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Gamal A. Amhalhel, Piotr Furmański

Institute of Heat Engineering

PROBLEMS OF MODELING FLOW AND HEAT TRANSFER IN POROUS MEDIA

The paper presents an up-to-date overview of the theoretical and the engineering application aspects of the porous media. Problems which arise in modeling the flow and heat transfer phenomena by momentum and energy equations, and which are not present in the corresponding free-fluid flow, are discussed in more detail. Such problems arise from the higher order terms in the momentum equation and from thermal dispersion or local thermal equilibrium in the energy equation. The general class of boundary conditions which differentiate the flow phenomena from that of corresponding free-fluid flow have also been discussed in more detail. The channeling phenomena caused by the nonhomogeneity of porosity near the wall is presented and models which approximate the porosity variation are demonstrated. The boundary effects and their influence on the porous media have also been analyzed. The important aspects of engineering applications of the porous media and other applications of the macroscopic approach typical for porous media were also discussed. Finally conclusions corresponding to problems met in the modeling of flow and heat transfer in porous media are drawn.

NOMENCLATURE

a – specific surface area (fluid to solid contact) [m^2]
c_p – specific heat [$kJ/kg^{-1} \cdot K^{-1}$]
d_P – spherical particle diameter [m]
h – fluid to solid heat transfer coefficient [$kW \cdot m^{-2} \cdot K^{-1}$]
k – thermal conductivity [$kW \cdot m \cdot K^{-1}$]
K – permeability [m^2]
P – pressure [kN/m^2]
t – time [s]
T – temperature [°C]
v – velocity vector [$m \cdot s^{-1}$]
W – characteristic length of the porous bed (e.g. height and/or width and/or diameter [m])
r – coordinate vector [m]

Greek symbols

α – total diffusivity tensor $[m^2 \cdot s^{-1}]$
ε – the porosity of the porous media
μ – dynamic viscosity $[kg \cdot m^{-1} \cdot s^{-1}]$
ρ – density $[kg \cdot m^{-3}]$

Superscripts

α – phase
f – fluid phase
s – solid phase

Subscripts

e – effective
f – fluid phase
s – solid phase
sf – solid to fluid interface

Other symbols

$\langle \cdot \rangle$ – phase volume averaging
$\langle \cdot \rangle^{\alpha}$ – intrinsic volume phase averaging over the α phase
$\{\cdot\}$ – ensemble averaging

INTRODUCTION

The study of transport phenomenon in porous media have primarily been initiated by the research activity in geophysical systems and chemical engineering industry. But since the recognition of the engineering importance of the porous media a new area of research have been established which is equivalent to that of the free-fluid flow. Therefore, the subject of the transport phenomena through fluid-saturated porous media represents an important area of rapid growth in the contemporary heat transfer research. The study of transport phenomena in porous materials has attracted considerable attention, and has been motivated by a broad range of engineering applications including:

1. Agricultural applications: e.g. fermentation process in food industries, freeze drying of food products, grain storage, soil heating to increase the growing season.
2. Environmental applications: e.g. ground water pollution, ground water systems, storage of radioactive waste, water movement in geothermal reservoirs.
3. Industrial applications: e.g. artificial freezing of ground as a structural support and as a water barrier for construction and mining purposes, crude oil pro-

duction and recovery systems, porous radiant burners (PRBs), post accident heat removal (PAHR), solidification of castings, study of heat transfer phenomenon of buried electrical cables and transformer cables, fluidized bed combustion.

4. Thermal conversion and storage systems: e.g. catalytic reactors, geothermal systems, packed beds, fluidized bed, heat pipes, sensible, latent and thermochemical energy storage systems.

These diverse applications have made it essential for the thermal engineering community to focus its research interest on understanding the fundamentals of transport phenomena in porous media. Moreover, porous media provide a new approach to formulate transport phenomena that otherwise been modeled using the well known classical approaches.

1. STUDY OBJECTIVE

Porous media are proven to operate in most of the corresponding free fluid ranges. They can be used as an insulator (for all temperature ranges) and can be used as a heat transfer promoter for either sensible or latent heat transfer. This makes the porous media a kind of a super material which needs to be thoroughly studied in order to be fully utilized. Different transport models are available in the literature which are used to model energy and momentum transport in porous media. These models are phemonologically based upon governing equations which are inherited from the corresponding free-fluid flow. These inherited equations are modified by the use of the macroscopic concept for the porous media to meet its modeling demand. This brings new transport phenomena e.g. local thermal equilibrium and thermal dispersion in the energy equation and higher order terms and interfacial boundary problems in the momentum equation which are not present in modeling of the corresponding free-fluid flow and which need to be studied thoroughly experimentally and theoretically in order to know their role and contribution. Phase change, combustion and radiation heat transfer in porous media provide another class of problems which should be studied in detail in order to fully understand the thermal behavior of the porous media. Consideration of all of these phenomena and other modes of heat transfer makes mathematical modeling of transport phenomena in porous media very tedious and difficult to solve and simple modeling provide a challenge which need to be resolved. In spite of the extensive research activity, the study of the transport phenomena in porous media can be considered to be in the stages which only in future may provide a complete theory. This study is intended to

give a thorough overview of up-to-date state of the art in modeling the flow and heat transfer in porous media. Special attention is given to ambiguities found in modeling a well as to unsolved problems.

This study is primarily assembled to refer to:

1. Saturated porous media, i.e. the voids are completely filled with fluid.
2. Two phase flow system, i.e. solid and a single fluid phases only, however, the discussion is of great importance to multiphase flow porous system, for which the governing equations can be extended without any difficulty.
3. The porous media are assumed to be connected (i.e. the pores are assumed to be in communication between each other and so is the fluid flow through them).
4. Rigid (nondeformable) and fixed porous media are considered.
5. No heat generation is assumed.

2. CHARACTERISTIC FEATURES OF THE POROUS MEDIA

There is a large variety of natural and artificial porous materials encountered in practice, such as: soil, sandstone, limestone, ceramics, foam, rubber, bread, lungs, and kidneys. Aquifers (from where water is pumped), sand filters (for purifying water), reservoirs (which yield oil or gas), packed and fluidized beds in the chemical engineering and the root zone in agricultural industry may serve as additional examples of porous media domains. Common to all of these examples is the observation that part of the domain is occupied by a persistent solid phase, called the solid matrix. The remaining part is called the void space, which may be occupied either by a single phase fluid or a number of fluid phases [4]. In the last case each phase occupies a distinct separate portion of the void space. Therefore, a porous material may be regarded as a material where the solid portion is continuously distributed throughout the whole volume to form a loosely connected matrix and voids (pores) inside the solid matrix which is filled with fluids [4]. For a connected porous medium the porosity (ε) is defined as the fraction of the total volume of the medium that is occupied by void space. Therefore, ($1 - \varepsilon$) is the fraction that is occupied by the solid [4]. For a disconnected porous media (i.e. some of the pore space is separated from the remainder) an effective porosity, which is the ratio of the connected void to the total volume, has to be defined. A system of identical spherical particles of small radius and equal sizes affords a simple model of a porous body [26]. The spherical particles may be arranged in different ways, e.g. they may have a loose packing (cubic arrangement, $\varepsilon = 47{,}64\%$) or dense packing (hexagonal arrangement, $\varepsilon = 25{,}95\%$).

3. MICROSCOPIC AND MACROSCOPIC CONTINUUM DESCRIPTION OF POROUS MEDIA

Flow through porous media is characterized by complex geometry and by intimate contact between the solid matrix and the fluid. The extent of this contact depends on the characteristic features of the porous media, e.g. porosity. The microscopic nature of the flow is extremely complicated and random. The description the heat transfer inside the tortuous void passages in such a medium by taking into account the interaction between different mechanisms of transport does not appear to be possible analytically. Therefore, the solutions obtained by applying the classical models of fluid mechanics to both the fluid and solid phases are misleading and have no interest in practice. This is also due to the lack of information concerning the microscopic configuration of the interface boundaries and the fluid paths as it moves inside the porous media [4]. Therefore, a continuum model has to be defined in order to simulate mathematically the transport phenomena occurring in these media. A quasihomogeneous continuum model is defined for the porous media for which the phases are assumed to behave as a continuum that fills up the entire domain (i.e. each phase occupies its own continuum). The space occupied by these overlapping continue is referred to as the macroscopic space [4]. This continuum model of the porous media has the following advantages [4]:

1. It does not need the exact configuration of the interface boundaries to be specified; acquiring the knowledge of which is an invisible task anyway.
2. It describes processes occurring in porous media in terms of differentiable quantities, thus enabling the solution of problems by employing methods of mathematical analysis.

These advantages are at the expense of the loss of detailed information concerning the microscopic configuration of the interface boundaries and the actual variation of quantities within each phase [4]. But the macroscopic effects of these factors are still retained in the form of coefficients, whose structure and relationship to the statistical properties of the void space (or phase) configuration can be analyzed and determined.

4. METHODS OF AVERAGING

Two different averaging methods are broadly used in formulation of the macroscopic equations for heat and fluid flow in porous media. These are volume and ensemble averaging which will be demonstrated below. Because the volume averaging is more widely accepted, the governing equations for flow and

heat transfer in porous media, in this monograph will be demonstrated by using the volume averaging approach.

4.1. VOLUME AVERAGING

In the volume averaging approach, the mean values of phase variables are taken over an elementary volume (**EV**) centered at a point ($\boldsymbol{x}$) within the macroscopic space. The averaged values are referred to as macroscopic values of the considered variables. Therefore, the **phase volume averaging** of any arbitrary field quantity $\Omega_\alpha(\boldsymbol{x}, t)$ of the α-phase is obtained by performing an integration over the elementary volume (U_0) surrounding the point in question ($\boldsymbol{x}$) (Fig. 1), this is given by [4]

$$\langle \Omega_\alpha(\boldsymbol{x}, t) \rangle = \frac{1}{U_0} \int_{u_{0\alpha}(x,t)} \Omega_\alpha(\boldsymbol{x}', t:\boldsymbol{x})\, \mathrm{d}U_\alpha(\boldsymbol{x}') \tag{1}$$

This indicates that the total amount of the extensive quantity of the α-phase is averaged over the entire volume U_0 of the EV. When the volumetric averaging of the extensive quantity is taken over the volume $U_{0\alpha(x,t)}$, which is

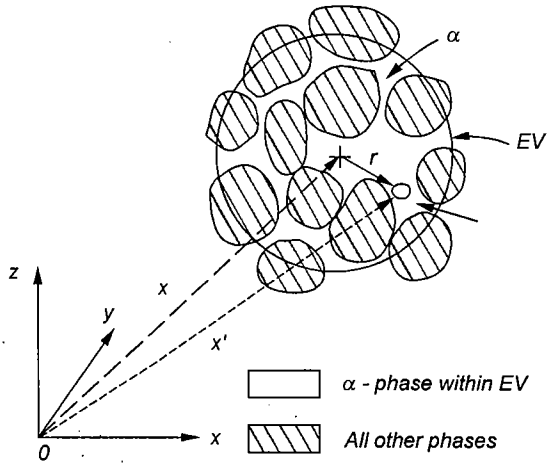

Fig. 1. Definition of the elementary volume (EV) [4]

occupied by the α-phase only, then the **intrinsic phase averaging** is defined, which is given by [4]

$$\langle \Omega_\alpha(\boldsymbol{x}, t) \rangle^\alpha = \frac{1}{U_{0\alpha(x,t)}} \int_{u_{0\alpha}(x,t)} \Omega_\alpha(\boldsymbol{x}', t:\boldsymbol{x})\, \mathrm{d}U_\alpha(\boldsymbol{x}') \tag{2}$$

Both averages are related by [4]

$$\langle \Omega(\boldsymbol{x}, t)_\alpha \rangle = \varepsilon_\alpha \langle \Omega_\alpha(\boldsymbol{x}, t) \rangle^\alpha \tag{3}$$

In general, the intrinsic phase volume averaging is used to develop the basic governing equations, particularly when there is more than one fluid phase and/or fluids saturated the porous media. Whereas the phase volume averaging is used when there is only one fluid phase saturated in the porous media. However, in this case intrinsic phase volume averaging may be used when two-energy equation model is applied.

4.2. THE ENSEMBLE AVERAGING

In the ensemble averaging approach the development of the macroscopic equations implies averaging over an ensemble of configurations. The configuration (A) is regarded as a specific distribution of the constituent in the porous media. Any change in the position of constituents with respect to the boundaries of the medium is recognized as a different configuration. This set of configurations constitutes an ensemble of realizations (**sample space**). If a random probabilistic function $p(A)$ is defined for this sample space, then the ensemble average of any function $\Omega_\alpha(t, \boldsymbol{x})$ can be written as [14]

$$\{\Omega_\alpha(t, \boldsymbol{x})\} = \int \Omega_\alpha(t, \boldsymbol{x}/A)\, p(A)\, \mathrm{d}A \tag{4}$$

This ensemble averaging approach has the following advantages [3]:

1. It conceptually defines the properties in space at a given point, without having to define a volume over which these properties must be integrated.
2. It provides means for studying the inherent heterogeneity and variability of these properties in space, and for evaluating the uncertainty of any method of estimation of their values.
3. It enables to introduce information about the statistical description of the microstructure of the porous media in an easy way.

5. MOMENTUM TRANSPORT

Several momentum flow models have been proposed to model the fluid flow through a porous media. The aim was to develop a macroscopic equation which matches the empirical observations and which converges to the corresponding free-fluid flow when the porosity of the porous media tends towards unity. Problems of momentum transport in porous media arise from the fact

that mismodeling of the velocity distribution will strongly affect the temperature distribution. Therefore, there is a need to focus on these momentum models before considering any model describing the temperature distribution.

The first momentum equation which describes the transport phenomenon of the fluid flow through porous media were deduced experimentally in 1856 by Darcy. Since then several flow models which are based on phenomenological observations rather than analytical approaches have been developed to match the same purpose.

5.1. DARCY FLOW MODEL

The overwhelming majority of existing studies pertinent to the phenomena of heat and fluid flow through porous media makes use of the Darcy Flow Model (**Darcy law**). This model features a linear momentum equation, which states (in the absence of gravity force) that the volumetrically averaged velocity in any direction is proportional to the pressure gradient in that direction. Mathematically it can be written as [18]

$$\langle \boldsymbol{v} \rangle = \frac{\boldsymbol{K}}{\mu_f} \left(\langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle \right), \tag{5}$$

where $\boldsymbol{G} = \langle \rho_f \rangle \boldsymbol{g}$ is the body force due to gravity, where: $\boldsymbol{g}$ is the gravitational force vector, P_f, μ_f and ρ_f are the pressure, viscosity and density of the fluid phase, respectively. The hydraulic conductivity (**permeability**) K can be best understood as an analogue to the thermal conductivity. For example, for a porous media, composed of identical spherical particles of diameter d_p, the value of the permeability can be calculated using the Cozeny-Karman model given by [8]

$$K = \frac{d_p \varepsilon^3}{36\, C(1 - \varepsilon)} \tag{6}$$

The value of the constant (C) depends on the type of packing of the porous bed.

Darcy law represents the resistance which is the friction offered by the solid particles to the fluid flow [20]. Since the development of Darcy flow model several attempts have been made to derive it analytically starting from the Navier-Stokes equation. These derivations show that the Darcy flow model is restricted to flow in which the **viscous forces** dominate over the **inertia forces**. Assuming an appropriately defined Reynolds number, Re_{d_v} (based on the average pore diameter d_v), the general criteria for the applicability of the Darcy flow model is [31]

$$\mathrm{Re}_{d_v} \equiv \left| \rho_f \langle \boldsymbol{v} \rangle d_v / \mu_f \right| < 1. \tag{7}$$

One of the main advantages of the Darcy law is that it makes the momentum equation linear and thus removes a great amount of difficulty in solving the governing equations. However, because the Darcy flow model is of order one less than the Navier-Stokes equation, the no-slip hydrodynamic boundary condition cannot be applied and therefore the maximum velocity is predicted to occur at the impermeable surface.

5.2. FORCHHEIMER-DARCY FLOW MODEL

As mentioned above, deviations from Darcy law are known to occur when Reynolds number based on the mean pore diameter exceeds unity. This limitation encouraged Forchheimer to propose a velocity square term in addition to the Darcy term to account for the inertia effects in the pressure drop, as the fluid makes its way through the porous media. Forchheimer-extended Darcy flow model is considered to be the earliest non-Darcian flow models. Mathematically it can be written as [8]

$$\frac{\mu_f}{K}\langle \boldsymbol{v} \rangle + \frac{\langle \rho_f \rangle b}{K} |\langle \boldsymbol{v} \rangle| \langle \boldsymbol{v} \rangle = \langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle, \tag{8}$$

where b is the Forchheimer inertial coefficient. For porous media composed of identical spherical particles it is given by [8]

$$b = 0{,}0117\, d_p / (1 - \varepsilon) \tag{9}$$

5.3. BRINKMAN-DARCY FLOW MODEL

Brinkman argued that the momentum equation for the porous media must reduce to the viscous flow limit and advocated that classical frictional terms should be added to Darcy law as the permeability is high. Brinkman considered the viscous force exerted on a dense swarm of particles by a fluid flowing through them. The force acting on a single particle in a slow stream is calculated from the Stokes flow velocity field, while the flow through a swarm of particles is described by the Darcy flow model for flow through a porous media. Brinkman reasoned that the force on a particle situated in a swarm of particles could be calculated as if it were a solid particle imbedded in a porous medium. Therefore, Brinkman represented the porous medium by modifying Stokes equation, adding a Darcy resistance term to it, so that the effect of the other particles is treated in an average sense [27]. Mathematically, the Brinkman-extended Darcy flow model can be written as [30]

$$\frac{\mu_f}{K}\langle \boldsymbol{v} \rangle = \langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle + \mu_e \nabla^2 \langle \boldsymbol{v} \rangle \tag{10}$$

The Brinkman-extended Darcy flow model removes the deficiency of Darcy law in the sense that it is applicable to media with high permeability and can account for all boundary conditions at a solid surface or at a fluid interface. The use of the Brinkman-extended Darcy flow model has been hampered, because of the uncertainty associated with the coefficient (μ_e), which acts as an effective viscosity. In the core of flow through a porous media the effective viscosity can be either greater or smaller than the fluid viscosity. The usual practice is to assume that ($\mu_e = \mu_f$). Nield [30] argued that the value (μ_e/μ_f) was a function of porosity where its value rose slightly above one as the porosity decreased from unity, and attained a maximum at about $\varepsilon = 0{,}8$ and decreased rapidly when $\varepsilon < 0{,}7$. The effective viscosity has also been determined experimentally [16] for a steady flow through a wall-bounded porous media. Its value has been shown to lie within the range of $\mu_e = (5{,}1 \text{ to } 10{,}9)\,\mu_f$.

5.4. DARCY-BRINKMAN-FORCHHEIMER FLOW MODEL

During further studies of fluid flow through porous media, the Forchheimer--Darcy and Brinkman-Darcy extended flow models were joined together and generalized as the Darcy-Brinkman-Forchheimer flow model (abbreviated DBF Flow Model) which can be written in the following form [24]

$$\frac{\langle \rho_f \rangle}{\varepsilon}\left[\frac{\partial \langle \boldsymbol{v} \rangle}{\partial t} + \frac{(\langle \boldsymbol{v} \rangle \cdot \nabla)\langle \boldsymbol{v} \rangle}{\varepsilon}\right] = \langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle + \mu_e \nabla^2 \langle \boldsymbol{v} \rangle + - \mu_f \frac{\langle \boldsymbol{v} \rangle}{K} - \langle \rho_f \rangle \frac{b\,|\langle \boldsymbol{v} \rangle|\,\langle \boldsymbol{v} \rangle}{K} \tag{11}$$

It has been found that the ratio of the permeability to the kinematic viscosity (K/υ_f) interpreted as the **viscous time** is very small (for example, a typical value of the viscous time for water flowing through a packed bed of sand is about 10^{-5} s). Therefore, the time derivative term in the above equation can be neglected [31]. However, the presence of the time derivative is necessary when stability of the porous bed is concerned as shown by Georgiadis and Catton [15]. The scaler product of the velocity and its gradient known as the **convective term** is due to Wooding [23], who introduced this term based on analogy with the Navier-Stokes equation for a pure fluid medium [23]. However, this term also contributes to the inertia effects it is clear that its reasoning is different from that of the Forchheimer inertia term. Although it is important at high velocity and/or high porosity media its role is not as clear as that of the Forchheimer inertia term and can be best understood as to that of the corresponding free-fluid flow. Its inclusion makes the solution of the momentum equation more difficult. And so forth the discussion of the inertia effect will be related to the Forchheimer inertia term only. Choi and Kulacki [10]

have studied the influence of the inertial and viscous terms on the velocity profile. Their results (Fig. 2) show that the viscous term contributes mostly to the deviation from the result obtained when Darcy's flow model is used, while the contribution of the inertia term is negligible.

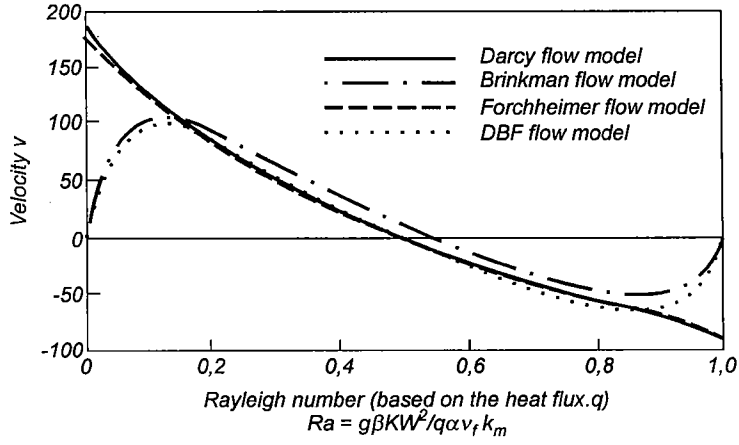

Fig. 2. Effects of inertia and viscous terms on the velocity distribution for natural convection in a porous layer [10]

Starting from the Navier-Stokes equations and utilizing the volume averaging technique, another form of the generalized flow model was proposed by Hsu and Cheng [20]

$$\frac{\langle \rho_f \rangle}{\varepsilon}\left[\frac{\partial \langle \boldsymbol{v} \rangle}{\partial t} + \frac{(\langle \boldsymbol{v} \rangle \cdot \nabla)\langle \boldsymbol{v} \rangle}{\varepsilon}\right] = \langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle + \mu_f \nabla^2 \langle \boldsymbol{v} \rangle + \boldsymbol{B} \tag{12}$$

where $\boldsymbol{B}$ is the total drag force per unit volume (body force) due to the presence of the solid particles. A closure scheme were developed to determine the drag force due to the solid particles and it is given by [20]

$$\boldsymbol{B} = -\left[\mu_f \frac{\langle \boldsymbol{v} \rangle}{\boldsymbol{K}} + \langle \rho_f \rangle \frac{b \langle \boldsymbol{v} \rangle |\langle \boldsymbol{v} \rangle|}{\sqrt{\boldsymbol{K}}}\right]. \tag{13}$$

Therefore, equation (12) can be rewritten as

$$\frac{\langle \rho_f \rangle}{\varepsilon}\left[\frac{\partial \langle \boldsymbol{v} \rangle}{\partial t} + \frac{(\langle \boldsymbol{v} \rangle \cdot \nabla)\langle \boldsymbol{v} \rangle}{\varepsilon}\right] = \langle \boldsymbol{G} \rangle - \nabla \langle P_f \rangle + \mu_f \nabla^2 \langle \boldsymbol{v} \rangle + - \left[\mu_f \frac{\langle \boldsymbol{v} \rangle}{\boldsymbol{K}} + \langle \rho_f \rangle \frac{b \langle \boldsymbol{v} \rangle |\langle \boldsymbol{v} \rangle|}{\sqrt{\boldsymbol{K}}}\right]. \tag{14}$$

In eq. (11) the inertia term is proportional to the permeability, whereas in

Forchheimer term in eq. (14) is more pronounced when inertia effect is considered. As the permeability $K \to \infty$ and consequently the porosity is unity, therefore the far right-hand sides of both eq. (11) and eq. (14) vanish and give access to the completely vectorial Navier-Stokes equation for Newtonian constant property flow. Therefore, the Darcy flow model provides the upper limit to the resistance offered by the porous matrix, while the modified Navies-Stokes equations (11 and 14) provide the lower limit.

As noted, Darcy model is the simplest one to use, but is limited to either slow flow and/or low porosity porous media. However, its main limitation is that the no-slip boundary condition cannot be imposed. Extension of the Darcy Flow model to high velocity and/or high porosity media provided by the Forchheimer-Darcy flow model still retains these deficiencies. The Brinkman-Darcy model removes the main deficiency inherited from the use of the Darcy model and enables the no-slip condition to be imposed, however it is not suitable for high velocity and/or high porosity media. Consideration of the inertia effects results in the DBF flow model which is a more general flow model, eq. (11) and eq. (14), but its solution is not an easy task.

The limitations of these models are justified since they are formulated on phenomenological observations. Therefore, the choice of which flow equation to be used to model the flow in porous media, is a problem which need to be resolved. In general it depends upon the physical model under study and the assumptions made.

6. ENERGY TRANSPORT

The knowledge of the heat transfer characteristics in porous media is of great importance in many applications. For example, in chemical reactor design it is important to know the thermal transport characteristics of the porous media in order to make accurate predictions of the variation in reaction rate caused by the inlet temperature disturbances. Temperature histories are also important for the design of packed bed thermal storage system. Therefore, it is desirable to have the proper values of the heat transfer coefficient and effective thermal conductivity, so that the time required to heat up the solid particles can be estimated [17]. Two different macroscopic descriptions: heterogeneous and homogeneous are available in the current literature.

6.1. HETEROGENEOUS (MIXTURE) FORMULATION OF HEAT FLOW IN POROUS MEDIA

In this model the internal state of the porous media is characterized by two co-existing temperature fields associated with the solid and fluid phases, res-

pectively. Because of the presence of two different temperatures, the heat transfer in the porous media is accompanied by a heat exchange between the two phases. For the heterogeneous description, the energy equation falls basically into three categories as classified by Wakao et al. [44]. Therefore, if an **intrinsic phase volume** averaged temperature for the f-phase (fluid) and the s-phase (solid) are denoted by $\langle T_f \rangle^f$ and $\langle T_s \rangle^s$, respectively, then the resulting macroscopic models for the energy equation are [44]:

(1) SCHUMANN MODEL

In this model the heat conduction in both phases is neglected and the energy equation can be written as:
– for the fluid phase

$$\varepsilon (\rho c_p)_f \frac{\partial \langle T_f \rangle^f}{\partial t} + \varepsilon (\rho c_p)_f \langle \boldsymbol{v}_f \rangle^f \cdot \nabla \langle T_f \rangle^f = h_{sf} a_{sf} \left[\langle T_s \rangle^s - \langle T_f \rangle^f \right] \tag{15.1}$$

– for the solid phase

$$(1 - \varepsilon)(\rho c_p)_s \frac{\partial \langle T_s \rangle^s}{\partial t} = h_{sf} a_{sf} \left[\langle T_s \rangle^s - \langle T_f \rangle^f \right] \tag{15.2}$$

(2) CONTINUOUS SOLID PHASE (C–S) MODEL

In this model heat conduction in each phase is included and, if the effective thermal conductivities of the fluid and solid phases are denoted by k_e^f and k_e^s, respectively, then the energy equation may be written as:
– for the fluid phase

$$\varepsilon (\rho c_p)_f \frac{\partial \langle T_f \rangle^f}{\partial t} + \varepsilon (\rho c_p)_f \langle \boldsymbol{v}_f \rangle^f \cdot \nabla \langle T_f \rangle^f = \nabla \cdot \left(k_e^f \cdot \nabla \langle T_f \rangle^f \right) + h_{sf} a_{sf} \left(\langle T_s \rangle^s - \langle T_f \rangle^f \right) \tag{16.1}$$

– for the solid phase

$$(1 - \varepsilon)(\rho c_p)_s \frac{\partial \langle T_s \rangle^s}{\partial t} = \nabla \left(k_e^s \cdot \nabla \langle T_s \rangle^s \right) + h_{sf} a_{sf} \left(\langle T_s \rangle^s - \langle T_f \rangle^f \right) \tag{16.2}$$

It should be noted that the effective thermal conductivity of the fluid phase k_e^f in eq. (16.1) also includes a thermal dispersion effect, which is discussed

(3) DISPERSION-CONCENTRIC (D–C) MODEL

This model also uses an equation for the average fluid temperature, but it couples this equation to the energy equation for the heat conduction in a single particle. The particle is usually treated as being spherical with uniform flux boundary condition along the surface [44]. The energy equation for this model can be written as:

– for the fluid phase

$$\frac{\partial \langle T_f \rangle^f}{\partial t} + \langle \boldsymbol{v} \rangle^f \cdot \nabla \langle T_f \rangle^f = \alpha_{ax} \nabla^2 \langle T_f \rangle^f + \frac{h_{sf} a_{sf}}{\varepsilon (\rho c_p)_f} \left(\langle T_s \rangle^s - \langle T_f \rangle^f \right) \tag{17.1}$$

– for the solid phase

$$\frac{\partial \langle T_s \rangle^s}{\partial t} = \alpha_s \left(\frac{\partial^2 \langle T_s \rangle^s}{\partial r^2} + \frac{2}{r} \frac{\partial \langle T_s \rangle^s}{\partial r} \right) \tag{17.2}$$

with a constraint that

$$\text{at } r = R, \; k_s \left(\frac{\partial \langle T_s \rangle^s}{\partial r} \right) = h_{sf} \left(\langle T_s \rangle^s - \langle T_f \rangle^f \right) \tag{17.3}$$

The fluid axial thermal diffusivity α_{ax} (i.e. along the bulk flow direction) includes *a* thermal dispersion term dependent on the bulk velocity of the flow.

For a porous media composed of identical spherical particles, the axial thermal diffusivity α_{ax} for the original D–C model was given as

$$\alpha_{ax} = (0.6 - 0.8)\alpha_f + 0.5 d_p \langle \boldsymbol{v} \rangle \tag{18}$$

whereas for the modified D–C model was given as [44]

$$\alpha_{ax} = \frac{k_m}{\varepsilon (\rho c_p)_f} + 0.5 d_p \langle \boldsymbol{v} \rangle \tag{19}$$

where $k_m = \varepsilon k_f + (1 - \varepsilon) k_s$ is the upper bound to the axial stagnant thermal conductivity of the porous medium and α_f is the fluid phase thermal diffusivity.

The specific surface area a_{sf} and the fluid to solid heat transfer coefficient h_{sf} for a porous media composed of identical spherical particles are given by eqs. (20) and (21), respectively [1]

$$a_{sf} = 6(1 - \varepsilon)/d_p \tag{20}$$

$$h_{sf} = k_f \left[2 + 1.1 \, \mathrm{Pr}^{1/3} \left(\rho_f \langle \boldsymbol{v} \rangle d_p / \mu_f \right)^{0.6} \right] / d_p \tag{21}$$

where $\mathrm{Pr} = \mu_f c_{pf} / k_f$ is the Prandtl number and k_f is the fluid thermal conductivity.

Of the three models, the Schumann model is the simplest one to analyze since the heat conduction is neglected in both the fluid and solid phases. Moreover, the continuous solid phase (C–S) model is much easier to analyze than the dispersion-concentric (D–C) model since both the fluid and solid phase temperature are function of the bed position and time. The dispersion concentric (D–C) model is the most complicated as additional variables associated within the individual particles are included in it.

A slightly different forms of macroscopic energy models are also available in the literature, the difference is being due to the constitutive equations which are used for the closure scheme. These macroscopic models require the introduction of length-scale and time-scale constraints which are based on pore--scale and local-scale characteristics length, and on the physical parameters describing the porous media. If these constraints are not satisfied, then memory effects have to be included in the analysis as suggested in [2].

6.2. HOMOGENEOUS (SINGLE EQUATION) FORMULATION OF HEAT FLOW IN POROUS MEDIA

By tracing the effect of a pulse disturbance (Δ) on the temperature of both the fluid and solid phases (Fig. 3), a considerable difference in temperatures of both the fluid and the solid phases can be observed at the initial stage after

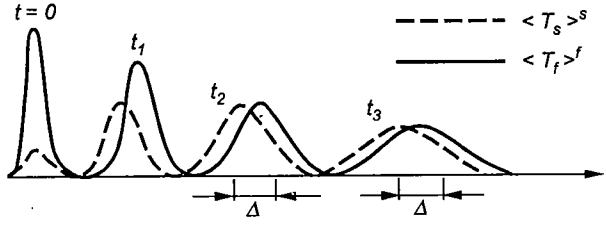

Fig. 3. Response of fluid and solid phases to a pulse disturbance in temperatures at time $t = 0$ [25]

applying the disturbance. As the time increases, the difference in the temperatures dies away and both phases converge to the same temperature. Therefore, if the response time for the local heat transfer between the fluid and the solid is several orders of magnitude smaller than the characteristic time for variation of the mean temperature of the phases, then the fluid and solid phases temperatures can be assumed to be the same at each point, i.e. the local thermal equilibrium (abbreviated as **LTE**) holds [25]. This makes a heat transfer model between the fluid and solid phases unnecessary and provides an unambiguous definition of the temperature at any point. Therefore, under the assumption of the **LTE** only a single energy equation is required to describe the

porous media. Mathematically the single energy equation model can be written as [34]

$$\left[\varepsilon + (1-\varepsilon)\frac{(\rho c_p)_s}{(\rho c_p)_f}\right]\frac{\partial \langle T\rangle}{\partial t} + \langle \boldsymbol{v}\rangle \cdot \nabla\langle T\rangle = \nabla\cdot[\alpha \cdot \nabla\langle T\rangle] \qquad (22)$$

Wong and Dybbs [46] have experimentally investigated the **LTE** for different boundary conditions. Their results show that **LTE** holds for flow rate where the Reynolds number Re_{d_y}, based on the pore diameter, is smaller than 10. However, Amiri and Vafai [1] pointed out that the Darcy number (Da), which is defined as the ratio of the permeability to the characteristic length squared (i.e., $Da = K/W^2$, where W is the width and/or height and/or diameter of the porous bed), is the most influential parameter in determining the validity of the **LTE**. Therefore, it is assumed that the **LTE** becomes less pronounced as both the particle Reynolds number and Darcy number increase. Whitaker [45] reported that certain additional constraints must be fulfilled for the homogenous model to be used with confidence. These constraints should be however treated as estimates and may only be useful to provide a guideline for detailed studies of both models before any final choice can be made.

6.3. THERMAL DISPERSION

In the single energy equation model or in the double equation model (i.e. C–S and D–C models) the effective thermal diffusivity tensor (α), the fluid thermal conductivity k_e^f and the axial thermal diffusivity α_{ax} include a thermal transport term due to **dispersion**. Dispersion here is understood in the Taylor's sense and it is proven from two mechanisms with distinct phenomenology [4]. The first is due to **heat conduction**, which is present in almost every transport process, and it depends on the local temperature gradient in the medium. Fluid flow causes local deformations of the temperature field, thus increasing the temperature gradient and enhancing heat conduction (Fig. 4). **Mechanical dispersion** is the second known source of dispersion. It

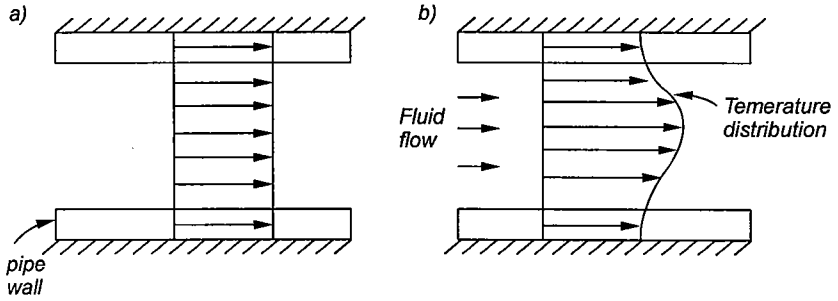

Fig. 4. Effect of molecular thermal dispersion to the over-all heat transfer: a) stagnant fluid, b) moving fluid

results from the mixing of the local fluid streams as the fluid follows via **tortuous paths** around the solid particles. In general the heat flux in porous media can be expressed by three terms. The first term accounts for the pure conduction, the second term is connected with dispersion and the third one corresponds to convection. This can be written as

$$\langle \boldsymbol{q} \rangle = -\boldsymbol{k}_e \cdot \nabla \langle T \rangle - \boldsymbol{D}(\langle \boldsymbol{v} \rangle) \cdot \nabla \langle T \rangle + \langle \rho c_p \rangle_f \langle \boldsymbol{v} \rangle \langle T \rangle \tag{23}$$

where $\boldsymbol{q}$ is the heat flux vector [$kW \cdot m^{-2}$], $\boldsymbol{k}_e$ is the local thermal conductivity tensor and $(\boldsymbol{D}(\langle \boldsymbol{v} \rangle))$ denotes the dispersion tensor which accounts for mechanical dispersion and heat conduction enhanced by fluid flow.

It should be noted that the mechanical dispersion (Fig. 5) is similar to the thermal eddy diffusivity in turbulence and is a direct result of the transport occurring at the length scale smaller than the selected local elementary volume used in the averaging [34].

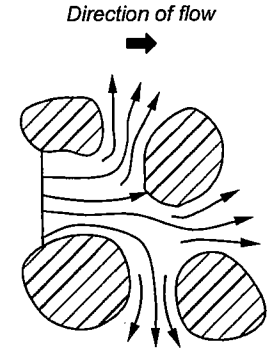

Fig. 5. Dispersion due to mechanical spreading

As with the eddy diffusivity in plain media, the dispersion in porous media is also anisotropic because of its dependence on the flow direction (**the flow anisotropy**) and on the presence of anisotropy in the solid phase direction (**structural anisotropy**) [34]. Therefore, in order for the thermal dispersion to be considered, the energy equation should include a total diffusivity tensor (α), the latter is assumed to be a superposition of a local effective thermal conductivity tensor $\boldsymbol{k}_e$, which represents the local volume averaged molecular conduction through both phases (**stagnant term**), and a tensor $\boldsymbol{D}^d$ which represents the local volume-averaged dispersion (**dispersive term**). Mathematically it can be written as [34]

$$\alpha = \frac{\boldsymbol{k}_e}{(\rho c_p)_f} + \varepsilon \boldsymbol{D}^d \tag{24}$$

Since dispersion results from the simultaneous presence of a temperature and a velocity gradient within a pore and because of the anisotropy and nonuniformity of the solid matrix structure both $\boldsymbol{k}_e$ and $\boldsymbol{D}^d$ are in general anisotropic and nonuniform [34]. The thermal dispersion contributes to the increase in the rate of heat transfer. Moreover, the dispersion concept helps to explain the difference often observed between transport parameters measured along and across the principle direction of fluid flow in simple geometries [37].

7. POROSITY VARIATION

In many applications, e.g. fixed bed catalytic reactors, packed beds heat exchangers, the constant porosity assumption does not hold because of the presence of an impermeable wall (Fig. 6). The nonhomogeneity due to the nonuniformity of porosity can be found particularly in packed bed of spheres. This is because, for perfect spheres, only point contact with the solid boundary can exist. The porosity should thus approach a limiting value of unity at the boundary. Therefore, there is a need to focus on the variable porosity effects on heat transfer in the vicinity of an impermeable boundary. This region close to the impermeable boundary is of practical importance since for most applications the heat flux at the boundary is of practical interest. The experimental results demonstrate that the porosity increases from an average value of ε_∞ (in the core region) to nearly unity at the wall. The variation in the porosity (Fig. 7) usually takes the form of a damped oscillatory function with the oscillation damped out at four to five sphere diameters (dp) away from the wall [9, 43].

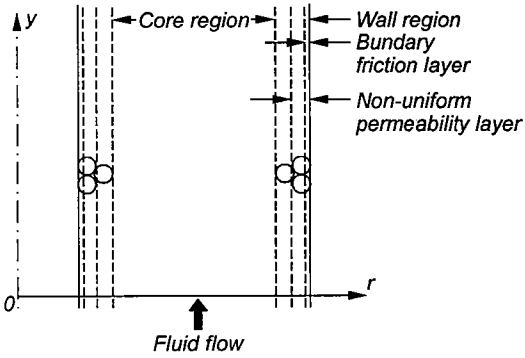

Fig. 6. Regions of variable porosity [9]

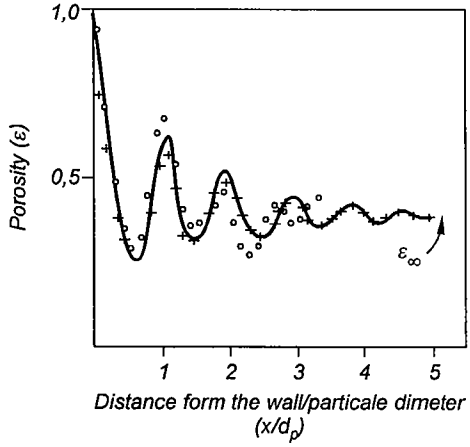

Fig. 7. Variation of porosity in a bed of spheres [43]

The nonhomogeneous effect of porosity with the no-slip boundary condition causes the velocity profile to peak near the solid surface resulting in the **channeling phenomenon** [39]. Channeling is therefore referred to the occurrence of maximum velocity in a region close to an external boundary. The inclusion of the variable porosity profile and the no-slip boundary condition dramatically alters the Darcean velocity profile. Another parameter of significant importance which also causes porosity variation and therefore contributes in the channeling effect is the dimensionless particle diameter ($\gamma = d_p/W$: which is the ratio of the spherical particle diameter to the system characteristic length) [9]. The effect of the dimensionless particle diameter for a fully developed forced convection flow through an annular packed-sphere bed is shown in Fig. 8. It can be observed that its effect is similar to the porosity effect since both contribute to the wall effect.

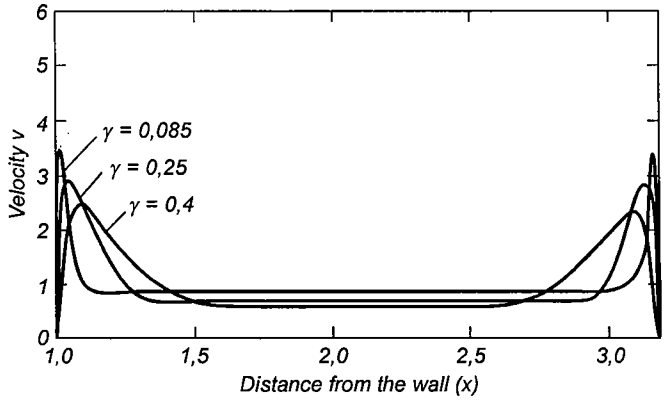

Fig. 8. Influence of dimensionless particle diameter (γ) on the channeling effect [9]

Different variable porosity models are available in the literature which approximate the porosity variations near the porous medium boundary:

(1) EXPONENTIAL-DAMPED MODEL

$$\varepsilon = \varepsilon_\infty \left[1 + c_1 \exp\left(-c_2 Z/d_p\right)\right] \tag{25}$$

where c_1, c_2 are constants, which depend on the packing of the spherical particles near the solid boundary. The symbol Z denotes the normal distance from the wall for either a rectangular or a cylindrical packed beds [9, 39]. For an annulus packed bed it is assumed as $Z = |r_o - r|$ for $Z/2 \le r \le r_o$ and

$Z = (r - r_i)$ for $r_i \leq r \leq Z/2$ where r_i and r_o are the inner and outer radius of the annular packed bed, respectively [9].

(2) EXPONENTIAL-DAMPED MODEL FOR CYLINDRICAL BEDS

$$\varepsilon = \varepsilon_\infty + (\varepsilon_o - \varepsilon_\infty) e^{-N(r-r_o)/d_p} \tag{26}$$

where ε_o is the porosity at the wall, N is a constant and, r and r_o are the radial distances normal to the wall and the radius of the cylindrical packed bed, respectively [19].

(3) OSCILLATORY-DAMPED MODEL

$$\varepsilon = \varepsilon_\infty [1 + a_1 \exp(-a_2 x/d_p) \cos(2\pi x/d_p)] \tag{27}$$

where a_1 and a_2 are the constants and x is the normal distance from the wall [11].

(4) MEAN POROSITY MODEL

$$\varepsilon_m = \varepsilon_\infty [1 + 0.5(d_p/W)^3] \tag{28}$$

In all of the above models ε_∞ is the porosity far away from the wall (or in the core region) [13].

8. INTERFACIAL BOUNDARY CONDITIONS

The momentum and the energy equations for the porous media must be supplemented with boundary conditions in order to formulate a full mathematical model for flow and heat transfer in porous media. Solution of this mathematical model thus depends strongly on the type of the interface separating the porous media from its surroundings.

In general, the interface can be porous/impermeable boundary, porous/permeable (free fluid) interface, and porous/porous media interface, which are

shown in Fig. 9. In the following section the problems arising from the interfacial boundary conditions for either the momentum or the energy equation will be discussed.

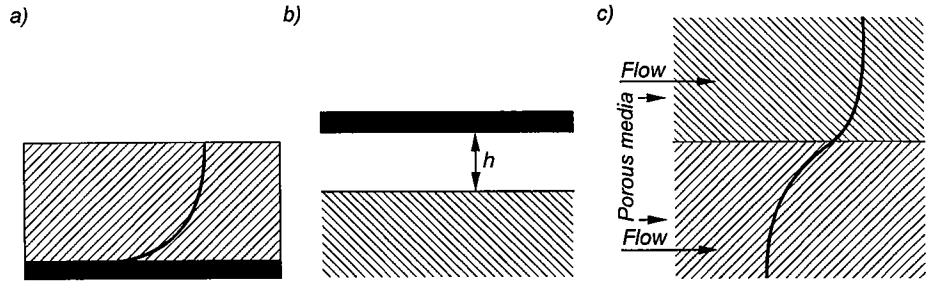

Fig. 9. The general class of the interfacial problems: a) porous/impermeable boundary, b) porous/-fluid and c) porous/porous interfaces

8.1. INTERFACIAL BOUNDARY CONDITIONS FOR THE MOMENTUM EQUATION

In general, the interfacial flow for porous media does not pose any difficulty when the Brinkman and/or the DBF flow models are used, since they are of the same order of magnitude as the Navier-Stokes equation. However, they provide a challenge when either Darcy or Forchheimer flow models are applied. This is because of the fact that these models are one order of magnitude smaller than the Navier-Stokes equation and therefore, the no-slip velocity on an impermeable boundary cannot be imposed. Moreover, the properties continuation across a permeable boundary (porous or free fluid interface) cannot be satisfied.

(1) POROUS/ IMPERMEABLE BOUNDARY

For this case of the interface, viscous effects are confined in such a thin layer that experimental observations are very difficult. For this reason most of the existing experimental information have been primarily limited to gross effects, such as pressure drop and flow rate correlation. In most flow experiments the viscous effects have indeed been found insignificant. Thus assumption of the no-slip tangential velocity has little effect on the pressure drop, because the major part of the pressure drop is due to the porous media alone. For low porosity media the thickness of the viscous layer which is caused by the no-

-slip tangential velocity can be safely neglected and the Darcy flow model is well satisfied. However, for high porosity media the boundary effects can be quite important. For this situation the Darcy and/or Forchheimer-extended Darcy flow models will incorrectly over-predict the flow rate [40, 42]. A comparison of the velocity profiles at mid-height of an enclosure obtained from the Darcy flow model and from utilizing the Brinkman-extended Darcy flow model are shown in (Fig. 10).

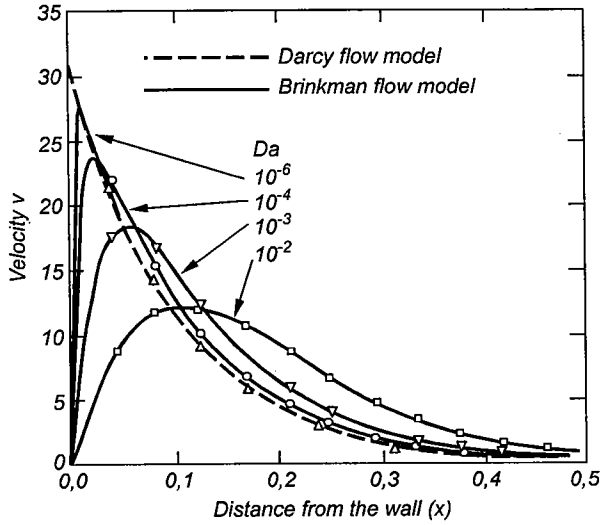

Fig. 10. Comparison of velocity profiles for Darcy and Brinkman flow models [42]

(2) POROUS/FLUID AND/OR POROUS/POROUS INTERFACES

Flow or recharge may occur when either a porous medium (with the same or different permeability) or a free fluid constitute an interfacial boundary with the porous layer of interest and also when a fluid flows over or across a porous medium. Despite the fact that there is a discontinuity of material properties at the interface region, the properties of the fluid flow normal and tangential to the interface need to satisfy conditions of continuity across the interface. The problems arising from such interfacial boundary conditions for the different momentum transport models will be discussed below.

(A) DARCY AND/OR FORCHHEIMER FLOW MODELS

The fact that the Darcian velocity is defined as an average velocity flux rather than a velocity at a point as in the classical viscous flow theory, and because

of the slip boundary condition, a kinematic boundary condition cannot be imposed. Therefore, a dynamic boundary condition can be employed for which the velocity distribution can be determined from the pressure distribution on each side of the interface. Experimental work [5] shows that when a viscous fluid flows over a porous media the tangential stress moves the fluid close to the interface with a tangential velocity $u_{f,i}$ slightly greater than the velocity of the fluid in the bulk of the porous media (i.e. the tangential component of the Darcian velocity vector $\langle u \rangle$, which is parallel to the interface). Beavers & Joseph [5] have measured this difference by confining the fluid above the porous surface to a narrow channel of height (h) (Fig. 11). A slightly greater flow would occur if these two velocities are equal. Beavers & Joseph have assumed that the difference $u_{f,i} - \langle u \rangle$ is related to the surface drag $\left[\mu_f (du_f/dy)\right]_i$ on the fluid side by the following relation [5]

$$\left[du_f/dy\right]_i = \left(\beta_{hd}/K^{1/2}\right)\left(u_{f,i} - \langle u \rangle\right) \tag{29}$$

where β_{hd} is the hydrodynamic slip coefficient, $u_{f,i}$ is the tangential fluid velocity component along the x-direction, and the subscript i indicates that the slip (interface) velocity is evaluated at the interface.

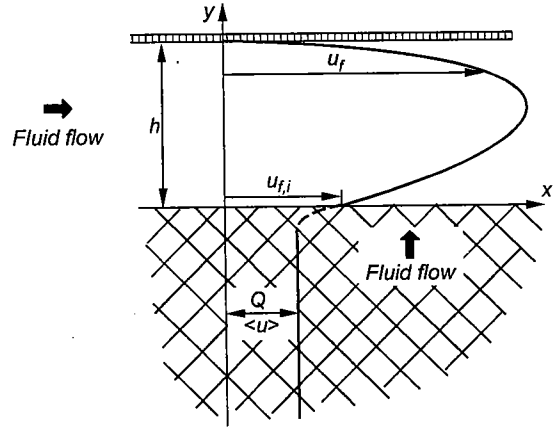

Fig. 11. Velocity profile for a flow in a horizontal channel formed by a porous/fluid interface [5]

The hydrodynamic slip coefficient depends on the bulk direction of flow (with respect to the interfacial plane), the channel height, the porosity, the value of the Darcian velocity and the porous media structure [32].

(B) BRINKMAN AND/OR DBF FLOW MODELS

Since these flow models are of the same order of magnitude as that of the Navier-Stokes equation, therefore boundary conditions are needed to match the

governing equations at the porous and the fluid interface. For a porous/fluid interface these matching conditions for a 2-dimensional flow and with reference to the coordinate axis of Fig. 11 can be written as [6]

$$\langle \boldsymbol{v} \rangle = \boldsymbol{v}_f \tag{30}$$

$$\mu_e (\partial \langle u \rangle / \partial y + \partial \langle v \rangle / \partial x) = \mu_f (\partial u_f / \partial y + \partial v_f / \partial x) \tag{31}$$

$$-\langle P_f \rangle + 2\mu_e \partial \langle v \rangle / \partial y = -P_f + 2\mu_f \partial v_f / \partial y \tag{32}$$

where $\langle \boldsymbol{v} \rangle$, $\boldsymbol{v}_f$, $\langle \boldsymbol{P}_f \rangle$ and $\boldsymbol{P}_f$ are the Darcian velocity vector, the fluid velocity vector and the pressures on the porous medium and fluid side respectively, and the symbols $\langle u \rangle$, $\langle v \rangle$ and u_f, v_f denote the tangential and normal components of the Darcian velocity vector and the fluid velocity vector relative to the interface on the porous and the fluid sides, respectively.

It should be noted that for a porous/porous interface the interfacial boundary conditions can be written in a similar manner.

8.2. INTERFACIAL BOUNDARY CONDITIONS FOR THE ENERGY EQUATION

For a porous/impermeable interface the nonhomogeneity of the porous media caused by the porosity variations contribute significantly to the variation in the effective thermal conductivity and thermal diffusivity of the porous media particularly near the boundary. This variation is difficult to estimate and greatly complicates the solution of the energy equation. An alternative simplified approach is to assume that the thermal properties are constant (and identical to the interior of the medium), and to apply a slip boundary condition at the porous/impermeable interface. Sahraoui and Kaviany [33] have developed the following slip boundary condition

$$\left. \frac{dT}{dy} \right|_i = \frac{\beta_{th}}{\lambda} \left(T_i - \langle T \rangle_i \right) \tag{33}$$

where β_{th} and λ are the dimensionless thermal slip coefficient and a pore-level length-scale, respectively, and the symbols T_i and $\langle T \rangle_i$ denote the temperature of the solid (rigid) wall and the porous media, at the interface, respectively.

The thermal slip coefficient depends on the porous media structure and the thermal conductivities of both the solid and fluid phases constituting the porous media, and on the thermal conductivity of the solid wall [33].

For a porous/fluid and a porous/porous interfaces the energy equations on both sides are matched by assuming that the temperature and heat fluxes are

continuous across the interface [7]. These assumptions can be mathematically written as

$$\langle T \rangle = T_f \tag{34}$$

$$k_m \partial \langle T \rangle / \partial x = k_f \partial T_f / \partial x \tag{35}$$

where $\langle T \rangle$, k_m and T_f, k_f are the temperature and the thermal conductivity on the porous media side and the fluid side, respectively.

Summing the above discussion on the interfacial boundary conditions it can be said that:

In order for the heat transfer rate not to be overpredicted, higher order flow models (Brinkman and DBF flow models) have to be used to eliminate the slip boundary condition inherited in the Darcy and/or Forchheimer flow models, particularly at higher flow velocities, in the case of a porous/impermeable boundary.

For the porous/porous and/or porous/fluid interface, the slip boundary conditions inherited by using the Darcy and Forchheimer flow models do not permit any kinematic boundary condition to be imposed and only the pressure boundary conditions can be employed to match the governing equations at both sides. To account for the no-slip effect, one alternative is to use the B–J boundary condition. The B–J boundary condition have received certain amount of theoretical foundation and several subsequent experimental investigations have provided further support [32]. However, the B–J boundary condition have been verified for a parallel flow over a porous media, it can also be used for other porous media interfacial configurations [6]. Using the B–J boundary condition, the deficiency inherited by using the Darcy or Forchheimer flow models for these type of interfacial boundary conditions can be resolved. Its main limitation is that it is suitable for one dimensional parallel flow (i.e. Poiseuille or Couette flows). This is because experiments with oblique (2-dimensional) flows are very difficult to handle and yet have not been studied as those of the parallel flows [32]. To account for the 2-dimensional effect the vertical component of the velocity vector with respect to the interface at both sides may be assumed equal which is consistent with eq. (29) [6]. Moreover, a vertical velocity gradient term ($\partial v_f / \partial x$) needs to be added to the left-hand side of eq. (28) and a boundary condition which accounts for matching the normal stress at both sides with analogous to eq. (31) needs also to be employed [6]. However, the effect of this extension is small and therefore can be neglected [32].

In the case of the higher order flow models the usual practice is that the velocity components at both sides are assumed equal. This is physically justified if a mass balance across the interface is performed and the conservation of shear and normal stresses justifies, the eq. (31) and eq. (32), respectively, since both sides are mechanically in equilibrium with each other.

The temperature slip boundary condition enables constant effective properties to be used for the porous media which ease the modeling of the porous system and thereby its solution. Therefore, the increase of the heat flux near the impermeable wall can be accounted for. For a porous/porous and/or porous/fluid interface the constant temperature assumption across the interface is physically justified if the resistance to heat flow across the interface is neglected. The constant heat flux is also physically justified if a heat balance around the interface is performed.

9. IMPORTANT ASPECTS OF ENGINEERING APPLICATIONS OF POROUS MEDIA

Porous medium is characterized by a very large surface area to a volume ratio. This peculiar feature of the porous media can be utilized to either distribute heat energy uniformly or to enhance the heat transfer in heat exchange systems. This characteristic feature of the porous media and the engineering importance of the macroscopic approach of the porous media will be discussed in detail in the following section.

9.1. ENHANCEMENT OF HEAT TRANSFER

When a porous material is inserted in a coolant passage or a porous substrate is attached to a hot surface (Fig. 12), a uniform temperature at the hot surface

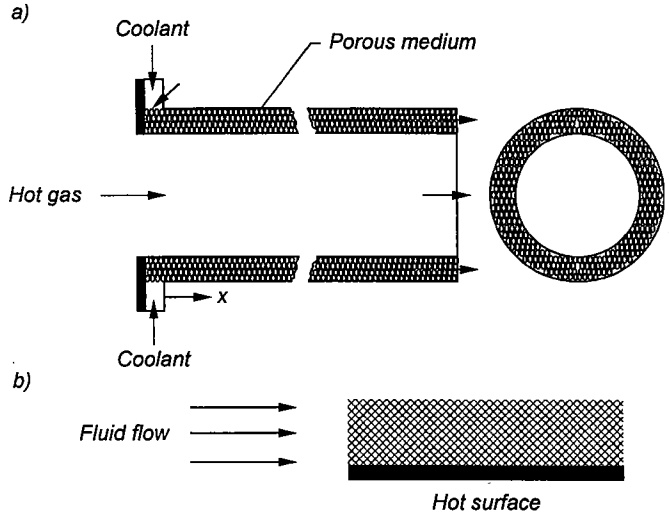

Fig. 12. Enhancement of heat transfer: a) porous channel, b) porous substrate

will be established. Furthermore, if the fluid trapped in the porous media is allowed to circulate or discharged to the environment, then a significant increase in the heat transfer rate will occur and consequently the surface temperature will be reduced [22]. This arrangement may thus be considered as a super heat exchanger. It is shown in Fig. 13 that for approximately the same wall heat flux a larger amount of coolant mass flux is needed in order to achieve a considerable decrease in the wall temperature in the case without porous material than when the porous material is applied. Whereas a considerable decrease in the wall temperature and the coolant mass flux is achieved for approximately the same wall heat flux, due to the presence of the porous media coolant passage.

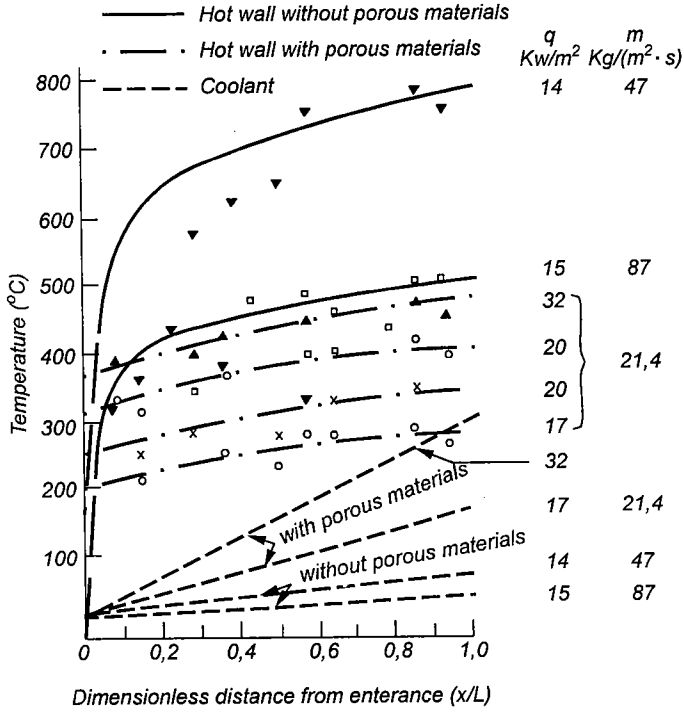

Fig. 13. Effects of porous materials on wall temperature and coolant mass flux in a coolant passage [22]

The cooling effectiveness can be further enhanced by the elimination of the interface resistance between the wall and the porous material and by the use of high conductivity porous material and high conductivity wall material. In general a porous media can be designed so that they either retard (acting as an insulation) or enhance the heat transfer.

The other alternative to enhance the heat transfer is transpiration [35], which is achieved by making the metal porous and forcing a coolant through it from

a reservoir toward the boundary exposed to the high temperature (Fig. 14). The transpiring gas greatly increases the thickness of the thermal boundary layer and reduces the temperature gradient at the surface. The reduction of the heat flux to the surface results in a corresponding enthalpy increase of the transpiring gas in the boundary layer. Possible applications embrace cooling of rocket nozzles, arc electrodes, reentry bodies and turbine blades [35].

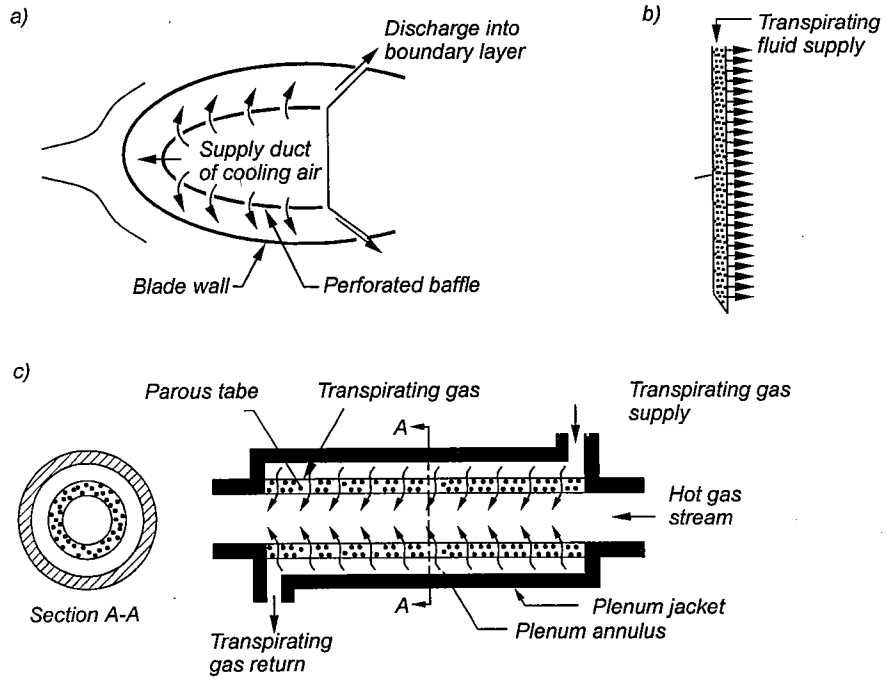

Fig. 14. Possible physical models for a transpiration cooling, e.g. (a) cooling of turbine blades

For phase change heat transfer, a very small wall overheating can start nucleate boiling on a porous surfaces. Bubble generation raises the heat flux to a higher order of magnitude in a range of small wall superheats, where natural convection is normally the sole mode of heat transfer. Thereby, the porous medium raises the heat transfer coefficient of order of magnitude higher than that of convective heat transfer for the corresponding free-fluid alone. This makes the porous media very attractive to high efficiency heat exchangers [29].

9.2. ENERGY CONVERSION SYSTEMS

The use of porous media, particularly packed and fluidized beds, are very attractive as conversion and storage systems. This is because of their large

surface area which provides a very compact structure and therefore greatly enhances the heat storage capacity as compared to the systems that utilize energy transporting fluid which alone acts as the storage medium [41]. Energy conversion and storage in porous media include the use of:

1. Sensible heat storage systems.
2. Phase change material (PCM): The principle advantage of the PCM in packed beds is that the storage density of the bed is increased significantly. Therefore, the size and mass of storage system required for a particular application are reduced proportionally [41].
3. Solid-to-solid phase transition (STSPT) materials: Plastic crystals are such materials which exhibit mesocrystalline phases and high transitional enthalpy during solid-to-solid phase transition, between the crystalline and plastic crystalline states [28]. These materials reversibly absorb large amount of energy during solid state phase transformation at specific temperatures below their melting temperature. Therefore, they can be used for energy storage without being confined.
4. Thermochemical energy conversion: This method utilizes a material which undergoes a reversible chemical reaction where the thermal energy can be stored in the form of reaction enthalpy in endothermal reactions (forward reaction) to yield products. The input thermal energy can be recovered with the exothermic reaction [12]. This method features a very high energy density and storage of thermal energy can be made possible at room temperature and therefore, there is no need for thermal insulation. The method appears to be promising, especially for large term storage and transport of thermal energy. Among the different methods of thermo-chemical energy conversion are: the production of hydrogen from water and the decomposition of inorganic substances [12, 21].

Porous media can be also used for direct conversion of chemical energy to low (infra-red) radiation. Porous radiant burners (PRBs) (Fig. 15) can serve as an example of such an application, they have many advantages over the conventional burners.

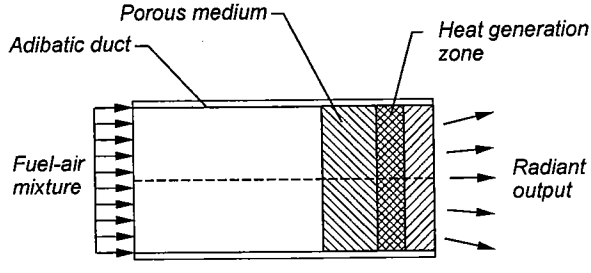

Fig. 15. Schematic diagram of a porous radiant burner [38]

These are [38]:

a) high thermal efficiencies, due to reduced enthalpy loss through the fuel gas, which therefore reduce the energy consumption;
b) elimination of hot spots in heat transfer devices, because of more uniform heating, which thereby increases the life time of the heat transfer equipment;
c) lower emission of toxic nitrogen oxides, as a result of lower flame temperature, which therefore improves the quality of environmental air.

9.3. MODELING OF CLASSICAL PROBLEMS BY THE POROUS MEDIA APPROACH

The macroscopic approach, which is utilized to study problems of transport phenomena in porous media, was applied to analyze other complex systems. This might ease the difficulty of mathematical modeling and gives a guide for better understanding of the system which may result in improvement of the system performance and therefore in energy and material saving. This class of problem formulation could contribute to the energy transport in systems, such as: heat exchanger, air flow around building, pre-cooling of foodstuffs, transport of chemical spices in biological bodies and the modeling of the phase change problems (e.g. solidification of alloys). For example, Sinivasan et al. [36] have modeled flow and heat transfer through a spirally fluted tubes using the porous media approach. The model divided the flow domain into two regions. The flutes were modeled as a porous substrate with direction-dependent permeability, excellent agreement was reported for the numerical and experimental results.

10. CONCLUSIONS

This critical review of modeling momentum and energy transport in porous media allows the following conclusions to be drawn:

Different momentum equations are used to model the fluid flow in porous media; each of these models has its own deficiency. The popular Darcy flow model is the simplest flow model, but it is restricted to either low porosity or to a slug flow. The other flow models which include higher order terms accounting for inertia and viscous effects should be used to model the fluid flow for high porosity and/or high velocity flow. The criterion for their use should however be clearly stated.

Modeling the thermal behavior of the porous media depends on whether the homogeneous or the heterogeneous formulation is to be used. When the heterogeneous formulation is considered, Schumann model provides the simplest thermal energy model since in this model the heat conduction is neglected in both phases. Accounting for the heat conduction in the phases requires the more complicated (C–S) and (C–D) models to be used. The other alternative is the homogeneous formulation, but it needs the local thermal equilibrium assumption to be verified. In both formulations the thermal dispersion effect should be included, but still very little is known about this phenomenon.

Porosity variation has to be considered in order for the channeling effect, occurring at the wall, to be accounted for so that the velocity and temperature profiles will not be affected. This can be done by choosing an appropriate porosity model. Porosity variation also effects the porous media properties, e.g. the Brinkman effective viscosity, which needs to be modeled with this damping oscillatory in porosity, and also contributes to the variation of the effective thermal conductivity which results in a temperature slip at the bounding wall.

Although the porous media exhibit some characteristics of that of the corresponding free-fluid flow, its behavior is totally different, particularly for a porous/porous interface and porous/fluid interface. This behavior brings another difficulty in modeling interfacial boundary conditions. As the porosity of the porous media approaches unity its behavior should resemble that of the fluid. However, as the porosity approaches zero, its behavior should approach that of the solid. But actually this is not the case, since the porous media are proven to enhance the heat transfer and they are also found to retard the heat transfer (insulation) in some cases. There are also complex systems which resemble the characteristics features of the porous media, e.g. the mushy zone during the solidification of alloys, pre-cooling of food products and heat exchangers. These systems can be satisfactorily modeled by the macroscopic approach of the porous media.

The wide range of porous media applications, particularly for the enhancement of heat transfer, makes the porous media a kind of a super-material which promotes the idea of the super compact heat exchangers. Other modes of heat transfer, e.g. phase change and radiative heat transfer, are needed to be studied in order for the behavior of the porous media to be fully understood.

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PROBLEMY MODELOWANIA PRZEPŁYWU PŁYNU I WYMIANY CIEPŁA W OŚRODKACH POROWATYCH

W pracy przedstawiono przegląd problemów teoretycznych i inżynierskich zastosowań ośrodków porowatych. W szczególności rozpatrzono te zjawiska towarzyszące przepływowi płynu i wymianie ciepła w ośrodkach porowatych, które nie występują w przepływie samego płynu. Zjawiska te związane są z występowaniem członów wyższego rzędu w równaniu pędu oraz ze zjawiskiem dyspersji termicznej i lokalnej równowagi termicznej w równaniu energii. Rozpatrzono ogólną klasę warunków brzegowych oraz zjawisko kanałowania spowodowane niejednorodnością porowatości w pobliżu brzegu ośrodka. Opisano zasadnicze inżynierskie zastosowania ośrodków porowatych wiążące się z wymianą ciepła.