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UNIVERSAL CORRELATIONS FOR PREDICTING COMPLETE PUMP PERFORMANCE CHARACTERISTICS

The aim of this paper is to introduce a systematic approach for the prediction of pump performance characteristics of different specific speeds when the experimental data are not available. To accomplish this task, a set of equations representing the best fits of the available experimental data is developed. The equations provide a data source useful in the design, simulation and checking of the pumps. The correlations obtained should avoid carrying out complex experimental programs in a test loop. Numerical results indicated that the proposed method favorably predicted the nuclear-grade coolant pump performance at minimal cost. Application of the method has demonstrated its viability as a tool of obtaining pump data useful to light water reactor safety analysis codes.

INTRODUCTION

Pumps play a major role in the design and operation of nuclear power plants and other industrial facilities. Their operating characteristics are believed to play a significant role in determining the thermal and hydraulic behavior of the nuclear reactor following transients [7]. The renewed interest in nuclear power plant safety – after Chernobyl accident – and related transients caused by equipment failure have created a need adequate models and appropriate simulation based on accurate data.

To calculate precisely the transient response of a reactor following a pump coastdown, it is necessary to identify the key parameters governing the operation of the pump. The only problem mentioned by many code developers and users in this area is the lack of characteristics data of the modeled pump [6, 17, 12, 2]. Therefore, in the context of light water reactor analysis the need arises to develop a reliably accurate method to generate the performance characteristic of any pump at minimal cost [2].

In spite of the long history of pump development, the empirical formulae remain the only valuable asset to be applied quite adequately in the industry.
Based on this fact, the main problem solved in this paper is to find a suitable set of correlations covering a wide range of performance characteristics of any pump. The development of the correlations is based on the experimental data available and involves no major assumptions other than the law of dynamic similarity [3, 4].

1. PUMP PERFORMANCE REPRESENTATIONS

In the various safety analysis codes, the pump is described by a separate model that accounts for the interaction of the system fluid with a centrifugal pump. The model calculates the pump head and the rotational speed based on the pump performance characteristic curves. These curves should be constructed by the pump manufacturers on the basis of performance tests in a test loop.

The similarity in performance characteristics between two pumps is termed "homologous" and results from two pumps having similar specific speed [9, 13]. In its basic form, the specific speed of a pump is an index that is numerically equal to the rotative speed at which a geometrically similar prototype of that pump would operate in order to deliver one unit of capacity against one unit of head.

\[ n_q = \frac{N \sqrt{Q}}{H^{0.75}} \]

where,

- \( n_q \) — specific speed,
- \( N \) — rotative speed (rpm),
- \( Q \) — volumetric flow rate (m\(^3\)/s),
- \( H \) — pump head (m).

To obtain the transient results with sufficient accuracy it is necessary to have the complete characteristics of a pump of the same specific speed as that being under study. In most cases of practical interest, this is not available, and the question has often been raised about the error introduced in transient studies by the use of improper pump performance characteristic [1].

Although complete pump performance characteristics are difficult to obtain, those ones were usually described by the classical four quadrant curves or the homologous curves. The complete four-quadrant curves representation is extremely inconvenient for digital computer calculations, since it requires large tables and complicated approximations to simulate transient systems [3, 5, 13].
The homologous curves representation is adopted by current codes such as RELAP5[12], TRAC[17]... etc, because of the relatively simple method of tabulating the input data. The usual homologous curves are obtained by the following dimensionless relationships [9, 4, 12].

$$\frac{h}{\alpha^2} \text{ versus } \frac{v}{\alpha}, \text{ or } \frac{h}{v^2} \text{ versus } \frac{\alpha}{v}$$

$$\frac{\beta}{\alpha^2} \text{ versus } \frac{v}{\alpha}, \text{ or } \frac{\beta}{v^2} \text{ versus } \frac{\alpha}{v}$$

in which

$$h = \frac{H}{H_R}, \quad v = \frac{Q}{Q_R}, \quad \alpha = \frac{N}{N_R}, \quad \beta = \frac{T}{T_R} \quad (2)$$

where:

- $H$ – total dynamic head produced by the pump,
- $Q$ – volumetric flow,
- $N$ – rotational speed,
- $T$ – hydraulic torque,

$h$, $\alpha$, $v$, and $\beta$ are dimensionless quantities of head, rotational speed, volumetric flow and hydraulic torque respectively.

The subscript $R$ refers to the rated value of the variable parameter.

Although this method is relatively simpler and more convenient for computer calculations, it is complicated by the fact that $h$, $\alpha$, $v$, and $\beta$ could all change sign, moving through zero and thus create solution and stability problems. To overcome these difficulties, a third representation of pump performance characteristics is introduced.

Based on the fact that $v$ is proportional to $\alpha$, and that all possible kinds of machine operation lies in the limited interval $0 \leq x \leq 2\pi$ [3, 4, 5], a set of dependent variables are worked out, refined [14] and presented in the following forms:

$$WH(x) = \frac{h}{\alpha^2} \frac{1}{1 + \frac{v^2}{\alpha^2}} \text{ or } \frac{h}{v^2} \frac{1}{1 + \frac{\alpha^2}{v^2}} = f_1(x) \quad (3)$$

$$WT(x) = \frac{\beta}{\alpha^2} \frac{1}{1 + \frac{v^2}{\alpha^2}} \text{ or } \frac{\beta}{v^2} \frac{1}{1 + \frac{\alpha^2}{\mu^2}} = f_2(x) \quad (4)$$
where:

\[ WH(x) \] — head function,
\[ WT(x) \] — torque function.

The single independent variable \( x \) is the angle that depends on the dimensionless speed (\( \alpha \)) and dimensionless volumetric flow (\( v \)) given as follows:

\[
x = \pi + \tan^{-1} \frac{v}{\alpha}
\] (5)

The values of \( \tan^{-1} \frac{v}{\alpha} \) is between \(-\pi\) and \(\pi\), for this cause \(\pi\) is added to define the range of \( x \) between 0 and \(2\pi\).

For the sake of better accuracy for small values, the functions \( WH(x) \) and \( WT(x) \) are read off for all values of \( x \) from 0 to \(2\pi\). In this manner, the complete characteristics through all zones of pump operation can be obtained, with the normal pump zone given by \( \pi \leq x \leq \frac{3\pi}{2} \).

The functions \( WH(x) \) and \( WT(x) \) may be readily derived from conventional homologous curves or from the characteristics data (or curves) furnished by the pump manufacturers.

Based on the literature data, the head functions \( WH(x) \) for four pumps of different specific speeds \( (n_q = 18, 35, 147, 262) \) are given in figure (1). The corresponding torque functions \( WT(x) \) for the three pumps \( (n_q = 35, 147, 262) \) are given by figure (2).

![Fig. 1. Function \( WH(x) \) for four pumps of specific speeds \( n_q = 18, 35, 147, 262 \)](image-url)
The advantages of this representation are:
• Small storage over a limited interval and simpler algorithm for computer calculations.
• The dimensionless quantities $\alpha$ and $\nu$ near zero cause no solution problem, as finite, unique $WH$ and $WT$ are defined.
• The possibility and simplicity of representing and comparing more than one pump on a one plot as in figures (1) and (2).

2. PUMP UNIVERSAL CURVES

Results of the figures (1) and (2) provide, if not a direct indication of how all pumps behave, at least an idea of how to develop universal curves to predict any pump characteristics.

The assumption is made, that if the manufacturers do not furnish homologous or pump data, curves and correlations based on available data can be developed.

Literature [17, 12, 16, 10, 11, 9, 14, 2, 13] presents experimental data for the pumps of specific speeds ($n_q$) = 18, 28, 35, 58, 100, 147, 262, and to a limited extent for the pumps of specific speeds ($n_q$) = 20, 30, 40, 50, 80, 110, 250. These experimental data are worked out and reduced to third representation functions $WH(x)$ and $WT(x)$. These functions are plotted versus specific
speed as an independent variable for each value of $x$. The procedure is repeated for all values of $x$ until the universal curves representing the pump operation zones of interest are obtained [4].

Fig. 3. Universal $WH$ curves

Fig. 4. Universal $WT$ curves
Figures 3 and 4 show the universal curves representing the characteristic data ranging from the shut-off head (and torque) to the point where the pump speed is zero. In a similar manner, the curves can be extended to the other zones of pump operation.

3. PUMP UNIVERSAL CORRELATIONS

Most of the well known reactor safety analysis codes such as RELAP[12], TRAC[17]... etc. used only one or two built-in pump performance homologous curves data of strictly determined specific speed. These data are usually that of the LOFT (Loss Of Fluid Test facility) \((n_q = 35)\), or the semiscale pump \((n_q = 18)\) [12]. For full-scale pressurized water reactor pump (e.g. \(n_q > 100\)) transient analyses, there are several restrictions and limitations in the use of these curves instead of that of the proper pump. Therefore, in the absence of detailed experimental information, simple universal correlations are obtained by best fitting the universal pump curves covering a wide range of specific speeds as given in figures 3 and 4.

Some useful correlations to find the functions \(WH(n_q)\) for some values of \(x\) in the pump zone \(\pi \leq x \leq \frac{3}{2}\pi\) (at an interval of \(\frac{1}{44}\pi\)) are given below in table 1.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(WH(n_q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi)</td>
<td>(1.09458 + 0.00881416n_q - 9.56E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{45}{44}\pi)</td>
<td>(1.08551 + 0.00771208n_q - 8.2443E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{46}{44}\pi)</td>
<td>(1.08127 + 0.0062589n_q - 5.3857E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{47}{44}\pi)</td>
<td>(1.04857 + 0.00488787n_q - 2.65355E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{48}{44}\pi)</td>
<td>(0.994794 + 0.00375009n_q - 1.00626E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{49}{44}\pi)</td>
<td>(0.95079 + 0.00213921n_q + 2.79052E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{50}{44}\pi)</td>
<td>(0.904765 + 0.00113164n_q + 4.18123E(-06)n_q^2)</td>
</tr>
<tr>
<td>(\frac{51}{44}\pi)</td>
<td>(0.850766 + 0.000715628n_q + 3.4325E(-06)n_q^2)</td>
</tr>
</tbody>
</table>
In a similar manner, the universal correlations for the functions $WT(x)$ are obtained.

The experience in using these correlations has showed their adequacy for most applications. They increase the flexibility of the nuclear reactor safety analysis codes and permit a substantial reduction in the time and labor that is generally expended in the determination of pump characteristic data. These correlations are developed and adopted by the authors for generating pump characteristic data used in pressurized water reactor pump transients code (TRAPU) [5].

<table>
<thead>
<tr>
<th>52</th>
<th>0.745394 + 0.00120084$n_q$ - 7.62472E(-07)$n_q^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>53</td>
<td>0.62403 + 0.0017068$n_q$ - 2.19598E(-06)$n_q^2$</td>
</tr>
<tr>
<td>54</td>
<td>0.549354 + 0.00115668$n_q$ - 1.99221E(-06)$n_q^2$</td>
</tr>
<tr>
<td>55</td>
<td>0.5</td>
</tr>
<tr>
<td>56</td>
<td>0.451514 - 0.00061576$n_q$</td>
</tr>
<tr>
<td>57</td>
<td>0.363014 - 0.00100$n_q$</td>
</tr>
<tr>
<td>58</td>
<td>0.259533 - 0.0021052$n_q$ + 7.11626E(-06)$n_q^2$ - 1.74007E(-08)$n_q^3$</td>
</tr>
<tr>
<td>59</td>
<td>0.213805 - 0.00254378$n_q$ + 5.87277E(-07)$n_q^2$ + 1.04944E(-08)$n_q^3$</td>
</tr>
<tr>
<td>60</td>
<td>0.13752 - 0.00378183$n_q$ + 2.6036E(-06)$n_q^2$ + 9.21642E(-09)$n_q^3$</td>
</tr>
<tr>
<td>61</td>
<td>0.121829 - 0.0063026$n_q$ + 1.23903E(-05)$n_q^2$</td>
</tr>
<tr>
<td>62</td>
<td>0.0200282 - 0.00646691$n_q$ - 2.84207E(-07)$n_q^2$ + 4.99101E(-08)$n_q^3$</td>
</tr>
<tr>
<td>63</td>
<td>-0.19752 + 0.00221572$n_q$ - 0.000175575$n_q^2$ + 1.16208E(-06)$n_q^3$ - 2.14434E(-09)$n_q^4$</td>
</tr>
<tr>
<td>64</td>
<td>0.0129996 - 0.0142948$n_q$ + 3.81799E(-05)$n_q^2$ + 1.09954E(-08)$n_q^3$</td>
</tr>
<tr>
<td>65</td>
<td>0.0541045 - 0.0183515$n_q$ + 5.78271E(-05)$n_q^2$ - 8.71503E(-09)$n_q^3$</td>
</tr>
<tr>
<td>66</td>
<td>0.270364 - 0.0345706$n_q$ + 0.000200409$n_q^2$ - 3.29514E(-07)$n_q^3$</td>
</tr>
</tbody>
</table>
The error involved in these correlations is of an order of magnitude that may be expected and accepted in hydraulic calculations.

CONCLUSIONS

Nuclear power plant design and simulation of anticipated transients caused by pump malfunction would require a reliable source of performance data of the pump involved in the process. With this powerful motivation universal curves and correlations to obtain any pump performance characteristics have been developed without restrictive assumptions. These correlations are based on the experimental data available. Their distinctive features and predictive capabilities as well as their importance in nuclear thermal-hydraulic codes and to pump manufacturers have been explained. The developed correlations provide a data source to check pump experimental test results and to predict any pump performance at minimal cost.

REFERENCE


