allowed MV feeder voltage drop, |
| $X_j$ | Reactance of *j*-th MV feeder, Ω/m |
| *ALDF* | Average annual load factor of a DT or HV substation |
| *ALSF* | Average annual loss factor of a DT or HV substation |
| *APF* | Average power factor |
| *B* | Number of all load blocks |
| *CC* | Cost of construction, $/kVA for HV substations and DTs; and $/m for feeders |
| *CL* | Cost of resistive and core loss of a DT or HV substation, $/h |
| *ELCF* | Energy loss cost factor, $/kWh |
| *I* | Current of feeder, A |
| *PDF* | Probability density function |
| *PF* | Power factor |
| *P* | Demand of a load block, kW |
| *Size* | Size of DTs or HV substations kVA |
| *SPL* | Supplied load by a DT or HV substation, kVA |
| *TL* | Current of a DT or HV substation at the operation time, p.u. |
| *VD* | Voltage drop of a feeder, |